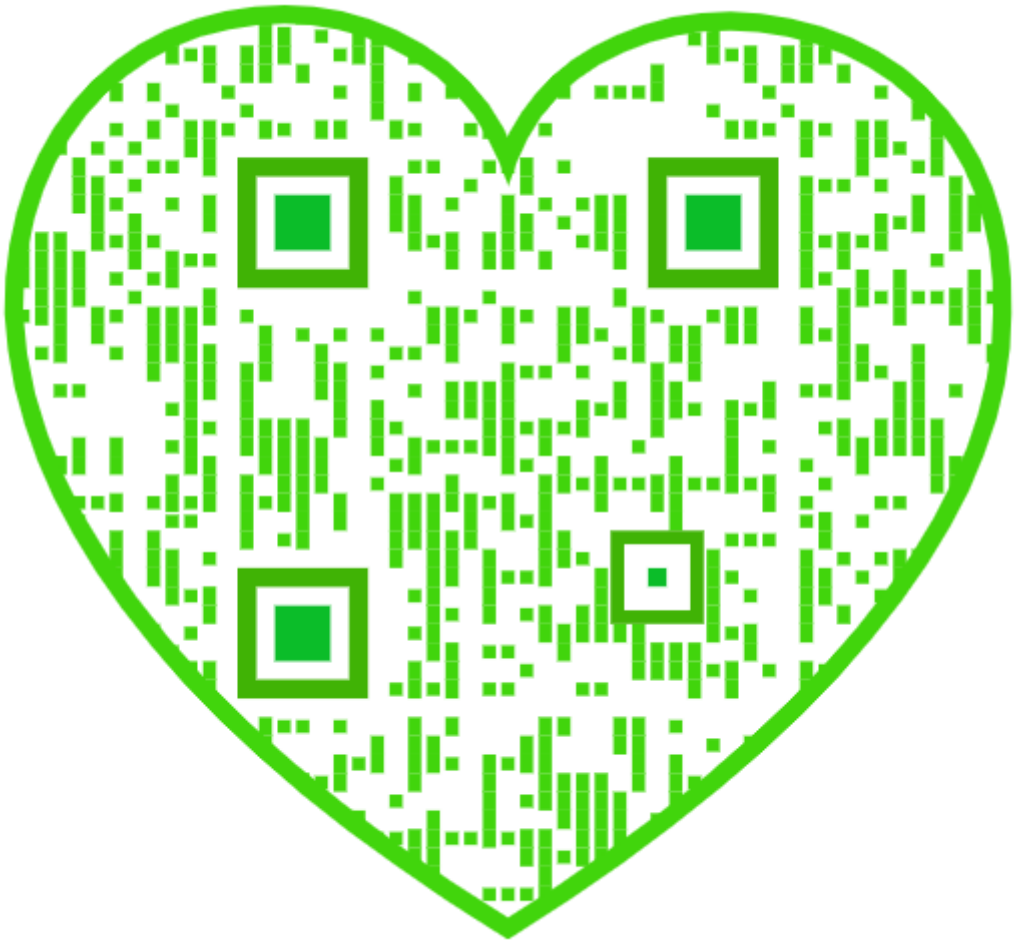

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THE

STEAM ENGINE

BY

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PREFACE.

THE progress of technical education in this country during the last few years has rendered necessary the production of an elementary text book on the Steam Engine, containing information upon branches of the subject which have hitherto received but scant notice in works of this nature. I have endeavoured, as far as the limits of space in this small volume permitted, to make good these deficiencies, which were for the most part brought under my notice by engineering students.

There are four important points in which I venture to hope this book will be found to contain information, put in a form suitable for beginners, which has hitherto only been accessible in works of a more advanced character or in those which only profess to treat special branches of the subject.

They are as follows :—

1. The modern science of thermodynamics, which is the foundation of all knowledge of the steam engine considered as an apparatus for converting heat into mechanical work.

2. The very important effects exercised on the motion of quick running engines by the inertia of their reciprocating parts.

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3. The geometrical methods of fixing the dimensions and the setting of slide valves.

4. The investigation of the methods in use for diminishing the losses of efficiency in expansive engines, due to the cooling of the cylinders by the expanding steam, the principal of which methods are, superheating, steam jacketing, and compounding.

The space required for even an elementary treatment of the above subjects could not be gained without a certain sacrifice, and after full consideration I came to the conclusion to sacrifice altogether the historical part of the subject, partly because there are already in existence many elementary works full of historical information, and partly because I doubted whether a history of the steam engine has any legitimate place in a text book for students. I have endeavoured throughout this work to make the descriptions as simple as possible, and their sequence as systematic as the nature of the work allowed. I believe that fully one half of the difficulties experienced by students in mastering new subjects is due to the want of system which characterises too much of our older technical literature. It is the rule rather than the exception in many books to present to the student ready-made formulæ without indicating the steps by which they are reached. I have carefully avoided this source of difficulty to beginners, for I conceive it to be the duty of all who attempt to teach even the most elementary subjects to husband the powers of their readers by saving them all unnecessary trouble.

I cannot claim anything original in the book, but I do claim that I have endeavoured to render the information which it contains very easy to understand, so that it can be

followed from first to last by any student who possesses a slight acquaintance with elementary mathematics. Wherever it has been advantageous to do so I have used geometrical instead of analytical methods of demonstration. I have not assumed the slightest acquaintance on the part of the reader with the sciences of heat or of motion, and have consequently devoted many pages to the explanation of such parts of these sciences as are necessary for the proper understanding of the working of engines. In this I have followed the precedent set in many excellent works included in this series.

If I were to acknowledge in detail all the sources of information from which I have freely drawn, I fear this Preface would run to an inordinate length ; but I cannot forbear to express my deep obligations to my old friend and private tutor at Cambridge, Professor James Stuart, M.P., who has kindly revised the proofs of the entire work, and to the Editors and Proprietors of 'Engineering' and the 'Engineer,' who have allowed me free use of many of the illustrations and of the inexhaustible stores of information which have appeared in their journals. Students of thermodynamics would be in a bad way without the writings of the late Professor Rankine, F.R.S., and of Professor Cotteril, F.R.S., and I have availed myself freely of the information contained in their invaluable books. I have also found much that was valuable in the published papers of the Institutions of Civil Engineers, Mechanical Engineers and Naval Architects, and am greatly indebted to the Councils of these Societies for permission to make use of many drawings which are reproduced in these pages. Among the other authors whom I have consulted, I may mention Mr. Arthur

Rigg, whose very ingenious system of circular diagrams of twisting moments on crank-shafts I have adopted in Chapter V.; Professor Zeuner, whose invaluable system of valve diagrams is explained in Chapter VII.; Mr. Cowling Welch, Mr. Porter, Professors Galbraith and Haughton, Clerk Maxwell and Cawthorn Unwin, Mr. A. E. Seaton, and lastly Mr. R. Sennett, to whom I am indebted for several illustrations and for much valuable information on the subject of the distribution of the steam in Compound Engines.

GEORGE C. V. HOLMES.

5 ADELPHI TERRACE:
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THE STEAM ENGINE.

CHAPTER I.

INTRODUCTION.

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THE complete study of the steam engine is, in its nature, somewhat complex, involving as it does an acquaintance with the sciences of heat, of chemistry, and of pure and applied mechanics, as well as a knowledge of the theory of mechanism and the strength of materials. It is proposed, therefore, to begin this work by showing, in a very simple case, how steam can be used to do work, and then to proceed to describe an actual steam engine of the most modern construction, but one which at the same time is remarkably free from complexity. When studying this description, the student will soon find out how it is that the perfect knowledge of the steam engine involves an acquaintance with so many branches of science; and the order in which these subjects must be studied, so far as they bear on the matter in hand, will naturally be suggested by the description.

Take a hollow cylinder (fig. 1) of indefinite height, the bottom of which is closed while the top remains open, and fill this cylinder to the height of a few inches with water.

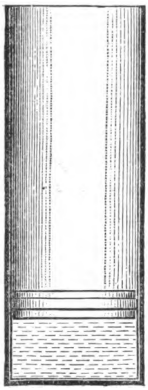

Fig. 1.

Next cover in the water by means of a flat plate, or piston, which fits perfectly the interior of the cylinder, and then apply heat to the water; we shall witness the following phenomena. After the lapse of some minutes the water will begin to boil, and steam will accumulate at its upper surface between it and the piston, which latter will be raised slightly in order to make room for the steam. As the boiling process continues, more and more steam will be formed, and the piston will be raised higher and higher, till the whole of the water is boiled away, and nothing but steam is contained in the cylinder. Now this apparatus, consisting of cylinder, piston, water, and fire, is an elementary form of steam engine of the simplest kind. For a steam engine may be defined as an apparatus for doing work by means of heat applied to water; and it is manifest that the appliance just described, inconvenient and clumsy though it may be, perfectly answers to the definition; for the piston is a weight, and this weight has been raised to a certain height by the formation of steam from the water. Now the raising of a weight through a height is a particular form of doing work, and consequently this combination is an apparatus capable of doing work by means of heat applied to water.

If, instead of a simple piston, we had taken one loaded with weights, and applied heat as before, the result would have been similar but not precisely the same. The water would not have begun to boil so soon, and when it was all boiled away the loaded piston would not have risen to the same height as did the simple one. The reason of this will be amply explained in the chapters on heat. Supposing

that, having raised the weight to the utmost height it would go, we then removed it from the piston, and wished to employ the apparatus in order to raise a similar weight to the same height, we should have to bring back the steam to its original condition of water. This we could do by removing the fire and by surrounding the cylinder instead with cold water. The result would be that the steam would all condense into water, and fall back to its original place, the piston following it, and everything would be ready for a fresh start. Now, though this apparatus answers the definition of a steam engine, it is, nevertheless, a very bad one, for the following reasons. The only kind of work it can do is the raising of weights through certain heights. When we want to repeat the operation we have to remove the fire and surround the cylinder with cold water, and then replace the fire, which is a most cumbrous process. While condensing the steam we made the cylinder cold, and a large quantity of heat is wasted in warming it again. Moreover, when, at the cost of a considerable amount of fuel, we have heated the water and turned it into steam, we allow the whole of the heat in the steam to escape into the cold water, and thus become wasted, though it is capable of doing much more work if properly used. Thus we see that our elementary engine is limited in its scope, clumsy in use, and extremely wasteful of fuel. It is in obviating these disadvantages that actual engines differ from the one we have described.

It will have been observed that this engine consists of four principal elements, viz.: the fire, or source of heat; the water, or medium to which the heat is applied, and by the conversion of which into steam the work is done; the cylinder with movable piston, which contains the water and steam, and which prevents the latter from escaping into the air when formed and becoming lost; and, lastly, the source of cold, or the water by means of which the steam was condensed and brought back to its original condition. The great majority of actual engines consist of precisely the same

elements, more advantageously arranged, with the addition of certain mechanism for changing the straight line movement of the piston into circular, or any other kind of motion. This mechanism has also to effect other subsidiary objects which will be fully described hereafter. It should also here be mentioned that if, instead of condensing the steam by means of cold water, we had opened a temporary communication between the steam space inside the cylinder and the open air, we should have equally well succeeded in bringing the piston back to its original position, when, by introducing into the cylinder a fresh quantity of water, we could have again raised the weights.

In practice the arrangement adopted is as follows :—

1. The source of heat, and the vessel containing the water to be boiled, are kept quite separate and distinct from the cylinder. These parts of the apparatus are called respectively the furnace and boiler. The steam is supplied from the boiler, where it is generated, to the cylinder where it is used, as it is wanted, by means of a pipe, called the steam pipe.

2. The steam, after doing its work in the cylinder, is led away through a second pipe, called the exhaust pipe, into the open air, or else to be condensed in a separate vessel kept quite apart from the cylinder, and which is called the condenser.

3. The cylinder, instead of being open at one end, and of indefinite length, is closed at both ends, and in length seldom exceeds twice the diameter of the piston.

4. The steam, instead of being used only on one side of the piston, is admitted alternately to and exhausted from each side in succession, so that when the engine is in use, the piston is constantly travelling backwards and forwards from one end to the other of the cylinder.

5. Suitable openings are made at each end of the cylinder, to allow the steam alternately to enter and escape, and valves driven by suitable mechanism are provided in

order to ensure that the admission and escape of the steam shall take place at the proper moments.

6. Instead of placing the weights to be lifted directly upon the piston, a cylindrical bar or rod called the piston rod is attached firmly to the centre of the piston, and is continued through one end of the cylinder to the open air, so that the outside end of the rod moves backwards and forwards in a straight line, exactly as the piston does. By means of suitable mechanism, which will be fully described hereafter, this straight line motion of the piston rod end is changed into rotatory or circular motion, so that the engine can be used, not only for lifting weights up in a vertical direction, but for doing any kind of work which may be required of it.

The manner in which all this may be accomplished in practice will be shown in the following description and drawings of an engine and boiler, which are here selected for description on account of their simplicity of construction. We will commence with the source of heat, and apparatus for turning the water into steam ; then go on to the engine proper, i.e. the cylinder with the mechanism belonging to it. The abstracter of heat, or condenser, will be considered in a separate chapter. Fig. 2 is an elevation of the boiler, fig. 3 a vertical section through its axis, and fig. 4 a horizontal section through the furnace bars.

The type of steam generator here exhibited is what is known as a vertical tubular boiler. The outside casing or shell is cylindrical in shape, and is composed of wrought iron or steel plates riveted together as shown in fig. 2. The top, which is likewise composed of the same material, is slightly dome-shaped, except at the centre, which is cut away in order to receive the chimney, *a*, which is cylindrical in shape and formed of thin wrought-iron plates. The interior is shown in vertical section in fig. 3. It consists of a furnace chamber, *b*, which contains the fire. The furnace is formed like the shell of the boiler of wrought iron or steel

plates in the form of a cylinder, the top of which is covered by a flat circular plate, *cc*, firmly attached to the cylindrical

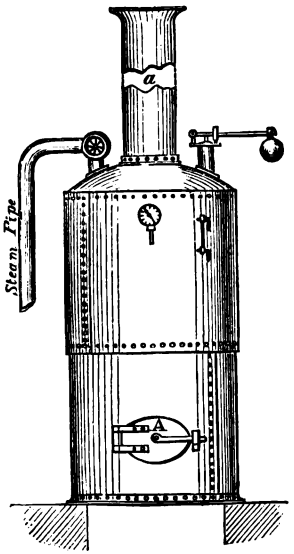

Fig. 2.

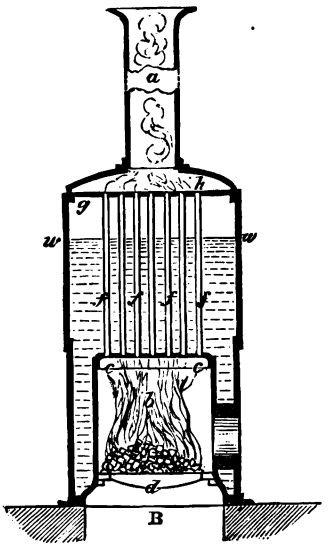

Fig. 3.

portion by flanging and riveting. The bottom is occupied by the grating, on which rests the incandescent fuel. The grating consists of a number of cast-iron bars, *d* (fig. 3), and shown in plan in fig. 4, placed so as to have interstices between them like the grate of an ordinary fireplace. The bottom of the furnace is firmly secured to the outside shell of the boiler in the manner shown in fig. 3. The top covering plate, *cc*, is perforated with a number of circular holes of from one and a half to three inches diameter, according to the size of the boiler. Into each of these holes is fixed a vertical tube made of

Fig. 4.

brass, wrought iron, or steel, shown at *fff* (fig. 3). These tubes pass through similar holes, at their top ends in the plate *gg*, which latter is firmly riveted to the outside shell of the boiler. The tubes are also firmly attached to the two plates, *cc*, *gg*. They serve to convey the flame, smoke, and hot air from the fire to the smoke box, *h*, and the chimney, *a*, and at the same time their sides provide ample heating surface to allow the heat contained in the products of combustion to escape into the water. The fresh fuel is thrown on to the grating when required through the fire door, A (fig. 2). The ashes, cinders, &c., fall between the fire bars into the ash pit, B (fig. 3). The water is contained in the space between the shell of the boiler, the furnace chamber, and the tubes. It is kept at or about the level, *ww* (fig. 3), the space above this part being reserved for the steam as it rises. The heat, of course, escapes into the water, through the sides and top plate of the furnace, and through the sides of the tubes. The steam which, as it rises from the boiling water, ascends into the space above *ww*, is thence led away by the steam pipe to the engine. Unless consumed quickly enough by the engine, the steam would accumulate too much within the boiler, and its pressure would rise to a dangerous point. To provide against this contingency, the steam is enabled to escape when it rises above a certain pressure through the safety valve, which is shown in sketch on the top of the boiler in fig. 2. The details of the construction of safety valves will be found fully described in Chapter IX., which is devoted exclusively to the consideration of boilers and their fittings. In the same chapter will be found full descriptions of the various fittings and accessories of boilers, which it would be out of place here to describe in detail, such as the water and pressure gauges, the apparatus for feeding the boiler with water, for producing the requisite draught of air to maintain the combustion, and also the particulars of the construction of the boilers themselves and their furnaces,

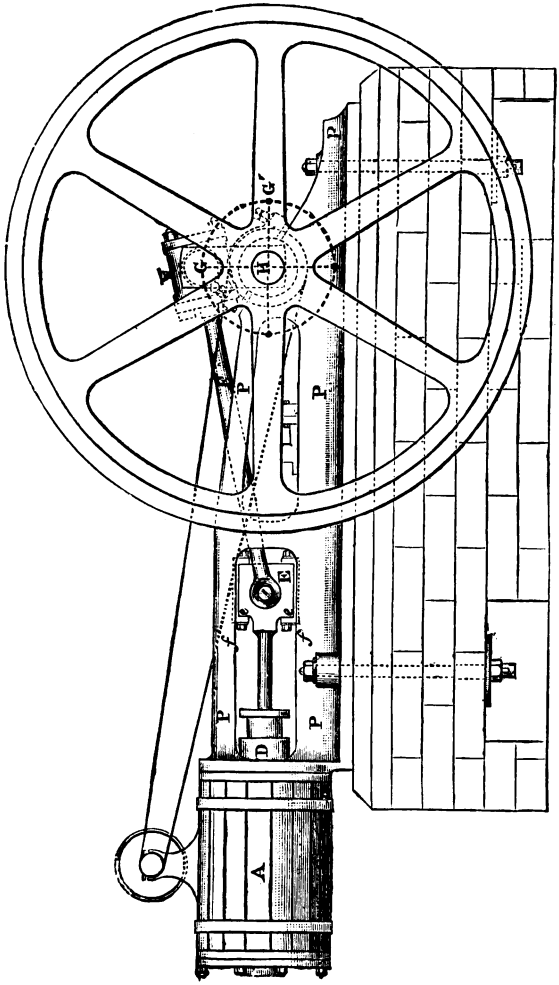

Fig. 5.

and the principles on which their strength is determined, and their various parts proportioned, so as to fully realise the effects intended.

We now come to the description of the engine, and the type selected for illustration is that usually called horizontal single cylinder, direct acting.

Fig. 5 is an elevation of the exterior. Fig. 6 is a horizontal section of the cylinder, piston, and valve box. Fig. 7 is a plan. The cylinder is shown at A, figs. 5, 6, 7; its construction is best seen from the section, fig. 6. It is formed of cast iron, the ends being flanged to allow of the cylinder cover or end plate, *aa*, and the frame, PP, being

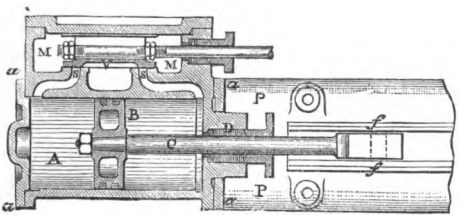

Fig. 6.

bolted to it. The piston is shown at B; it is a circular cast-iron disc, made to fit the cylinder in a steam-tight manner. Into the piston is fixed the piston rod, C, which passes through the front cylinder cover, the place where it passes through being made steam-tight by the stuffing box, D. The front end of the piston rod is fastened to the crosshead, E (fig. 5), which is a joint used for connecting the piston rod to the connecting rod, F, in such a manner as to allow the latter to swing in a vertical plane as the piston travels backwards and forwards. The crosshead is also provided with two slides, *ee* (fig. 5), which move between the guide bars, *ff* (figs. 5 and 6), and which prevent the piston rod from being bent, and from moving otherwise than in a straight line. The connecting rod, F (figs. 5 and 7), joins the end of the piston rod to the crank pin, G. The crank axle in which

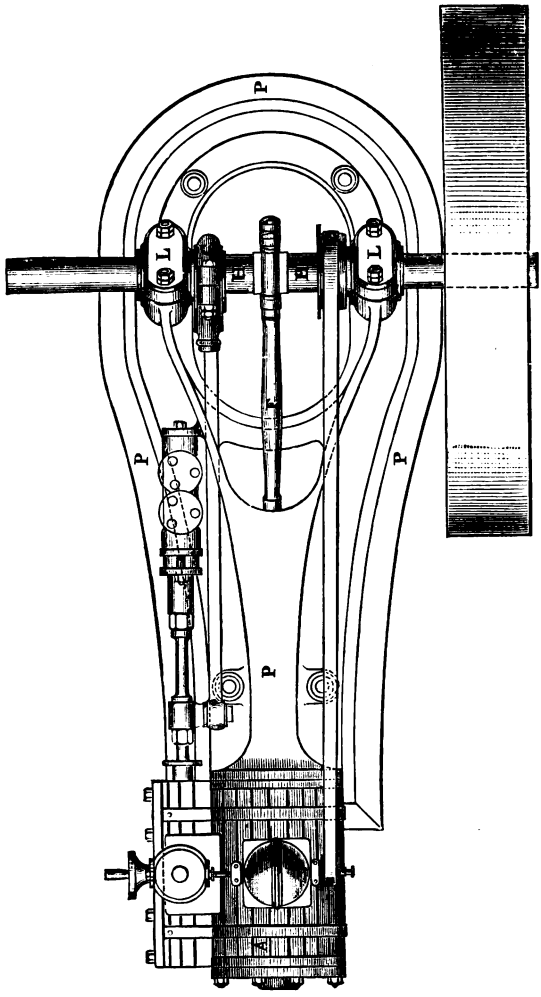

Fig. 7.

the crank is formed is shown in section at H (fig. 5), but is seen more clearly in the plan, fig. 7, where it is shown passing through the two bearings, LL. The distance between the centre of the crank pin, G, and the centre of the crank axle, H (fig. 5), is called the length of the crank arm, and is exactly equal to half the distance which the piston moves from one end to the other of the cylinder.

Supposing now that steam were allowed to flow from the boiler into the cylinder in such a manner as to obtain admission behind the piston, B ; this latter would commence to travel towards the front cover of the cylinder, and in doing so would push forward the piston rod and the crosshead. The end of the connecting rod next the crosshead would also be pushed forward, but the other end of the connecting rod which encircles the crank pin, not being free to move simply forward, would describe an arc of a circle round the centre of the crank axle, H, and in so doing the direction of the rod would become inclined so as to form an angle with the axis of the cylinder. By the time the piston has travelled to the front end of the cylinder, the crank pin will have been turned round into the position G′ (fig. 5), diametrically opposite to its initial position. Suppose that, just before this takes place, the steam is shut off from the back of the piston, and the steam already in the cylinder is allowed to escape, while at the same time fresh steam from the boiler is allowed to enter the cylinder at the *front* side of the piston, this latter will commence to travel back to its original position,[1] and in doing so will cause the crank pin to revolve from the position G′ (fig. 5), through a semicircle, till it reaches its original position, it having thus described a complete revolution round the centre of the crank axle, while the piston was making a double stroke backwards and forwards. This operation may be repeated as often as we like provided we have a suitable apparatus

[1] For the sake of simplifying the description, no account is here taken of the action at the dead centres. See p. 14.

for admitting the steam alternately on each side of the piston, and then allowing it to escape either into the open air or a condenser.

The manner in which the steam admission is regulated is as follows. By referring to the section (fig. 6), it will be seen that a box-like casing, MM, is cast in one piece with the cylinder and on one side of it. This box contains the valve, V, which controls the flow of the steam. It will be noticed that the side of the cylinder next the valve box contains two passages, *ss*; these are called the steam ports because the steam by means of them gains access to and escapes from either end of the cylinder. For the sake of clearness the following diagram, fig. 8, is given, showing the

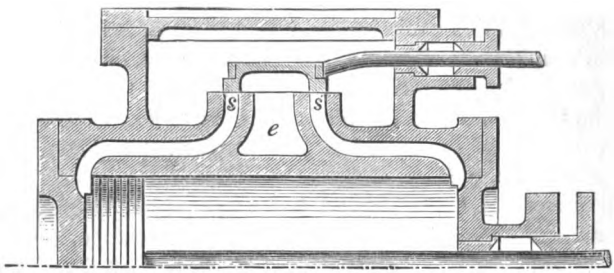

Fig. 8.

valve and side of a cylinder to a larger scale. The cast-iron box containing the valve is always filled, when the engine is at work, with steam from the boiler. If the valve occupies the position shown in fig. 8, the steam cannot enter the cylinder at all, because both ports are covered up by the valve. If the latter, however, be moved a little to the right so as to uncover the steam port *s*, two things will happen. The steam will be enabled to pass through the port *s* into the cylinder, and push the piston forward from left to right, while at the same time the port *s'* will be uncovered by the inner edge of the valve, and any steam which may be contained in the cylinder on the right-hand side of the piston

will be enabled to escape through the port *s'* into the interior hollow of the valve, and thence into the exhaust passage *e*, whence it can escape to the air of the condenser. This condition of things is shown by fig. 9.

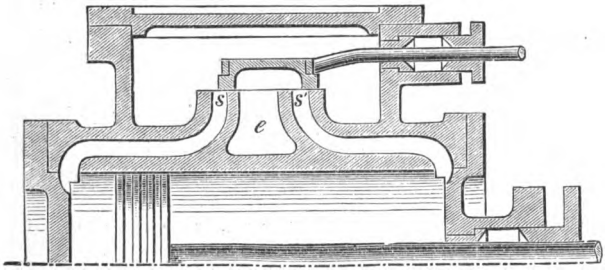

Fig. 9.

If when the piston has reached the end of its forward stroke the valve be moved backwards into the corresponding position on the other side, the steam port *s'* will then be uncovered and will allow the boiler steam to enter the cylinder, and force the piston back from right to left, while the steam on the left-hand side of the piston will be enabled to escape into the exhaust passage.

The foregoing remarks must be looked upon as merely an elementary sketch of the working of this particular sort of valve (which is commonly called the D slide valve). The proper way of proportioning the parts of the valve, the widths of the steam ports, and the methods of driving the valve so as to admit and cut off the fresh steam and release the exhaust steam precisely at the right moments during the stroke of the piston, are points of the greatest nicety and require the most careful study, and are fully described in Chapter VII. ; but enough has been now said to illustrate the method of working in a general way without going into complexities.

It will be noticed that the valve is connected by a rod (see fig. 7) with a cam, C, fixed to the crank axle of the engine. This cam, which is called an eccentric, drives the valve

backwards and forwards; its manner of working will be found described in the chapter already referred to.

When the centre of the crank-pin occupies either the point G′, fig. 5, or the diametrically opposite position, the centre line of the crank is in the prolongation of the axis of the cylinder and connecting rod, and it is evident that when in either of these positions, which are called the dead centres, the steam would only tend to press the crank axle against its bearings, LL, fig. 7, and would exercise no rotating effect whatever. Consequently unless some means can be devised for getting the crank over the dead centres the engine will stick fast.

The plan invariably adopted with a single cylinder engine is to provide a heavy fly-wheel, shown in elevation in fig. 5, and in plan in fig. 7. The momentum acquired by this fly-wheel during the stroke carries the crank over the dead centre. In addition to the above the fly-wheel exercises other useful functions which are explained in Chapter V., but which need not be dwelt upon at present.

The engine which has been described above is mounted on the heavy combined bed plate and frame PPP, shown in elevation fig. 5, and in plan fig. 7. The bed plate is bolted down to a solid mass of masonry as shown in fig. 5.

For our present purposes it is not necessary to examine into the other details of the mechanism, such as the governor and feed pump shown on fig. 7.

The engine which we have just described belongs to a type which is very much employed to drive machinery on land. It must not be supposed, however, that the steam engine, as originally invented, was anything like so simple a machine. On the contrary, it has taken two centuries of time to attain its present degree of perfection. We have no intention of entering into the history of the steam engine; indeed, the limits of this volume would preclude any such idea; moreover, the historical part of the subject has been dealt with over and over again in special

books, and in the biographies of the great engineers. At present we are only concerned with the engine as we actually find it, and with its possible future. The past will only be referred to for the purpose of showing what increase in efficiency has been attained in more modern times.

The importance of the accurate study of the steam engine will not be disputed when it is remembered to what purposes the engine is applied now-a-days, and to what an extent this manufacturing and sea-trading country is dependent upon its efficiency. Foremost among these purposes are :—

1. Locomotion on railways. The steam engine is employed in effecting nearly the whole of the internal goods and passenger traffic of the country. At the present moment there are in this country over 18,000 miles of railway opened for traffic, and the various railway companies employ between them many thousands of locomotives.

2. Marine locomotion. For this purpose the engine is employed in propelling the numerous steam vessels, which effect the greater part of the ocean-carrying trade of the world. Another important use of marine engines is the propulsion of those ships of war on which we depend for the protection of our coasts and our mercantile navy from foreign enemies.

3. The driving of machinery in our factories. The importance of the engine for this purpose can hardly be overestimated, when it is remembered that we depend on our factories, and on the export of our manufactures, for the means of maintaining our present population, which is far too large to be supported by the produce of the country.

4. The winding of coal and other minerals, and the pumping of water out of mines.

5. The tillage of the soil, and the preparation of its produce for the use of mankind. This is a comparatively novel purpose for which the steam engine is employed, but one which is daily increasing in importance.

Each of these purposes requires a different type of engine for itself. In a small volume like this, it would be out of the question to describe every variety of engine at present in use. It will only be possible, at best, to explain the principles on which they should all alike be designed. The great importance of an accurate study of the subject is this: that without this study we cannot make our engines economical in the use of fuel. This economy should be one of the first objects of every constructor of a steam engine; for even if our supply of fuel at present prices were inexhaustible, nevertheless in many cases economy is of paramount importance. Take only the case of steam vessels which have to make long voyages. Up to a comparatively recent period it was not found commercially practicable to run merchant steamers on the longest trade routes, such as to China; for the mere coal required to develop the power necessary for propulsion would have occupied so much of the carrying capacity of the vessel as to leave insufficient room for a remunerative cargo. Thanks, however, to the fuel economies introduced during the last twenty years, steamers can now be employed with advantage on the longest voyages. Similarly the magnificent passenger steamers which now cross the Atlantic owe their high speed mainly to the modern improvements which have enabled great power to be attained with a comparatively moderate weight of machinery and fuel.

The ultimate object of all study of the steam engine is this:—to enable us to attain the maximum economy in the use of fuel with the greatest efficiency of the machinery. Hence the theoretical portion of the subject naturally divides itself into two principal parts. First, the study of the engine as a heat engine; that is, as an apparatus for the conversion of the heat supplied to it into mechanical work. Second, the study of the engine as a piece of machinery.

The study of the heat engine involves a knowledge of the nature of heat, and the laws of its conversion into mechanical work; hence we shall have the following divisions:—

A chapter (II.) in which is explained the nature of heat, and the mode of measuring it. This chapter will only deal with the subject so far as it bears directly upon the heat engine, and all reference to other branches of this science will be avoided.

A chapter (III.) which deals with the conversion of heat into mechanical work, by its application to gas and water. This chapter will give an exact account of the physical properties of these bodies, and an explanation as to how the heat supplied to them under given circumstances is actually spent. It will also contain a description of the theoretically perfect heat engine, and show what proportion of the total heat supplied to it can, under the most favourable circumstances, be in theory turned into work, and also the conditions to be observed in order that this ratio of work done, to heat supplied, may be realised. It will, lastly, show how to apply the principle of the theoretically perfect heat engine to actual steam engines, and will explain why these latter are comparatively wasteful of heat.

We now come to the consideration of the engine, as a piece of machinery, and the student will require to study in detail, both theoretically and practically, the nature of the mechanical means, or mechanism, by which the pressure of the steam is transformed into work. The study of this part of the subject is contained in the following divisions.

Chapter IV. shows the connection between the size of the cylinder, the pressure of the steam, the velocity of the piston, the useful work to be done, and the incidental resistances which have to be overcome; and will show practically how to proportion the size of the cylinder to the work it has to do.

Chapter V., on the laws of motion, as applied to the separate moving parts of an engine, so that the effects of their weights, velocities, and directions of motion, on the working of the whole may be understood.

The practical part of the book contains descriptions of the various organs of which different types of engines and boilers are made up, and the rules for proportioning them to their several purposes.

Chapter VI. is on the practical details of the mechanism employed. This chapter will contain illustrations of the working parts of various sorts of engines.

Chapter VII., on a part of the mechanism—viz., the valves and valve gear—which, on account of its importance and complexity, requires a separate detailed description.

Intimately connected with the subject of valve-gearing is the instrument which is used in practice in order to ascertain if the valves are effecting a proper distribution of the steam. This instrument is called the indicator, and it is used not only for the above-mentioned purpose, but also to measure the power which is being exerted by the engine. The indicator records the performance of the engine by inscribing a geometrical figure called an indicator diagram on a piece of paper.

Chapter VIII. is devoted to the consideration of indicators and the interpretation of their diagrams, and is illustrated by numerous diagrams taken from actual engines, each of them being remarkable for some peculiarity.

Chapter IX. deals with the means of generating steam in practice, and contains an account of the nature of combustion, the constituents of fuel, and the various descriptions of furnaces, boilers, and their fittings.

The subject of the condensation of steam, and the various forms of condensers, air and circulating pumps, are dealt with in Chapter X.

In the chapters containing descriptions of the mechanism of steam engines, several arrangements, which may be looked upon in the light of refinements, have been omitted. Most of these contrivances have been designed with the object of minimising the losses of efficiency of actual engines, as compared with those which are theoretically perfect. These

sources of loss are enumerated at the end of Chapter III., and a special chapter (XI.) is devoted to the various remedies, and contains an examination into the merits of steam jackets, super-heating, and the compounding of engines.

Students who approach this subject for the first time, or those who wish only to acquire a general knowledge of the construction of engines and boilers, are recommended to omit Chapters III., IV., V., and the latter part of Chapter VII.

CHAPTER II.

Nature of heat—The mode of measuring it—Its effects on gases and water—General ideas of nature of heat—Old notions regarding it—Material theory and its refutation by Davy—Modern theory that heat is a form of energy—Definitions and examples of energy and work—Example of conversion of heat into work—Measurement of heat—Temperature—Thermometers, their graduation and defects — Quantity of heat—Specific heat—British thermal unit—Capacity of substances for heat—Relation between heat and work —The mechanical equivalent of heat—Joule's experiments—Effect of application of heat to gases—Nature of gas—Boyle's law connecting the pressure and volume of gas—Graphic representation of Boyle's law—Definition of an Isothermal—Charles's law connecting the volume and temperature of gas—Dalton's law connecting the volume and temperature of gas—The air thermometer—Absolute temperature—Combination of Boyle's and Charles's laws—The specific heat of gases—Difference in the specific heats according as the gas is heated at constant volume or at constant pressure—External and internal work done when a gas is heated at constant pressure—Effect of application of heat to water and ice—Heat absorbed in liquefying ice—Heat absorbed in evaporating water at various pressures—External and internal work of evaporation—Law connecting the pressure and temperature of steam—Total heat of steam analysed—Specific volume and relative volumes of steam—Law connecting the pressure, volume, and density of steam—Graphic representation of the expenditure of heat in evaporating water—Expansion of gas and steam—Isothermal expansion of gas—Isothermal expansion of steam—Adiabatic expansion of gas—Adiabatic expansion of steam.

IN this and the following chapter it is not by any means proposed to go into the study of heat, otherwise than as it bears directly upon the heat engine. Consequently no reference will be made to theories and phenomena of heat, other than those which affect gases and water : nor will any attempt be made to describe the numerous experiments which are usually dwelt upon in treatises devoted exclusively to this branch of science. On the contrary, these chapters will be found to be mere summaries of certain

parts of the subject, inserted here because they are absolutely necessary to the correct knowledge of the heat engine.

Everyone is familiar with the sensations produced by heat on the human body, as, for instance, when the hand is exposed to the action of a fire, or plunged into boiling water. The agency which produces this sensation is called Heat. The nature of the agency has, ever since the physical sciences were first studied, been the subject of speculation with natural philosophers. In the last and the beginning of the present century, heat was supposed to be a kind of matter which differed from all other forms of matter with which we are acquainted, in that it had no weight. It was, in fact, supposed to be a subtle and imponderable fluid, which was capable of spreading and insinuating itself between all the elementary particles which constitute matter, and of flying from hot bodies to colder ones, no matter at what distance apart these bodies might be. This theory did good service in its time, in helping philosophers to account for many of the effects of heat; it had, however, ultimately to be discarded, because it failed altogether to account for the fact that heat, in apparently illimitable quantities, could be evolved from cold bodies, by rubbing them together; that is to say, by the process of friction, cold bodies could be made hot and could be made to communicate heat to any quantity of other cold bodies. This phenomenon was accounted for, by the believers in the material theory of heat, in the following manner: The bodies to be rubbed together possessed in their state of heat, or thermal condition, before the commencement of the experiment, a certain quantity of the fluid called heat, which caused them to be as hot as, say, for example, the human body. This was expressed by saying that the bodies when as warm as the human body had a certain capacity for heat, i.e. they required a certain quantity of the imponderable fluid to be absorbed between their particles, in order that they might become as warm as aforesaid. Now, when the bodies were rubbed together,

and became eventually hotter than the human body, this was accounted for by saying that their capacity for heat became diminished by the action of friction ; that is to say, they could not, when rubbed, retain the same amount of the imponderable fluid as before, without becoming hotter. If any experiment could be devised which should prove that the capacity of bodies for heat is not diminished by friction, then the material theory of heat would fail to account for the fact that bodies become hotter when rubbed.

The first absolutely conclusive experiment, which established the fact that friction makes bodies hot, while it does not diminish their capacities for heat, was made by Davy in 1799. His experiment consisted in rubbing together two pieces of ice till they melted into water, due care having been taken to prevent heat from entering the ice by any other means than friction alone. Now, according to the old theory, the resulting water ought to have a less capacity for heat than the original ice ; but it has been proved over and over again by experiment that the capacity of water for heat is not only not less than, but about double that of ice ; consequently the material theory failed completely to account for the facts, and Davy, after reasoning on his experiments for some years, came to the following conclusion, which we repeat in his own words :—

'Heat, then, or that power which prevents the actual contact of the corpuscles of bodies, and which is the cause of our own sensations of heat and cold, may be defined as a peculiar motion, probably a vibration of the corpuscles of bodies tending to separate them.' Again, in 1812, Davy thus states his theory :—'The immediate cause of the phenomenon of heat, then, is motion, and the laws of its communication are precisely the same as the laws of the communication of motion.'

Another way of stating the above is that heat is a form of *energy*. To make this point clear before going further into the nature of heat, we must first define what is under-

stood by the term *energy* and the involved term *work*, and illustrate the definitions by examples.

Energy is the power of doing work.

Work is the overcoming of a resistance through a certain space, and is measured by the amount of the resistance multiplied by the length of space through which it is overcome.

The simplest possible example of doing work is to raise a weight through a space against the resistance of the earth's attraction, that is to say, against the force of gravity. For instance, if a hundred pounds be raised vertically upwards, through a space of three feet, work is done, and, according to the above, the amount of work done is measured by the resistance due to the attraction of the earth or gravity, i.e. one hundred pounds, multiplied by the space of three feet, through which it is lifted. The product formed by multiplying a pound by a foot is called a foot-pound. Thus, in the above instance, the amount of work done is 300 foot-pounds. Had the weight been only three pounds, but the height to which it was raised been 100 feet, the quantity of work done would have been precisely the same, i.e. 300 foot-pounds.

In Great Britain, the unit of work is—a resistance, equal to the attraction of the earth upon a pound of matter, overcome through a space of one foot; or, in other words, one foot-pound.

Rate of Doing Work. Horse-Power.

The rate of work of any agent means the quantity of work which it performs in a given time, and is measured by the number of foot-pounds done in an hour, or a minute, or a second.

A quantity of work equivalent. to the raising of 33,000 *pounds through one foot, in one minute, is called a horse-power.* This is the unit generally employed to represent the rate

of work of a steam engine, and is adopted to avoid the use of the very high numbers which would result if foot-pounds per minute were chosen. Thus an engine which can overcome resistances equivalent to raising 10,000 pounds vertically upwards through 33 feet every minute is said to be an engine of 10 horse-power or 10 H.P. If the engine raised the same weight through the same height once every second, instead of every minute, then by the definition the work done would be equal to sixty times ten horse-power, or 600 H.P. Hence if r = the resistance expressed in pounds, h = the height in feet through which r is overcome, and t = the time in minutes which it takes to do the work, then

$$\text{The horse-power exerted} = \frac{h \times r}{33{,}000 \times t}.$$

The lifting of weights is only one special form of doing work, but there are also many other ways of doing it. For example: if a carriage be pulled along a level road, it is well known that its progress is resisted by the friction of its wheels against the surface of the road and against their own axles. Hence the pulling of such a carriage answers perfectly to the definition of doing work, for resistance is thereby overcome through a space.

Again, it is well understood by those who have studied the laws of motion that if a mass—as, for example, a stone—be projected upwards it will rise to a certain height, depending on the velocity with which it left the hand. The exact height to which it will rise is precisely equal to the height through which it must fall, under the action of gravity, in order that, at the end of its fall, it may have acquired a velocity equal to that with which it was projected upwards. Now the imparting of this velocity to the mass is evidently a way of enabling work to be done, for the mass is thereby caused to rise to a certain height, against the attraction of the earth, and the amount of the work done is measured by the weight of the mass multiplied by the height to which it rises.

It is not necessary to impart velocity to the mass in a vertical direction only, in order to do work. Whenever motion is given to a body in *any* direction the resistance due to the inertia of the body is overcome through a space, and consequently, by the definition, work is done. If, for instance, a train were capable of moving without friction on a level railway, in order to start it from a state of rest and give it a speed of, say, forty miles an hour, work would have to be done in order to overcome the mere inertia of the train. When once the given speed had been imparted to the train, it would, of course, move on for ever on a level railroad, provided it met with no frictional resistances. If in its course it came to an inclined plane, it would run up the plane till it had attained a vertical height above the level equal to the height through which the train must fall downwards, in order to attain the given speed of forty miles an hour. The measure of the work done in giving motion to the train is equal to the weight of the train multiplied by this height.

On actual railroads the work done by the engine partakes of the character of each of these examples. When starting the train from a station and giving it a certain speed, the resistance due to the inertia of the whole moving mass is overcome. The going up an incline corresponds to lifting a weight up a height; and throughout the entire run the friction of the wheels and axles and the resistance of the air are being overcome.

Energy.

It has been necessary to dwell thus at considerable length on the nature of Work, in order that the term Energy, i.e. the power of doing Work, might be thoroughly understood. This power of doing work exists in many different ways. For instance, a coiled spring is capable of doing work in driving a clock, and therefore possesses energy. Similarly a

weight raised to a height, and attached to a string passing over a pulley, is capable, during its fall, of raising another weight, or of driving machinery, and consequently it also possesses energy. Again, a body in motion, such, for instance, as a railway train, is capable of overcoming the friction of the brakes for a certain time till it is brought to a standstill, and therefore possesses energy. Similarly a projectile from a modern rifled gun possesses very great energy owing to the high speed at which it moves, so much so that before it is forcibly brought to a rest it can do work represented by piercing many inches of iron armour. It will be noticed that there is a great difference between the kind of energy of which the first two cases are examples and the last two. The first two are instances of bodies which, though themselves at rest, are capable at any moment of doing work. In the case of the coiled spring its energy was due to the relative position of its parts with regard to each other and to the mutual forces acting between them. In the case of the raised weight, its energy was solely due to its position with regard to the earth and to the forces acting between the earth and it. This energy, due to position, is called potential energy, a term which signifies that the energy is *capable* of being exerted. The last two instances on the contrary are cases of bodies possessing energy by virtue of their motion. This kind has been called actual or kinetic energy. The last term, which is derived from a Greek word signifying motion, is, perhaps, the most appropriate of the two.

Bodies may be possessed of both descriptions of energy at one and the same time. For instance, when the raised weight of the former example begins to fall, it possesses kinetic energy by virtue of the motion which it has acquired, while it still possesses potential energy, for it is capable of falling further still. For every foot which it descends its kinetic energy increases, while the potential diminishes. Just as it touches the earth its kinetic energy is a maximum,

while the potential has vanished altogether. Thus during the fall the energy has changed from being all potential into being all kinetic. Moreover, the kinetic energy acquired at the end of the fall is exactly equal in amount to the potential energy possessed at the commencement. For, before its fall the mass was capable of pulling up another mass of nearly equal weight with itself, to the same height above the ground which it occupied; while at the end of the fall it has acquired a velocity, sufficient, if reversed, to send itself back to whence it came. Moreover, it is clear that at any time during the fall, the sum of the potential energy left, and the kinetic energy acquired, are equal to the original energy, for what the one has lost in amount the other has gained.

This is an example of what is called the *transmutation of energy*, by which is meant that the energy is changed from one form into another, and also of the *conservation of energy*, by which is meant that the total energy of the two bodies—viz. the earth and the weight—is not altered in amount, but only in kind. It is one of the cardinal doctrines of modern science, and one which has done more to extend our knowledge of heat than any other, that energy, like matter, can neither be created nor destroyed by material agency, but can only be transmuted from one form to another. This doctrine is called the Principle of the Conservation of Energy. In books on Dynamics, the principle is proved by mathematical reasoning to be true for certain cases, and it has, moreover, been proved by experiment to be true in all cases which can be tested by experiment. Hence it is believed to be universally true. The following is a general statement of the principle.

The energy of any system of bodies cannot be altered in quantity by the mutual action of the bodies; it can only be transmuted in kind into one or more of the forms which energy takes.

We are now in a position to return to the subject of Heat, and to understand how it is that heat is a form of

energy—i.e. a form of the capability of doing work. For Davy's statement is, that heat may be defined to be a peculiar motion of the corpuscles of bodies ; now, we have seen that matter in motion is capable of doing work, and is therefore possessed of energy, and consequently if heat be motion, or the cause of the motion of the ultimate corpuscles of matter, heat is also a form of energy.

Heat a Form of Energy.

The reasons for believing that Davy's definition of heat is a true one are the following:—

1. It seems impossible to believe that heat is a substance; for if it be such, then no theory has yet been advanced which can account for certain phenomena, such for instance as the production of heat from bodies in boundless quantities by means of friction or other mechanical action.

2. Heat can always be generated by doing work upon bodies. For example, we have seen how Davy melted ice by friction. Again, let the student attempt to file a piece of metal, and after a very few strokes of the file, he will find that both it and the metal have become perceptibly warmer, and if he continues the action smartly for some time on a small piece of metal, he will not be able to touch it without burning his hand. As an example of another kind of mechanical action producing heat, it is well known that a smith can hammer a small piece of iron to a red heat. Again, if water be allowed to fall several times from a height into a nonconducting receptacle, and care be taken to prevent the escape of heat, it will be found that after its fall the water will be warmer than at the commencement of the experiment. Another and most important example is the effect of compression upon gases. If, for instance, a portion of air be inclosed together with a piece of easily inflammable tinder in a cylinder provided with a movable piston, and the piston be driven down suddenly, it will be found that

the contained air has become so warm that it can cause the ignition of the tinder. Now in all the above instances, unless we are prepared to admit that energy is destructible, that is, that it can be put out of existence altogether, we are forced to confess that it is merely transmuted into heat, for heat is apparently the only thing we have to show for the energy expended in the majority of these examples.

3. The converse of the above is also true, viz. heat can be made to produce work, and for every unit of work which it does, a certain amount of heat disappears, and there is nothing to show for the disappearance of the heat but the work done. As an example of this, we need only refer to the elementary steam engine described in the first chapter, where we saw that the heat of the fire, communicated to the water contained in the cylinder, was partially converted into work done; for by the agency of heat alone the piston with its weights was raised to a certain height. The conditions of this experiment were not such as to enable us to ascertain what heat, if any, had disappeared in consequence of the work done; but the following modification of the experiment will render this fact also demonstrable.

Instead of holding water, let the cylinder contain a portion of air of a certain warmth, and let the piston, instead of being loaded with common weights, have a vessel containing a quantity of water placed on it. Then, so long as the weight on the piston remains constant, and so long as no heat is communicated to the gas from outside, nothing will happen.

If, however, a little of the water be removed from the can, the pressure of the inclosed air will cause the piston with its load to rise through a small space, and again come to rest. For, the air occupying a larger space in the cylinder, its pressure becomes diminished by a well-known law, which will be explained hereafter (see page 43), and as soon as this diminished pressure on the bottom of the piston is equal to the diminished pressure of the piston on the in-

closed air (caused by some of the water having been taken away) then the whole must come to rest. Let now a little more water be abstracted; the piston will rise a little higher, and so on, till the whole of the water has been removed, when the piston will have risen higher still. A convenient way of abstracting the water, as fast or as slowly as we like, is by means of a syphon. If now we have any means for ascertaining the warmth of the air at the beginning and at the end of the experiment, it will be found to have lost heat at the end, after having done work, measured by lifting the piston, together with the pressure of the external atmosphere and the empty can through the whole height, and different portions of the water through different heights. Now, unless this heat has been spent in effecting internal changes in the constitution of the gas itself, it must have been spent in doing the above work, for no other effects have been produced. It is of course assumed that in the experiment no heat has been allowed to escape from the air in the cylinder to external bodies, or, *vice versâ*, to reach the air from external bodies.

Most elaborate experiments have been made on steam-engines when at work, in which the following quantities have been measured:—1. All the heat which enters the engine in the shape of steam; 2. All the heat which leaves the condenser in the shape of warm water; and 3. All the heat which escapes during the working of the engine in various ways; and it has been found that the quantities comprised under the 2nd and 3rd headings are not equal to but less than the heat which enters the engine; so that a certain quantity remains to be accounted for, the disappearance of which can only be explained on the supposition that it has been turned into the mechanical work done by the engine.

From all the above considerations we conclude that heat is a form of energy. It is further supposed that the special form which this energy takes is that of a motion of the

molecules which constitute matter. Into the nature of this motion, however, it is not proposed to enter here.

The next thing which we shall want to know is this: What is the exact relation between heat and work ; that is to say, What quantity of heat can be produced by the doing of a certain quantity of work, and, *vice versâ*, How much work is a given quantity of heat capable of doing? Before it is possible to answer these questions, it must first be explained what is meant by a quantity of heat, and how heat is measured at all.

Measurement of Heat. Temperature.

Everybody is familiar with the sensations caused by different *intensities* of heat. For instance, the sensation produced by plunging the hand into boiling water is very different from that caused by contact with cold water taken direct from a well. The quality of heat which causes these sensations is called *temperature.* In the first case the immediate cause of the sensation experienced was the heat leaving the boiling water and entering the comparatively cold hand ; while in the second instance exactly the reverse took place, heat entering the cold water from the comparatively warm hand. This communication of heat from one body to another depends on the differences between their temperatures ; so much so that temperature has been defined as follows: 'The temperature of a body is its thermal state considered with reference to its power of communicating heat to other bodies.'[1]

If now two bodies be so placed that they can freely communicate heat to one another, and are isolated from the influence of all other bodies, then if neither of them loses heat they are said each to have the same temperature, but if one of them loses and the other gains heat, then the body which loses is said to have the highest temperature.

[1] See Clerk Maxwell's *Theory of Heat*, p. 32.

Temperatures are measured and compared by noting the effects which heat has upon bodies. One of the most remarkable effects of heat is that it expands most substances to which it is communicated, so that the higher the temperature the greater the expansion. If then we want to compare the temperatures of two bodies, we have only to bring each of them in turn into thermal communication with some third substance which expands readily under the action of heat.

If care be now taken that each of the bodies in turn remains sufficiently long in contact with the third body, so that the latter may acquire, first, the exact temperature of one of them, and afterwards that of the other, and if its expansion in each case be carefully measured, then that body which causes the greatest expansion has the highest temperature. An instrument designed to serve the purpose of this third body is called a Thermometer.

THERMOMETERS.

A thermometer for practical use should be portable, readily acted upon by slight differences of temperature, and difficult to put out of order; it should be furnished with an index, or scale, for reading off differences of temperature, and should always give the same reading on the scale, for the same temperature, under the same circumstances. Thermometers are made of various substances, but we propose at present to describe only the one which is in most common use, viz. the ordinary mercurial thermometer. This instrument (see fig. 10) is made by taking a tube of glass, a few inches in length, having a capillary bore, that is to say a bore of very small calibre. A bulb is blown at one end of the tube, and while the bulb is warm, so that most of the air it contains is expelled, the tube is plunged into mercury. The effect of this is to cool the tube, and, as we shall see afterwards, to reduce the pressure of the air which remains in the

bore and bulb. Some of the mercury then enters the bore, and partly fills the bulb. By boiling this mercury while in the bulb, the remainder of the air is expelled, its place being taken by the vapour of mercury. If now the open end of the tube be again plunged into mercury, both tube and bulb will be completely filled, and while still warm the open end is closed hermetically. As soon as the tube and its contents have cooled down, the mercury will be found to have contracted, leaving part of the bore quite empty. The instrument is now ready for graduation. This is done by first marking on the tube the position at which the mercury stands for two different temperatures, and then dividing the intermediate space into an arbitrary number of equal spaces, each of which is said to represent one degree of temperature. The two temperatures always chosen are those of melting ice and boiling water. The temperature of melting ice is always the same, at the varying pressures of our atmosphere. The thermometer is plunged into a mixture of melting ice and water, and, after remaining immersed for some time, the point at which the mercury stands is marked on the tube. We may be certain that we have thus marked the exact temperature of melting ice ; for if, during the process of immersion, heat enters the mixture of ice and water, its effect will be simply to melt some more of the ice, and not to raise the temperature of the water. This action of heat will be explained

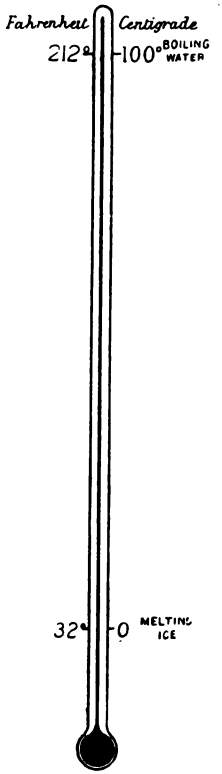

Fig. 10.

hereafter. It is more difficult to mark the point at which the mercury stands for boiling water, for it is known that water does not always boil at the same temperature. In fact, the greater the pressure under which the water boils, the higher will be its temperature. Consequently, on a day when the barometer stands high, boiling water is hotter than when the opposite is the case. It is therefore necessary to fix upon some one special atmospheric pressure in order to settle a standard boiling-point, and the pressure always adopted in this country is that marked by the barometer when the mercury stands at a height of 29·905 inches, the temperature of the mercury being that of melting ice. If then, on a day when the barometer indicates the above pressure, the thermometer be immersed in the steam of boiling water, it will be found that the contained mercury will rise to a certain point, and remain there ; and by marking the tube at this place we obtain a point which always shows the temperature of boiling water at the standard pressure of the air. This temperature is always fixed and invariable, so long as the pressure under which the water boils remains fixed ; for the effect of adding more heat to the water is only to turn more of it into steam, but not to raise its temperature. The reason of this will be explained hereafter.

Having now got two fixed points on the tube of the thermometer, we are at liberty to call them by any numbers we please, and to divide the space between them into any convenient number of divisions, or degrees, and to carry these divisions above the boiling and below the freezing points, as far as the length of the tube permits. There are three modes of numbering in common use in various countries:—

1. The Centigrade scale, in which the temperature of ice is called zero, or 0°, and the temperature of boiling water 100°. The space between these two is divided into one hundred equal parts, each of which, if the bore of the tube be per-

fectly even, is *assumed* to represent an equal increment of temperature, and the divisions are carried up above boiling-point, and down below freezing-point as far as the tube permits. Those degrees below freezing-point are called negative. This scale of temperature is in common use in nearly all the countries of the continent excepting Russia.

2. The Fahrenheit scale, used in the British Empire and the United States. On this scale the freezing-point is called 32°, the boiling-point 212°, and the intermediate space is divided into one hundred and eighty equal parts. The zero of this scale is 32° below freezing-point, and below this zero the numbers are negative.

3. The Réaumur scale is used chiefly in Russia. This scale differs from the Centigrade, in that the boiling-point is called 80°, and the space between it and zero or melting ice is divided into eighty equal parts. This scale is less in use than either of the others. Throughout this book the Fahrenheit scale is the one generally referred to. Whenever the Centigrade scale is made use of, it will be specially indicated by writing C. after the numeral showing the number of degrees. Fig. 10 shows the scales of the Fahrenheit and Centigrade thermometers side by side.

To compare degrees on the Fahrenheit and Centigrade scales it is only necessary to remember that the freezing-point on the Fahrenheit scale is 32°, and on the Centigrade 0°, while the number of degrees between this and boiling-point is in the former case 180, and in the latter 100. Consequently the length of one degree F. is $\frac{5}{9}$ of one degree C. Now the actual number of degrees F. above freezing-point is equal to the number on the scale minus 32°. Let T° stand for the number of degrees on either scale, then $T_C° = \frac{5}{9}(T_F° - 32°)$ and conversely $T_F° = \frac{9}{5}T_C° + 32°$.

It is not proposed to enter here into the refinements of thermometer-making, but it will be necessary now to point out how far the mercurial thermometer may and may not be trusted, as a measurer of temperature, and what errors

are inseparably connected with its use, no matter how perfectly it may be made.

For the mere purpose of ascertaining whether two or more bodies are precisely of the same temperature, or for stating generally in which of them the temperature is highest, the instrument is trustworthy enough. It is only when it is wanted to measure the thermal condition of bodies *quantitatively* that its indications can no longer be accepted. For instance we cannot be certain that a difference of temperature of one degree between say 32° and 33° in any body measured on a mercurial thermometer represents the same difference in its thermal condition as does a difference of one degree between, say, 200° and 201°. In other words, if we heat a certain quantity of water from 32° to 33°, and a similar quantity of water from 200° to 201°, we cannot by any means state that we have in each operation altered the thermal condition of the water by the same amount.

There are two reasons for this. The first has to do with the thermometer, and the second with the substance of which the differences of temperature have to be measured. It will be remembered that the way in which the length of degrees was arrived at when making the thermometer was by dividing the space between freezing and boiling point into 180 equal divisions, each of which was called one degree of temperature. Now in order that each of these degrees should represent an equal increase of heat of the mercury we should have to prove that if we add successive equal quantities of heat to the mercury, we thereby expand it by each operation by an absolutely equal quantity. Now we have no right to assume that this is the case, for the action of heat in causing some bodies to expand is known to be most irregular. If, for instance, the thermometer had contained water instead of mercury, then, commencing at freezing-point, it is known that the first effect of increasing the temperature is to cause the water to contract. This contraction would go on till the water had reached the tem-

perature of about 39°, after which further additions of heat would cause the water to expand. In the same way, careful experiments made with mercury have proved that its rate of expansion at high temperatures is considerably greater than at low ones, for equal increments of heat; consequently the errors in the high part of the scale become considerable.

The second reason has only to do with the substance the temperature of which has to be measured. Even assuming that our thermometer were quite perfect, we should still be unable to use it by itself alone to determine quantitatively the thermal condition of bodies; for the thermometer in the first instance shows only the temperature of its own mercury, and though its degrees might be so marked that each successive one would correspond with successive equal additions of heat to the mercury, still it does not follow that this would also be true of the substance whose temperature had to be ascertained. On the contrary, experiments on some substances. such as water, show that it takes more heat to raise their temperature by one degree at the high part of the scale than at the low part.

This last remark leads us at once to the object of all experiments on thermometry, viz. the measurement of quantities of heat. It might at first be supposed that the measurement of temperature was the same thing as the measurement of quantity of heat, but an easy experiment will prove that this is not the case. Take two vessels, one containing a pound of water and the other a pound of olive oil, each liquid having the temperature of the air of the room, say 55°. Take also two pieces of copper, each weighing a pound. For the purpose of the experiment sheet copper about one-sixteenth of an inch thick is best; and for convenience sake it should be bent round nearly to the form of a cylinder. Bring each of these pieces to a certain high temperature. This is best accomplished by boiling them for a short time in water, so that their temperature becomes 212°. Next, plunge one of the pounds of copper into the pound of

water, and the other into the pound of oil. The two liquids will of course receive heat, each from its own piece of copper, and they will therefore rise in temperature. Let the rise in temperature be carefully noted by means of two identical thermometers, one immersed in the water and the other in the oil. As soon as the mercury in the two thermometers has ceased to rise, we may assume that the pieces of copper have parted with their surplus heat, but it will be found that the temperature of the water is $68\frac{1}{2}°$, while that of the oil is nearly 92°. Here then we have a pound of copper at 212°, which is only capable of heating a pound of water, having the original temperature of 55°, up to $68\frac{1}{2}°$. In other words, while the copper has lost $212° - 68\frac{1}{2}° = 143\frac{1}{2}°$ the water has gained only $68\frac{1}{2}° - 55° = 13\frac{1}{2}°$. While in the case of the oil, the copper has lost $212° - 92° = 120$, and the oil has gained $92° - 55° = 37°$. Now the amount of heat lost by the copper in each case is of course exactly equal to that gained by the water in the one instance, and by the oil in the other; therefore it is evident that it takes less heat to raise the temperature of the oil by 37° than it does to raise that of the water by $13\frac{1}{2}°$, while the same quantity which suffices to produce this latter effect upon the water is competent to raise the temperature of the copper $143\frac{1}{2}°$.

These figures show conclusively how very differently the temperatures of different bodies are affected by different quantities of heat.

Specific Heat.—The amount of heat which a body of unit mass absorbs in order that its temperature may be raised by one degree; or, *vice versâ*, the amount of heat which the body parts with while its temperature is lowered one degree, is called its Capacity for Heat.

To compare this quantity for different bodies we must first fix upon some unit of quantity of heat. The unit generally adopted in Great Britain is the quantity of heat required to raise one pound of pure water from the temperature of 39° to 40°; or, *vice versâ*, the quantity of heat

parted with by the water in cooling from 40° to 39°. It is necessary thus to specify the temperature, because water, and indeed most bodies, have different capacities for heat at different temperatures. This quantity of heat is called the British Thermal Unit. The capacities for heat of other bodies are designated numerically, by comparing the quantities of heat necessary to raise their temperatures by one degree with unity.

The ratio of the quantity of heat required to raise the temperature of a body one degree, to the quantity required to raise an equal weight of water one degree, is called the Specific Heat of the body. Thus, if it take half the quantity of heat to raise one pound of ice from 20° to 21° that it does to raise a like quantity of water from 39° to 40°, then the specific heat of ice is said to be $\frac{1}{2}$ or ·5.

It is often necessary in questions connected with the steam engine to know how much matter at one temperature it will take, in order to raise a certain quantity of matter of another temperature to some third temperature. These questions are easily solved in the following manner:

Let M be the mass of one of the bodies.

„ M′ be the mass of the other body.

„ T° be the temperature of the body of mass M.

„ T′° be the temperature of the body of mass M′.

„ S be the specific heat of the body of mass M.

„ S′ be the specific heat of the body of mass M′.

When the bodies are mixed together the hotter of them will lose heat to the colder, till at last they attain some common temperature; and the quantity of heat lost by the one substance must be exactly equal to the quantity gained by the other, since the total quantity of heat remains unchanged. Let the body of mass M be the hotter of the two, and let the common temperature which both attain when mixed be called θ°. Then, the quantity of heat lost by one pound of the hotter body in cooling from T° to θ°$=$S (T°$-\theta$°), and consequently the quantity lost by M

pounds, is M.S $(T^\circ - \theta^\circ)$. Similarly, the quantity gained by the other body is $M'.S'\ (\theta^\circ - T'^\circ)$, and since these two quantities are equal, we obtain the equation

$$M.S.\ (T^\circ - \theta^\circ) = M'.S'.\ (\theta^\circ - T'^\circ).$$

This equation is only true provided the whole effect of heat upon bodies is the changing of their temperatures; but it is known that this is not the only effect. We shall afterwards see that a large quantity of heat may be added to bodies without changing their temperatures in the least, and that its effect is in these cases to change the state of constitution of the body; as, for instance, when ice of 32° is changed into water of 32°, or water of 212° changed into steam of 212°. For such cases, therefore, as the mixing of ice and water together, the above equation does not hold good. The equation is also only true on the supposition that the specific heat of bodies is the same at all temperatures. This also is not, strictly speaking, true, but for ordinary purposes the error thus introduced may be neglected.

Mechanical Equivalent of Heat.

Having thus examined the question of the measurement of heat, we are now in a position to revert to the subject of the equivalence of heat and energy. What we want to know is, firstly, how much heat can be got by the doing of a certain quantity of work. The converse question, viz., how much work can be got out of a certain quantity of heat, is of a more complicated character, and its discussion must be postponed till the following chapter.

The first question was accurately settled experimentally by Dr. Joule, of Manchester. He conducted an immense number of experiments on the friction of various solids and liquids, and on the compression of gases. His experiments on the friction of fluids were carried out in the following way. The work was done by causing a known weight to

descend through a given distance; the weight during its descent caused, by means of a suitable mechanism, a paddle to revolve inside a closed vessel filled with the liquid to be experimented upon. This paddle, by agitating the contents of the vessel, produced friction between the particles of the liquid, the walls of the vessel and the paddle, which friction would of course be converted into heat, and would raise the temperature of the vessel and its contents. Careful allowance was made for the resistance caused by the friction of all mechanism exterior to the vessel, and also for all the heat which escaped into the sides of the vessel or into the air. The temperature of the liquid was carefully noted, first before the experiment commenced, and next after the weight had descended through a given distance. The rise of temperature multiplied by the mass of liquid, multiplied by its specific heat, gave, after making all allowances, the quantity of heat generated by the descent of the weight.

The result was that a quantity of work represented by 772 foot-pounds is capable, when all converted into heat, of raising the temperature of a pound of pure water, weighed in vacuo, and having the temperature of 50°, through one degree Fahrenheit.

In other words: *The British Thermal Unit is equivalent to 772 foot-pounds of work.* This number is called the mechanical equivalent of heat.

This result has been fully confirmed by numerous other experiments, made on various substances and in various ways, and it constitutes by far the most important practical discovery which has yet been made in the science of heat. Another way of putting the above result is this. If a pound of water be allowed to fall in vacuo down a height of 772 feet, and if all the heat generated by its impact at the end of its descent be collected into the pound of water, its temperature will be raised one degree.

The equivalent of the units of mechanical work in thermal units can now be readily expressed. For example,

one foot-pound of work is equivalent to $\frac{1}{772}$nd part of a thermal unit. One horse-power exerted for a minute, or 33,000 foot-pounds, is equivalent to 42·74 thermal units.

It might at first be supposed that if by doing 772 foot-pounds of work on a pound of water, we thereby raise its temperature one degree, the converse of this must also be true, viz. that by cooling a pound of water by one degree we should thereby be enabled with a suitable apparatus to do 772 foot-pounds of work. It will be seen hereafter that this is not possible; but before this question can be thoroughly understood we must examine into the effect of heat upon gases and water, as it is in general through the medium of these substances that heat is converted into mechanical effect.

The Effect of the Application of Heat to Gas.

The effect of heat upon water has more to do with the subject-matter of this book than has its effect upon gases; but as steam is an imperfect gas, and as the laws which govern the behaviour of gases are much simpler than those for water and steam, we will commence with the subject of gases.

Gases differ from solids and liquids in that they have no tendency to keep to any fixed form and volume. A small portion of gas if introduced into a closed empty vessel will at once expand, so as to fill the whole of it, and will press with a certain equal pressure against every equal portion of the containing surface of the vessel. If by any means the vessel be enlarged, the gas will expand still further, so as to fill it completely as before, but its density or weight per unit of volume will be less, and the pressure which it exerts against the sides of the vessel will also be less.

Take for instance a cylinder closed permanently at one end, and containing a movable piston, by means of which the volume of the portion below the piston can be changed

at pleasure. Let the area of the horizontal section of the cylinder be one square foot, and let the piston, which is supposed to be without weight, be placed at a height of one foot above the bottom of the cylinder, so that the space beneath it is filled with one cubic foot of gas or air at the ordinary temperature and pressure (say 14·7 lbs. per square inch) of the atmosphere. The volume of the air is now one cubic foot ; and its pressure is 14·7 lbs. per square inch, or 2116·8 lbs. per square foot. Next, let the piston be raised by hand to a new position, two feet from the bottom of the cylinder, the temperature of the inclosed air being maintained constant. The volume of the air is now doubled or two cubic feet. If we had a proper instrument for measuring pressures, we should find that the pressure per square foot is only half what it was before, or 1058·4 per square inch, and as the same weight of air now occupies twice the original bulk, its weight per cubic foot is halved If instead of raising the piston we had weighted it, or pushed it down to within half a foot of the bottom of the cylinder, and if we had taken care to keep the temperature of the inclosed air constantly the same, we should thus have halved the volume, which would now be half a cubic foot ; and doubled the pressure to 4233·6 lbs. per square foot, and also doubled the density or weight per cubic foot.

These facts are expressed generally by Boyle's law of the pressure and volume of gases, which is as follows :

The volume of a portion of gas varies inversely as the pressure, so long as the temperature is constant.

This law may be expressed in other words as follows:

The product of the pressure multiplied by the volume of a portion of gas is a constant quantity, so long as the temperature is constant.

If we remember, that in the above experiment the weight of the gas per cubic foot diminished in proportion as the volume increased, we may express the law otherwise thus :

The pressure of a gas is proportional to its density.

If P represent the pressure in any units, say in pounds, per square foot, and V represent the volume in cubic feet, and C is a constant quantity, to be determined experimentally for one particular case, then the algebraical expression for Boyle's law is

$$P \times V = C \quad \text{or} \quad P = \frac{C}{V}.$$

Graphic Representation of Boyle's Law.

The law may be exhibited graphically in the following manner. Take a line OV, fig. 11, along which volumes are measured to any scale; and a line OP at right-angles to OV, along which pressures are measured, likewise to scale. Now, reverting to the former experiment, measure off along OV a distance Oa to scale, representing the original volume of the air beneath the piston, viz. one cubic foot, and draw aA at right-angles to Oa so as to represent to scale the original pressure, viz. 2116·8 lbs. per square foot, then the product of this volume and pressure is represented by the area of the rectangle OaAβ, which is the constant quantity in the above equation. Next draw Ob to represent the volume in the second stage of the experiment, viz. two cubic feet, and bB to represent the corresponding pressure, viz. 1058·4 lbs. per square foot. Then the product of these two quantities is represented by the rectangle ObBa which is equal to the original rectangle OaAβ because its base Ob = 2Oa, while its height bB = $\frac{1}{2}a$A. Next, for the third stage, measure Og equal to half a cubic foot, and gG equal to twice the original pressure, or 4233·6

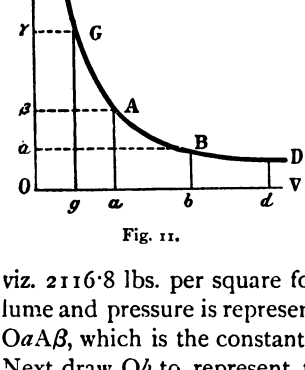

Fig. 11.

lbs. per square foot. Then the product of this pressure and volume is represented by the rectangle OgGγ, which likewise is equal to the original rectangle. In a similar way we can obtain the point D such that the rectangle $Od \times d$D equals the original rectangle, and similarly any other number of points. Now, a curve drawn through the angles G A B D &c. of any number of rectangles of equal area, arranged as in the figure, is called a rectangular hyperbola; and not merely for these points, but also for every other point along the curve, the condition holds good, viz. that the rectangle formed by drawing perpendiculars to the lines OV and OP is equal to the original rectangle.

If we designate all lines measured parallel to OV by the symbol v, and all corresponding lines measured parallel to OP by the symbol p, then it is evident that the curve GABD is expressed by the equation $pv=c$, in which c represents the area of the original rectangle. This equation corresponds exactly, in form, with the equation used to express Boyle's law. The lines OV and OP are called respectively the axes of volume and of pressure; they are also what are called the asymptotes of the hyperbola GABD. The lines drawn from any point on the curve, perpendicular to the axes, are called the ordinates of the point.

Isothermal Lines.

We see, therefore, that the volume and pressure of a portion of gas, when expanding or being compressed, the temperature remaining always the same, may be represented graphically by the ordinates of an hyperbola. Such an hyperbola as has just been drawn is called an *isothermal line of expansion or compression of a gas*, or briefly an *isothermal*. This term, which is derived from two Greek words signifying *equal* and *heat*, signifies that the temperature is constant throughout the expansion or compression represented by the line.

The system of representing graphically the varying pressures and volumes of a portion of gas, by means of a line should be carefully marked, for it is, as we shall afterwards see, the basis of the graphic representation of the working of all gas or steam engines, called indicator diagrams. The method may also be used in many theoretical investigations connected with the expansion of gases and steam under different circumstances, in place of complicated algebraical expressions, or also to supplement and explain these latter.

Effect of varying the temperature of a gas.—Let us now revert to the cylinder of the previous experiment, the piston being, as before, one foot above the bottom, and the air beneath it being at the pressure of the atmosphere, but at the temperature of melting ice. Let us now heat the air by any means so as to bring it up to the temperature of boiling water. During the rise of temperature the piston will gradually ascend, and when the temperature of 212° has been attained, it will have risen to ·366 feet or 4·39 inches above its original position, and will remain there so long as the temperature is maintained at 212°, and the other circumstances continue unchanged.

If the piston had not been allowed to rise, but the air beneath it had been heated as before up to 212°, its pressure would then have increased from 1 to 1·3654 atmospheres—i.e. from 14·7 to 20·08 lbs. per square inch. During the first of these operations the air is said to be heated at constant pressure, and during the second at constant volume.

The fact that air, and indeed all gases, increase in bulk when heated from freezing to boiling point by a fixed fraction (which is nearly the same for every gas) of their original volumes at freezing-point, was first discovered in France by Charles. Hence the numerical statement of the amount of the expansion is usually called Charles' law. The dilatation of gases was afterwards investigated more completely by Dalton in this country, and by Gay Lussac in France; hence

the statement is also often called the law of Dalton and of Gay Lussac. Though both these philosophers agreed as to the total increase in the bulk of a gas when raised in temperature from 32° to 212°, there was, nevertheless, an important difference between them in investigating the increase for each individual degree between these two temperatures.

Gay Lussac's view was that the increase for each degree is a certain fixed quantity, which quantity is a definite fraction of the volume at any temperature. Thus, if V_0 be the volume of the gas at 0°, and V the volume at any other temperature $t°$, and if a be the fraction of V_0 by which the volume is increased for each degree of rise of temperature (commonly called the coefficient of expansion), then, according to Gay Lussac,

$$V = V_0 + t.a.V_0 = V_0(1 + ta).$$

The total increase in volume between 32° and 212° is, as stated above, in the ratio of 1 to 1·3654, and the increase per degree is therefore ·3654 ÷ 180 = ·00203. If the original volume be taken at 0°, the corresponding increase per degree = ·00217.

In a similar manner, if the volume of a gas be kept constant and it be heated from 32° to 212°, the pressure will be increased in the ratio of 1 to 1·3654, and the increase per degree of temperature will be as before, ·00203. This experimental result can also be deduced by the aid of Boyle's law from the known increase of volume when the temperature is raised and the pressure kept constant. For, let the pressure be P and the original volume at 32° be V. When heated to 212° at constant pressure the volume becomes V × 1·3654. Let the gas now be compressed at the constant temperature 212° back to its original volume, and the pressure by Boyle's law will become $P \frac{V \times 1{\cdot}3654}{V}$ = P × 1·3654.

According to Dalton, however, if a be the coefficient of

expansion for a rise of one degree from 32° to 33°, so that the volume at 33° $=V+aV$; then at 34° the volume becomes $(V+aV)+a\,(V+aV) = V\,(1+a)^2$, and at 35° it is $V\,(1+a)^3$ and so on. In other words, a gas at a given temperature, say 50°, for a rise of one degree, say to 51°, increases in bulk by a fixed fraction of its volume at 50° and not of its volume at 32°—as stated by Gay Lussac—and so on for any other temperature. It will be seen that there is a most important difference between the two laws, and many of the theoretical calculations relating to steam would be largely modified by the adoption of Dalton's instead of Gay Lussac's statement. It might have been thought that the question of accuracy between the two could be easily settled by experimental investigation, and the subject has naturally received much attention at the hands of Regnault, Stewart, and others. It is, however, not possible absolutely to prove the truth of either law experimentally, because, as we have seen, no accurate experimental method exists of measuring temperature quantitatively. The experiments above referred to are, however, generally accepted as proving that Gay Lussac's statement is the nearer to truth of the two, and, in conformity with this generally received opinion, his law will be made use of throughout the remainder of this book.

It should here be stated that both Boyle's and Gay Lussac's laws are only true for perfect gases. For actual gases Regnault has found :

1. That the product $P \times V$ is not quite constant, especially for those gases which can be liquefied.

2. That the coefficients of expansion of air and other simple and compound gases are not quite identical.

3. That the coefficients of expansion of all gases, with the exception of hydrogen, increase somewhat as their density increases.

4. The coefficients of expansion of all gases become more nearly equal to each other as their pressures diminish.

The Air Thermometer.

One of the most important practical deductions from the above laws is the construction of the air thermometer. There are several reasons why air, or any other gas, is a better substance for measuring temperature than any liquid or solid. Foremost among these is the fact, that no two different liquids or solids will agree when used for measuring temperatures other than the two originally selected temperatures, which are invariably those of freezing and boiling water. On the other hand, air and all gases, when at the same pressures, will expand by almost exactly identical amounts when subjected to the same temperatures. Another reason for preferring gases to other substances for thermometrical purposes is that the specific heats of gases remain the same at all temperatures.

Without going into the question of air thermometers in actual use, a very simple theoretically possible form of such an instrument will now be described for the purpose of illustrating what is meant by *absolute temperature.*

Take a tube with a uniform bore, and inclose in it a portion of air or other gas, in such a way that, at the temperature of melting ice, the air will occupy a space of one foot measured from the bottom of the tube and will be separated from the external atmosphere by an air-tight piston, which, however, is

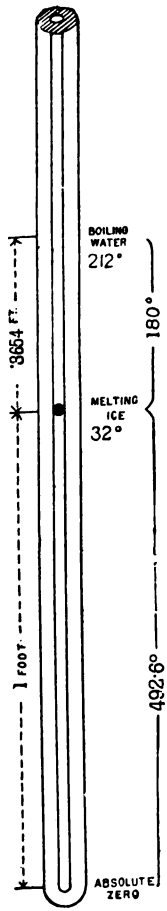

Fig. 12.

free to move up and down the bore of the tube (see fig. 12). The pressure on the inclosed gas is to be maintained constant. If, now, the tube be exposed to the steam of boiling water, the inclosed air will expand, causing the piston to rise to a point 1·3654 feet above the end of the tube. A mark at this point will indicate the temperature of 212°. If we divide the space between this and freezing-point, viz. ·3654 feet, into 180 equal divisions, each of these will, if Gay Lussac's law be true, represent one degree Fahrenheit. Hence we see that for each degree of rise of temperature the volume of the gas inclosed in the thermometer increases by a fraction of its original volume at $32° = \frac{·3654}{180} = ·00203$.

Absolute Temperature.

If we choose we can now extend these subdivisions below freezing and above boiling point as far as we like. As the space between freezing and boiling points, viz. ·3654 feet, contains 180 divisions, the space below freezing-point to the bottom of the tube, viz. one foot, will contain 492·6 divisions or degrees. In other words, the bottom of the tube is $492·6° - 32° = 460·6°$, or in round numbers, 461° below zero. As we shall see hereafter, great use is made in thermodynamical calculations of the above fact.

This point has been called the *absolute zero of temperature*, and the first conclusion which would be deduced from the foregoing would be, that by depriving a portion of gas of all its heat we should thereby reduce its bulk to nothing. It is needless to say that all reasoning as to the condition of a gas when deprived of all its heat is mere speculation, as we have no experimental knowledge of the subject whatever. Dalton's law, it will be observed, would lead to quite other conclusions. It is, however, very convenient in calculations respecting gases to measure tempera-

ture from the absolute zero, instead of from the zeros of any of the scales in common use. Temperature thus measured is called *absolute temperature.* To convert temperature measured on Fahrenheit's scale to absolute temperature, we have only to add 461° to the reading. Thus the absolute temperature of boiling water would be $461° + 212° = 673°$.

Combination of Boyle's and Gay Lussac's laws.—Let V_0 be the volume of a portion of gas at the pressure p and the temperature 0°.

Let V be the volume at any other temperature $t°$, the pressure remaining p.

Then by Gay Lussac's law $V = V_0(1 + at) \therefore V_0 = \frac{V}{1 + at}$.

Now by Boyle's law the volume V', at any pressure p', and temperature t', multiplied by its pressure p', equals the volume at pressure p, and same temperature t', multiplied by its pressure p,

or, $V'.p' = V_0(1 + at').p = V.\frac{1 + at'}{1 + at}.p$.

Now by reference to the description of the air thermometer and the definition of the absolute zero, it will be seen that $1 + at'$ and $1 + at$ bear the same relations to t' and t that the absolute temperatures do to the temperatures on the Fahrenheit scale; therefore writing τ' and τ for the absolute temperatures instead of $1 + at'$ and $1 + at$, we get the very simple equation

$$\frac{V'p'}{\tau'} = \frac{Vp}{\tau},$$

or in other words, *the product of the volume and pressure of a portion of gas is proportional to the absolute temperature.*

The above equation will be found of the greatest use in solving all questions as to the varying volumes, pressures and temperatures of a portion of gas when its condition at any one temperature is given.

The same result might have been arrived at in a simpler

way by the mere inspection of the air thermometer, for this latter is nothing more nor less than the record of a series of experiments on the varying volume of a portion of gas when the pressure remains constant and the temperature changes. The varying volume is in fact the exact measure of the varying temperature. Hence the product of the pressure and volume is exactly proportional to the length of the column of gas—i.e. to the absolute temperature. Now there is no reason why any one pressure should be chosen rather than any other, hence the above statement is perfectly general.

The specific heat of gases.—The specific heat of a gas, is, in accordance with the definition on page 39, the ratio of the amount of heat required to raise a given weight of it one degree in temperature, to the amount required to raise the same weight of water one degree. In heating a gas, it is possible to do so when the volume is kept constant, or the pressure constant, or partly in the one way and partly in the other. The first two cases are the most important. It will be seen that, if the mechanical theory of heat be true, it will take more heat to raise the gas one degree in temperature when the pressure is kept constant while the volume is variable than in the reverse case. For, take a cylinder closed at one end, having a section of one square foot, and confine a cubic foot of gas in it, by means of a piston free to move. The cubic foot of gas has then to sustain the pressure due to the weight of the atmosphere plus the weight of the piston. When, therefore, the inclosed gas is heated and expands, the whole weight of the atmosphere and piston is raised through a certain space, and work is done. Consequently, the heat supplied to the gas is partly expended in raising its temperature, and partly in doing external work. Now, in the case of heating at constant volume, the weight of the atmosphere is not raised at all, and no external work is done, and therefore less heat is required in this than in the former case.

The ratio which these two specific heats of a gas bear to each other can be easily ascertained. Reverting to the cylinder, let the cubic foot of gas be at the temperature of 32°, and let it be heated till its volume is doubled. To do this the temperature must be raised, according to Gay Lussac's law, by 492·6°.

Now, according to Regnault's experiments the specific heat of gas—say air—is, when heated at constant pressure, 0·2375, therefore the quantity of heat in thermal units required to effect the above operation is equal to the original weight of the cubic foot of air at the atmospheric pressure of 14·7 lbs. per square inch, and temperature of 32°, multiplied by the rise in temperature, multiplied by the specific heat. Now the weight of the cubic foot of air, under the above circumstances, is ·0807 lb. The rise of temperature is 492·6° and the specific heat is 0·2375 ; therefore, the quantity of heat required is,

$$\cdot 0807 \times 492 \cdot 6 \times 0 \cdot 2375 = 9 \cdot 422 \text{ thermal units.}$$

Now the external work done during the process is equal to the weight of the atmosphere, viz. 14·7 lbs. per square inch, resting on the surface of the piston, viz. 144 square inches, multiplied by the height through which it is raised viz. one foot.

$$= 14 \cdot 7 \times 144 \times 1 = 2116 \cdot 8 \text{ foot-pounds.}$$

Now 2116·8 foot-pounds is equivalent to $\frac{2116 \cdot 8}{772} = 2 \cdot 74$ thermal units. Therefore, of the above quantity of thermal units 2·74 have been expended in doing external work, and

$$9 \cdot 422 - 2 \cdot 74 = 6 \cdot 682 \text{ thermal units}$$

represent the quantity of heat expended in raising the temperature of the air, provided that no other effect has been produced by the heat.

Now, before we can establish a ratio from the above data between the two specific heats, we must ascertain whether

heat has been expended in any other way than in doing external work, and in raising the temperature of the mass of gas. It will be noticed, that the gas, at the end of the above experiment, is in a different molecular condition to what it was at the commencement, the particles composing the gas being much further apart. It might be thought that part of the heat was expended in effecting this molecular change. Whatever may be the case with other substances, it has been experimentally proved by Joule that no heat is expended in this way in the case of the permanent gases. The experiment consisted in warming a mass of gas in a closed vessel, and then, by opening a cock, allowing the confined and heated gas to expand into another vessel, in communication with the first, and which was in a condition of vacuum. In this case the gas evidently expanded without doing any work; therefore, no heat was consumed in this way. At the end of the experiment the temperature was found to be unchanged, showing that no heat had been expended in altering the distance apart of the particles.

Reverting now to the original experiment, we see that the only effects produced when the gas was heated at constant pressure were:—1. The doing of external work; 2. The raising the temperature of the mass of the gas; and 3. The further separation of the particles of the gas. We have seen that the last effect required no expenditure of heat at all, and consequently we may be sure that the above-named quantity of 6·682 thermal units represents the amount of heat expended in raising the temperature of the gas.

If, now, the mass of gas be heated at constant volume instead of at constant pressure, no external work is done, and no separation of the particles takes place, and the whole heat is expended in raising the temperature of the gas. The quantity of heat required for this purpose is, as we have seen, only 6·682 thermal units. The specific heat of the gas at constant volume is, therefore, less than the specific heat at con-

stant pressure in the ratio of 6·682 to 9·422, and consequently the numerical value of the specific heat at constant volume is

$$0{\cdot}237 \times \frac{6{\cdot}682}{9{\cdot}422} = 0{\cdot}167.$$

Our knowledge of the specific heats of various gases is derived chiefly from Regnault's experiments, which were all made upon gases at constant pressure. The experimental determination of the specific heat at constant volume is a very difficult operation, hence it is usually derived theoretically from the specific heat at constant pressure in the manner explained above. Though the true value of the ratio of the two specific heats has not been confirmed by direct experiment, still it has been assumed in calculations, such as the theoretical determination of the velocity of sound in air; and as the calculated velocity agrees practically with the result of experiment, we may assume that the value for the ratio has been indirectly confirmed by experiment.

The Effect of the Application of Heat to Water.

Having thus briefly considered the effect of heat when applied to gases, we must now consider its effect on water. This latter is the more complicated subject of the two, but is at the same time of the greater importance in the study of the steam engine.

It will be convenient to commence with the solid form of water or ice at the temperature of 0° F. In this condition the specific heat is about 0·5, that is, it takes half as much heat to raise a pound of ice one degree in temperature that it does to raise the same weight of water by the same amount. To raise the temperature of the ice up to 32° requires then $32 \times 0{\cdot}5 = 16$ units of heat. At this temperature its volume compared with water at its maximum density is as 1·0908 to 1.

If we continue to heat the pound of ice at 32° it will begin to melt, but the temperature will remain stationary

till the whole of the ice is turned into water. To effect this transformation 144 units of heat must be supplied, equivalent to $144 \times 772 = 111{,}168$ foot-pounds of work. In other words, it would require as much heat to raise a pound of ice at 32° through a height of *about* 21 *miles*, as it does to convert it into water of 32°. As the temperature remains stationary during the melting process, the question arises—what becomes of the heat which has been expended. The early discoverers of this phenomenon, being unable to account for the heat thus apparently lost, invented the theory that it had become *latent*, or concealed in the water, and, in accordance with this theory, it was said that the *latent heat* of water was 144°. In accordance with the mechanical theory, it is recognised that the heat thus expended is spent in doing internal work on the particles of the ice, which results in their cohesion being overcome so that the condition of the ice is changed from the solid to the liquid state. We should say, therefore, that 144 units of internal work, or of latent work, are done upon the ice in order to transform it into water. The term, internal or latent work, is used in contradistinction to the external work, which, as we have seen before, a body can perform when increasing in volume under the influence of heat.

It should here be noticed how enormous are the internal forces which have to be overcome in changing the molecular condition of the ice from the solid to the liquid state. The 111,168 foot-pounds of work necessary to effect the transformation in a pound of the substance being equivalent to about 595 tons raised one inch high. At the same time, the volume of the water is slightly less than that of the ice from which it was formed, being in the ratio of 1·0908 to 1·000127, thus showing that there is no great change in the distance apart of the molecules.

If we continue to apply heat to the water, its effect is to raise the temperature till the boiling-point is reached. This

point being 180° above the temperature of melting ice, the number of units of heat required to accomplish this rise in temperature would, if the specific heat were unity throughout the whole process, be 180. As, however, the specific heat is only unity at 39·1°, and after that increases slightly with the temperature, the actual number of heat units required has been found to be 180·9. During this rise in temperature the volume of the water decreases slightly to 39·1°, the point at which it is reckoned unity, and after that increases with the temperature till it reaches 1·04315 at 212°.

The further application of heat to the water will not increase its temperature, so long as the pressure is that of the atmosphere, but will only result in the formation of steam of atmospheric pressure. In other words, the water will boil and will continue to do so till it is all turned into steam. To effect this change from water of 212° into steam of the same temperature, 965·7 units of heat are required, equal to 745,520 foot-pounds. That is to say, to turn a pound of water having the temperature of 212° into steam of atmospheric pressure, heat has to be supplied to it, equivalent to the work involved in raising the weight of water up to a height of about 146 miles, or, in raising about 346 tons a foot high. When the whole of the water is turned into steam its volume is about 1650 times that of the water from which it was formed, and in this state it is called dry saturated steam, and in many of its qualities it resembles a gas. Its temperature is the same as that of the water from which it was formed, viz. 212°.

If more heat be added, the pressure remaining that of the atmosphere, the temperature of the steam will rise, and it will become what is called superheated, which means that it is of higher temperature than the water from which it was formed. The specific heat of steam is only 0·4805, so that for every unit of heat now supplied to it the temperature will rise 2°·08, and the volume will also increase directly as the absolute temperature.

Just as in the case of the liquefaction of ice, so with the vaporisation of boiling water, the 965·7 units of heat which have been supplied for this purpose, and which produce no effect on the thermometer, have all been expended in doing work. Part of the work so done is internal or latent work expended in overcoming the molecular resistances of the water, and part is expended in doing external work against the pressure of the atmosphere. To make this point clear, and to show how much heat is spent in each of these sorts of work, suppose a cubic foot of water at the temperature of 212° to be inclosed in a cylinder of indefinite length and of one square foot in section. Suppose the water to be covered in by a piston without weight, and free to move. The pressure of the air on this piston will be 14·7 lbs. × 144 = 2116·8. The cubic foot of water weighs 62·42 lbs. To turn it into steam requires, therefore, 62·42 × 965·7 = 60279 units of heat, equivalent to 46,535,388 foot-pounds. At the end of the process the piston is lifted 1,650 feet from the bottom of the cylinder, or 1,649 feet from its original position. The external work done is, therefore, this height multiplied by the pressure on the piston, or 1649 × 2116·8 = 3,490,603 foot-pounds ; while the internal or latent work is equal to the total minus the external work = 46,535,388 − 3,490,603 = 43,044,785 foot-pounds. The external is to the internal work, therefore, in the ratio of 1 to 12·33 nearly.

Suppose now the piston in the above experiment to be loaded with some other weight in addition to that of the atmosphere, say with another 14·7 lbs. to the square inch, and that heat be applied as before. The result would be that the water would rise in temperature to nearly 249° before steam began to form. When it did form, the steam would have the same temperature as the water, viz., 249°, and the same pressure as the piston sustains, viz. 29·4 lbs. per square inch. When the water was all turned into steam, the piston would have risen to a height of 858 feet above the cylinder bottom, i.e. to 857 feet above its original po-

sition. The quantity of heat requisite to effect this transformation is found by experiments to be 1157·85 units for each pound of water, whereas in the last example only 965·7 + 180·9 = 1146·6 units were required, thus showing an increase of 11·25 units. Also the way in which the heat is expended is different. For instance, in the second example the final temperature of the water is 249° instead of 212°; consequently, the heat expended in raising it from 32° is about 218·4 units against 180·9. The heat expended in merely vaporising the water is 1157·85 − 218·4 = 939·45 in the second example against 965·7 in the first, showing, therefore, a decrease of 26·45 units. Now, of these 939·45 units, a certain quantity is spent in doing external work, against the load on the piston. The whole heat thus expended on the cubic foot of water = 144 × 29·4 × 857 = 3,628,195 foot-pounds = 4699·7 thermal units. Consequently, the internal or latent work done per pound of water $= 939{\cdot}45 - \frac{4699{\cdot}7}{62{\cdot}42}$ = 864·15 units; and the ratio of the external to the internal work is 1 to 11·47 instead of 1 to 12·33, as in the first example.

We see, therefore, that by increasing the load on the piston we have changed everything, viz. the temperature at which the water boils; the temperature, pressure, volume, and consequently density of the steam; the total heat necessary to effect the change, and also the proportions of the heat which are expended in raising the temperature of the water, in vaporising it and in doing external and internal work.

The laws which regulate many of these changes are not yet perfectly understood, and consequently at present only empirical formulæ are available to express them. The formulæ are founded upon the results of experiments which have been carried out in the most exhaustive manner. The results of these experiments are recorded in tables, so that the student is, except for the purpose of analytical calculation, rendered independent of the formulæ.

Connection between pressure and temperature of steam.—The connection between the pressure and temperature of steam was determined by Regnault, and the numerical results are given in the Table, page 489, transformed into English measure for every degree between 100° and 401° Fahrenheit. Regnault's experiments were made at pressures varying from 3 lbs. to 200 lbs. per square inch. It will be seen by studying the table that the pressure increases with the temperature, but not in a uniform manner as in the case of gas. For instance, starting at 212°, the pressure is 14·7 lbs. per square inch and the increment of pressure per degree of rise of temperature is 0·29 lb. At 300°, however, the pressure is 67·22 lbs. and the increment of pressure per degree is 1 lb.; while at 408° the pressure is 270·99 lbs., and the increment of pressure per degree is 3 lbs.

Total heat of steam.—The connection between the temperature of the steam and the total quantity of heat required to raise the water from 32° and vaporise it was also determined by Regnault, and the numerical results are given both in thermal units and in foot-pounds in col. 4 of the Table. It will be seen that the total heat increases with the temperature, the rate of increase being about 0·305 of a thermal unit for each degree above 212°, so that if 1146·6 units is the total heat of one pound of steam at 212°, and if we want to know the total heat at any other temperature $t°$, it will be given by the expression

$$\text{Total heat} = 1146{\cdot}6 + {\cdot}305\ (t° - 212°).$$

Heat of vaporisation of steam.—The heat of vaporisation, as distinguished from the total heat, is easily calculated, if we know the total heat, by subtracting from this latter the number of units of heat required to raise the water from 32° to the boiling-point; see col. 3 of the Table. The Table shows that the heat of vaporisation diminishes as the temperature of the steam increases, but not by a constant rate. The rate of diminution increases with the temperature.

When a table is not at hand, very approximate results can be obtained by assuming that the rate of diminution is 0·71 of one thermal unit for every degree above 212°. Thus for steam of $t°$ temperature,

$$\text{Heat of vaporisation} = 965{\cdot}7 - {\cdot}71\,(t° - 212°),$$

965·7 units being the heat of vaporisation of steam of 212°.

Volume of steam.—The heat spent in external work depends of course on the volume which the steam occupies when formed. Experiments have also been made upon this point. The volume in cubic feet occupied by a pound of water when turned into steam is called the *specific volume* of the steam. The term *relative volume* is used to denote the comparison between the volume occupied by the steam, and that occupied by the water from which it is formed.

Connection between pressure and volume of steam.—The weight in pounds of a cubic foot of steam is called its density. In the case of a gas the connection between the pressure, the volume, and the density is, as we have seen, extremely simple. The equation pv = constant, giving the connection between pressure and volume, while the density is exactly proportional to the pressure. In the case of steam, the relationship is not so simple. No rational formula has ever been devised to express the relationship, but experiments have been made for each separate case, the results of which are given in col. 5 of the Table. An empirical formula has been given by Rankine, which very nearly gives the results of the experiments on pressure and volume, and is of the same form as pv = constant. Rankine's formula, connecting the pressure and volume of steam, is as follows,

$$pv^{\frac{17}{16}} = \text{constant}$$

where p is the pressure in pounds per square inch, and v is the volume in cubic feet, the value of the constant being 475.

External work done during vaporisation of water.—This formula enables the external work done during the vaporisation of water to be calculated, but except where it is necessary to use a formula in analytical investigations, the figures are best taken from the Table by multiplying the volume as given in col. 5 by the pressure per square foot. It will be noticed on studying the results that the external work done increases slowly with the temperature, but not by a uniform rate of increase. The rate diminishes as the temperature rises.

Internal work done during vaporisation.—The heat expended in doing internal or latent work during vaporisation, in altering the molecular constitution of the water, is the difference between the heat of vaporisation and the heat expended in doing external work. It may be deduced from the Table by subtracting the external work, plus the heat expended in raising the temperature of the water as given in col. 3 from the total heat as given in col. 4. It diminishes with the temperature by about 0·792 unit for every degree.

The heat necessary for turning water of 32° into steam, at constant pressure, is expended in the three following ways, which must be kept distinct from one another.

1. In raising the temperature of the water from 32° to the temperature of the boiling-point, which last depends upon the pressure.

2. In changing the physical constitution of the water from the liquid state to the condition of steam. This is what has been called above internal or latent work.

3. In doing external work, by overcoming the resistance of the atmosphere, or other external resistance through a certain space, corresponding to the volume which the steam occupies at the particular pressure.

It should here be noticed that when steam is formed in a boiler, in connection with a non-expansive engine at work, it is generated under the condition of nearly constant pressure; the piston which is constantly moving backwards

and forwards in a cylinder which is in communication with the boiler, corresponding to the piston, in the example given above, while the forces which in a steam engine oppose the motion of the piston correspond to the weights placed upon the piston in the example. The case of steam formed in a close vessel is different, for here no heat has to be expended in doing external work, for by the nature of the case none can be done.

The relative proportions of the three separate quantities of heat necessary to raise a pound of water from 32° to boiling temperature, and then to evaporate it, may perhaps best be exhibited by a graphical diagram. Draw a line OX (fig. 13) along which to measure the volume of one pound of water when turned into steam. Suppose the water as before contained in a cylinder having a section of one square foot.

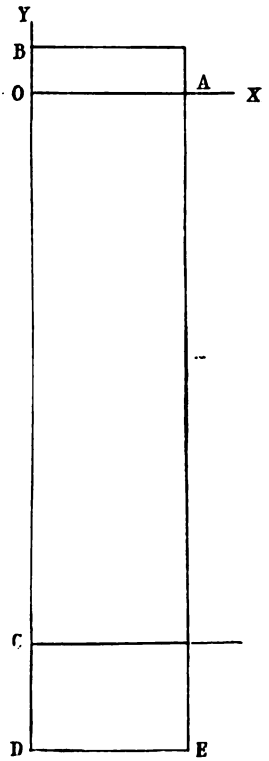

Fig. 13.

Draw a line OY along which to measure the total pressure in pounds on the piston. Let us take for the first illustration steam of 30 lbs. pressure to the square inch, the temperature of which, according to the Table, is about 250°, and the specific volume 13·49 feet. Now the original volume occupied by the pound of water is 0·016 cubic foot, therefore the space through which the steam lifts the piston when doing work is 13·49—·016=13·474 feet. Measure off a length OA to

scale along OX to represent 13·474 feet, and a length OB along OY to represent to scale the pressure on the piston $=144\times30=4320$ lbs. Then the exterior work done by the steam when formed is $=4320\times13\cdot474=58207\cdot68$ foot-pounds, and this is represented on the diagram by completing the rectangle AB, the area of which is of course OA × OB, and which therefore represents the above number of foot-pounds. Similarly the heat spent in internal work and in raising the temperature of the water may be represented by the areas of rectangles. For the sake of comparison these rectangles should have the same base OA as the original rectangle, and should be drawn below OX. Now the heat spent in internal work may be calculated from the Table to be 680,021 foot-pounds, which is about 11·6 times as much as the external work. Draw therefore OC downwards at right angles to OA and in length 11·6 times as much as OB. The area of the rectangle AC will then represent the heat calculated in foot-pounds expended in internal work.

Similarly the heat expended in raising the temperature of the water from 32° to 250° can be represented. This heat is 219·5 thermal units=169,454 foot-pounds, which is about 2·91 times as much as the heat spent in external work. From C therefore draw CD downwards, in length equal to 2·91 times OB, and complete the rectangle. Its area will then represent the amount of heat calculated in foot-pounds required to raise the temperature of the water from 32° to 250°.

An inspection of this diagram will show what a very wasteful kind of steam engine such a cylinder and piston would constitute; for the whole of the work done by the heat expended is represented by the rectangle AB, while the whole heat supplied is represented by the big rectangle BE, which is 15·51 times the area of AB. Therefore for every 15·51 units of heat supplied to such an engine only one unit can possibly be expended in doing useful work.

By constructing, with the aid of the tables, similar diagrams for every pressure of steam, we should be able to

see at a glance the proportion between heat supplied and useful work done.

Expansion of Gas and Steam.

Isothermal lines.—Boyle's law, connecting the volume and pressure of gas, viz. pv=constant, assumes that during the variation of pressure and volume the temperature remains constant. Suppose a portion of air or gas inclosed in a cylinder, provided as before with a movable weighted piston. The inclosed gas would attain a certain definite volume, pressure, and temperature, the pressure being, of course, in equilibrium with the weight of the loaded piston. If now the load on the piston be diminished exactly as in the example on page 29, the gas will expand, raising the remaining weights through a certain space, and consequently doing external work. This work is done at the expense of the heat contained in the gas below the piston. The result will be that the temperature of the gas will fall by an amount which can be easily calculated when we know the quantity of external work done and the specific heat and the weight of the gas. Such expansion, then, does not fulfil the condition laid down in Boyle's law, that the temperature should remain constant.

In order that this latter condition should hold, heat must be supplied to the gas from some external source. It was shown before, that when the pressure and volume vary according to Boyle's law, the different states of the substance as regards pressure and volume for any given temperature may be represented graphically by the ordinates of a rectangular hyperbola. Such a line is said to be an *isothermal curve of expansion*, or, shortly, an *isothermal*; so called from two Greek words which signify equal and temperature, because the temperature is supposed to remain the same throughout all the changes in pressure and volume indicated by the co-ordinates of the curve. There is, of course, a separate isothermal for every temperature: for, with a given

mass of gas, the variations in pressure and volume are different for different temperatures, though following the same law. For instance, if, at any given volume, the temperature is in one case 32°, and in another 100°, the pressure will be greater in the second than in the first instance, in accordance with Gay Lussac's law.

If we have a series of isothermal lines drawn to scale, as in fig. 14, for a portion of any gas, such as air, we can

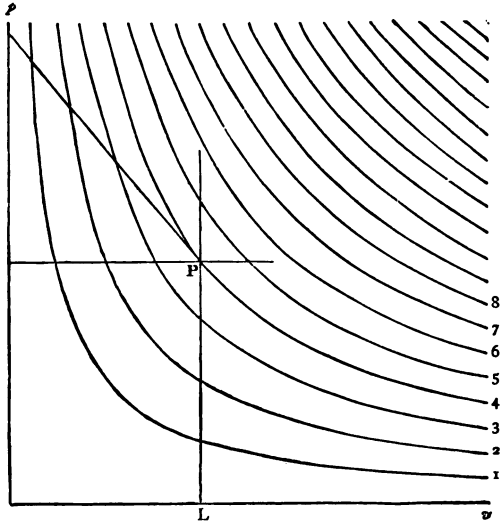

Fig. 14.

immediately find out by simple measurement either the temperature, the pressure, or the volume, when any two of these quantities are given. Each isothermal should be marked with the degree of temperature for which it is drawn. Suppose in the figure that there is a separate line drawn for each degree, and suppose that lines measured parallel to Op represent pressures, and those parallel to Ov volumes. Let

[1] Figs. 14 and 15 are taken from Clerk Maxwell's *Theory of Heat*.

the temperature 4°, and volume equal to OL, be given, and let it be required to find the pressure, we have simply to draw an ordinate LP vertically upward, till it meets the isothermal for 4°, then LP will be the required pressure.

The isothermals of dry saturated steam are very different to those of a gas. Suppose that a pound of water in a cylinder closed by a piston be turned into steam of atmospheric pressure, and that the piston be then pressed down, while the temperature is *maintained at* 212°, the pressure will not rise at all, while the volume will diminish,

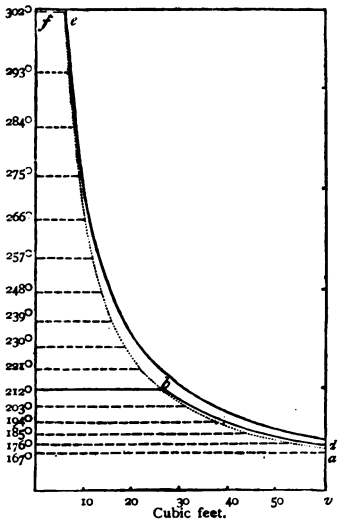

Fig. 15.

and, to permit of this taking place, part of the steam will be converted back into water. The reason of this is that dry saturated steam at a given temperature, say 212°, can exist at no *higher* pressure than the natural pressure of formation *at that temperature*, as shown by Regnault's tables. Take, for instance, steam of 341° This is the temperature at which the steam forms when the pressure is 120 pounds on the

square inch, and it can exist at no higher pressure so long as the temperature remains the same. If, however, instead of pressing the piston down, it were raised up, the volume of the steam would increase, and if the temperature were maintained constant the pressure would diminish as the volume increased, but not strictly in accordance with Boyle's law; that is, as has been before explained, the product pv would not be quite constant, though nearly so; and the isothermal curve would consequently not be a perfect hyperbola. Fig. 15 illustrates the isothermal lines of steam and water. Take for instance the full line *cba* which is the isothermal for the temperature 212°; we see that as the volume is enlarged the pressure diminishes as the ordinates of the curve *ba*, which is not a hyperbola. When, however, the mass of steam is compressed from the point *b*, the temperature remaining constant the isothermal is a straight line parallel to the base *ov*, showing that the pressure remains unaltered as the volume is diminished. The isothermal for the temperature 302° is shown by the full line *fed*. The dotted line shows the pressures and volumes at which condensation commences for the temperatures marked on the left hand side of the diagram.

Adiabatic lines.—In what has been said above regarding variations in the pressure and volume of gas and steam, the only condition observed when determining the shape of the curves was that the temperature was to be maintained constant throughout the changes. If, however, a portion of gas be inclosed in a cylinder as before and the cylinder and piston be supposed to be absolute non-conductors of heat, and to be also incapable of communicating any heat of their own to the gas, and if the pressure be then made to vary, the temperature will fall when the volume increases; for external work will be done, and, as this work can only be done at the expense of the heat contained in the gas, its temperature must fall. Now the pressure of the gas, other things being equal, depends on the temperature; conse-

quently, when a gas expands in a non-conducting cylinder doing external work, its pressure will be less for a given volume than when it expands isothermally to the same volume.

Let us now, instead of allowing the gas to expand in the cylinder, compress it by adding weights to the piston. The piston and weights, when closing in upon the gas, do work upon it, and consequently raise its temperature. As the cylinder is non-conducting, none of the heat represented by this increased temperature can escape, and its effect consequently is to increase the pressure of the gas to a greater degree than would be the case if the compression took place isothermally.

The effect of this difference in the conditions can easily be exhibited graphically. Take two cylinders of equal size and inclose in each of them an equal portion of gas, so that in their initial state the volumes, pressures, and temperatures shall be the same. Let the ordinates of the point of intersection of the two curves on fig. 16 represent the initial pressure and volume of each portion of gas. Let one of the cylinders be a non-conductor and in the other let the gas change its volume isothermally. In the latter case the variations in its pressure and volume will be represented by the isothermal curve or hyperbola. If, however, the gas be prevented from receiving or parting with heat by the non-conducting cylinder, as it expands from the point of intersection, the fall in the temperature will cause the pressure ordinates at each point to be less than the corresponding ordinates of the isothermal, and the new curve will consequently fall below the hyperbola. When, on the other hand, the volume is diminished, this can only be accomplished by doing work

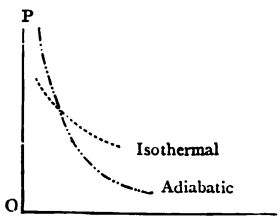

Fig. 16.

upon the gas, and, as no heat can escape, the temperature rises, and the pressure ordinates will consequently be greater than in the first case, and the curve will lie above the hyperbola. The second curve is represented on the diagram by the second dotted line. It is called an *adiabatic* line in contradistinction to the isothermal. The term *adiabatic* is derived from two Greek words signifying not passing through, and the line is so called because, in the operation above described, no heat passes through the cylinder either in or out.

Adiabatic lines of steam.—The adiabatic lines of dry saturated steam differ essentially from the corresponding lines for gases. Take a given mass of steam at the pres-

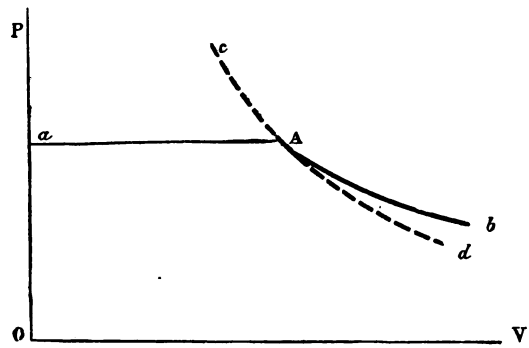

Fig. 17.

sure of the atmosphere, and the corresponding temperature 212°. Then, as we have seen, if the volume of the steam be diminished and the temperature preserved at 212°, the pressure will remain constant, and part of the steam will be changed back into water, because steam at the temperature 212° can exist at no higher pressure than that of the atmosphere. The isothermal will be then, as before explained, a horizontal straight line A*a*, fig. 17, while the volume is being diminished from the point A; and in order

to effect such a diminution and keep the temperature constant, heat must be abstracted from the steam. If, however, the compression be effected adiabatically, so that no heat can escape, the work done upon the steam by the act of compressing it will raise its temperature above 212°, and consequently enable it to exist at a higher pressure; and the more it is compressed, the greater will be the increase in the temperature, and consequently in the pressure. The adiabatic line therefore, during the diminution of volume of dry saturated steam, will be a curved line Ac, resembling very closely the corresponding line for a gas. It has been shown by Rankine and Clausius that if the cylinder, in addition to containing the steam, held also some water at the same temperature, the heat generated by the compression is sufficient to cause some of this water to become steam.

If we now return to the point A, and allow the volume to increase, so that the steam does external work, without gaining or losing any additional heat by conduction from outside bodies, the temperature will fall, for the external work is done at the expense of the heat contained in the steam. The heat will diminish so much that when the pressure is reduced to any given extent there will no longer be the quantity of heat present in the mass of steam, necessary, according to Regnault's experiments, to maintain it in the dry saturated condition (see p. 60 and the Table, p. 489), and consequently part of the steam will condense back into water, and in so doing will part with that portion of its heat which effected its transformation from water into steam. The heat thus liberated will maintain the remainder of the steam in the dry and saturated condition. Consequently, during expansion from the point A, the volume for a given pressure will be less—being reduced by the condensation of so much steam—than if the whole of the steam were maintained throughout in the dry and saturated condition, and less still than if the temperature were maintained uniform throughout.

The above example is applied to steam of the atmospheric pressure and corresponding temperature, but any other temperature might have been chosen, and the same reasoning would have applied. Just as in the case of the isothermals, so with the adiabatic lines, a separate one can be drawn for every separate degree.

We have now examined into the nature of the expansion lines of gases and steam for two separate cases, viz. first, the case of the temperature being maintained constant throughout the change, and second, the case of no heat being allowed to escape from or reach the gas and steam by conduction, radiation, &c., from or to other matter.

It is evident, however, that these are not the only possible cases, for we might, had we wished, have supplied or abstracted any quantity of heat we chose, to or from the gas, during the process of alteration of volume and pressure, and thus have made the shapes of the curves of expansion anything we pleased. The two cases above described are, however, the most important.

CHAPTER III.

THEORETICALLY PERFECT HEAT ENGINES.

Application of Boyle's and Charles' laws to gases—Specific heat of gases at constant pressure and at constant volume—Cycle of operations—Ratio of heat expended to work done—Graphic representation of external work done during the expansion of a gas—Nature of the curves of expansion of gases as influenced by the supply of heat—Heat supplies: (1) when the curve of expansion is a rectangular hyperbola, the equation for which is $pv = c$; (2) when the equation of the curve takes the form $pv^n = c$, where n has any value except unity—Nature of the curve when no heat is supplied or abstracted—The ideally perfect heat engine—Calculation of the efficiency of such engines—The reversed action of the ideal heat engine—Proof that no other engine has a greater efficiency than the ideal heat engine—Practical limits of efficiency in the ideal heat engine—Laws connecting the pressure, temperature, and volume of dry saturated steam—Specific heats of water and steam—Law connecting the pressure, volume, and temperature of superheated steam—Total heat expended in converting water into steam—Proportion of total heat expended in doing external and internal work—Expenditure of heat in a steam engine when the steam is not used expansively—Method of representing heat by an 'equivalent pressure'—Expenditure of heat in a steam engine when the steam is used expansively, 1st, when the curve of expansion is a rectangular hyperbola, 2nd, when the steam remains dry and saturated throughout whole stroke—To realise latter condition steam jacket is necessary—Rankine's formulæ for the expenditure of heat in a steam engine—Theory of the ideally perfect heat engine applied to steam—How actual steam engines differ from the ideal heat engine—Summary of laws and formulæ.

THE last chapter contained a sketch of the principles of the science of heat and an account of the effects of heat upon gases and water. The present chapter will deal with the conversion of heat into mechanical work through the instrumentality of heat engines, and will contain an account of an ideal heat engine which is perfect in theory; that is to say, no other conceivable engine can get more work out of the heat supplied to it than the one about to be described. Practical difficulties render the realisation of such an engine

impossible, but the study of it is nevertheless of the greatest importance, as enabling us to find out the deficiencies of existing engines, and to ascribe to each of these deficiencies its due share in causing waste of heat.

On account of the greater simplicity of gas, it will be found convenient, first to describe the mode of operation of the ideal engine when worked with gas or air, and afterwards to apply the results obtained to the case of steam. Before doing so, however, it will be necessary to recapitulate the laws affecting gases which were explained in the last chapter, but with greater numerical exactitude, and then, from these laws, to make certain deductions, which, as they refer to the power of doing work through the medium of gases, are commonly classed under the head of the Thermodynamics of Gases.

Numerical application of Boyle's law to gases —The first of the laws referred to is Boyle's law, connecting the pressure and the volume of the gas when the temperature is maintained constant. The algebraical expression for this law was shown to be $pv=c$.

If one pound's weight of air be taken, at the pressure p_0 of the atmosphere, equal to 2116·8 lbs. on the square foot, at the temperature 32°, then the volume of this pound of air, or v_0, multiplied by the pressure on the square foot has been proved by Regnault's experiments to be

$$p_0v_0=26,214 \text{ foot-pounds.}$$

This quantity, 26,214 foot-pounds, is therefore the value of the constant c, so long as the temperature remains 32°.

If the temperature be changed, the value of the constant is changed also. This leads us to Gay Lussac's law (see page 46) connecting the pressure and volume with the temperature. This law states that the product pv is increased when the temperature is raised from 32° to 212°, in the ratio of 1 to 1·3654; and that for each of the 180 degrees intermediate between 32° and 212° the increase is

$\frac{1}{180}$th part of the increase at 212°. If then p_0v_0 be the pressure and volume at 32°, and pv be the pressure and volume at any other temperature $t°$, then if $t°=212°$,

$$pv=p_0v_0+\cdot3654p_0v_0,$$

and if $t°$ be any other temperature then

$$pv=p_0v_0+\frac{\cdot3654}{180}(t°-32°)p_0v_0.$$

This rate of increase of course applies also when the temperature is raised above 212°.

It was also shown (see page 51) that if the temperature be reckoned from the bottom of the tube of the air thermometer, which was shown to be 492° below 32° Fahrenheit, the above law could be greatly simplified.

For, the product of the pressure and volume of a portion of gas is proportional to the absolute temperature, so that if $\tau°$ be the absolute temperature corresponding to $t°$; then, remembering that 492·6° absolute, corresponds to 32° on the ordinary scale, and attaching the same values as before to all the other symbols, we have—

$$pv : \tau :: p_0v_0 : 492\cdot6$$

$$\therefore pv=\tau\frac{p_0v_0}{492\cdot6}.$$

Now p_0v_0 as stated above=26,214

$$\therefore\frac{p_0v_0}{493}=53\cdot2.$$

Hence we get $pv=53\cdot2.\tau$,

which is a very simple expression, connecting the pressure, the volume, and the absolute temperature.

Specific heat of gases at constant volume, and at constant pressure.—The next law, which is now mentioned for the first time, relates to the specific heat of gases, and asserts that, if a gas be heated at constant pressure, it requires the same quantity of heat to raise its temperature from any point, say 212° to 213°, as it does from any other point, say 32° to 33°. In other words, the specific heat of a gas at constant

pressure does not change with the temperature, as is the case with water.

The capacity of air for heat, that is, the amount of heat required to raise one pound of it through 1° of temperature, the pressure being maintained constant, is, according to Regnault's experiments, 0·2375 thermal units, equal to 183·35 foot-pounds. This quantity of heat is, as has been shown before, not all expended in merely raising the temperature of the air; for, the heating having been accomplished at constant pressure, part of the heat has been spent in doing external work.

Let v_1, τ_1, v_2, τ_2 be the original and final volumes and absolute temperatures of a pound of air; and let p be the pressure which remains constant. Then the external work is measured by the increase in the volume, viz. $v_2 - v_1$ multiplied by the pressure p; therefore

$$\text{External work} = (v_2 - v_1)p;$$

and, as we have seen, $vp = 53{\cdot}2.\tau$; therefore

$$\text{The external work} = 53{\cdot}2\,(\tau_2 - \tau_1).$$

Also the total heat expended equals the specific heat multiplied by the number of foot-pounds in one thermal unit multiplied by the number of degrees of rise of temperature. The usual symbol for the specific heat at constant pressure multiplied by the number of foot-pounds in one thermal unit is K_p;[1] and as the rise in temperature is $\tau_2 - \tau_1$, we have

$$\text{Total heat expended} = K_p(\tau_2 - \tau_1).$$

Hence the heat expended in doing internal work—that is, in merely raising the temperature of the air—is the difference between the total heat expended and that part which is spent in doing external work

[1] K_p and K_v are spoken of hereafter for the sake of brevity and in accordance with a usual custom as specific heats; but in reality a specific heat is only a ratio, whereas K_p and K_v are absolute quantities.

Therefore the internal work $= (\tau_2 - \tau_1)\ (K_p - 53{\cdot}2)$.

Now this latter quantity within the right-hand bracket is also the value of the specific heat of air when heated at constant volume; because, as we know by Joule's experiment (see page 54), the mere separation of the particles of air requires no heat to effect it when no external work is done, and as the heat is only expended in doing external and internal work, and as, moreover, when the air is heated at constant *volume* no external work is done, therefore the specific heat of air heated at constant volume is the same as the internal specific heat at constant pressure, and, calling the specific heat at constant volume K_v, we have

$$K_v = K_p - 53{\cdot}2 = 130{\cdot}25 \text{ foot-pounds.}$$

Consequently the heat required to raise the temperature of one pound of air at constant volume from $\tau_1°$ to $\tau_2°$ is

$$K_v(\tau_2 - \tau_1).$$

From the equation $K_v = K_p - 53{\cdot}2$ we get, by simply transposing, $K_p - K_v = 53{\cdot}2$. That is to say, the difference in the two specific heats of air is equal to the constant quantity 53·2, which, as we have seen before, when multiplied by the absolute temperature, equals the product pv.

From the result given above for the value of the heat expended in internal work, when the air was heated at constant pressure, viz. $(K_p - 53{\cdot}2)\ (\tau_2 - \tau_1) = K_v(\tau_2 - \tau_1)$, we see that the internal work is proportional to the change of temperature, and is equal to the change of temperature multiplied by the specific heat at constant volume.

This result is true whether the air be heated at constant pressure, or at constant volume, or partly in the one way and partly in the other, or in fact in any way we can conceive of. For, as an example, first change the air from volume v_1, and temperature τ_1, to volume v_2, keeping the pressure constant at p_1; let the new temperature be τ; then by the above the heat expended in internal work is $K_v(\tau - \tau_1)$.

Next change the pressure from p to p_2, the volume being kept constant at v_2. To do this we must add heat to the gas, raising its temperature to τ_2; the heat spent is $K_v(\tau_2 - \tau)$, which is all internal work; adding to this the heat spent in doing internal work during the first part of the operation, we get

$$\text{Total heat spent in internal work} = K_v(\tau - \tau_1 + \tau_2 - \tau) = K_v(\tau_2 - \tau_1).$$

This result might be proved to be true for any other case which might arise, by similar reasoning to the above, but it may also be shown to be generally true from the following considerations.

Cycle of Operations.—If a substance such as gas or water be subjected to the action of heat, and be thus brought through a series of changes of state, and eventually brought back to its original condition, it is said to have undergone a *Cycle of Operations.* During these changes of state heat has been expended in doing two things only, viz. external work, and internal work of various sorts, such as altering the temperature or the molecular condition of the substance. When, however, the body is brought back to its original condition, the sum of all the quantities of heat expended in doing internal work must be nil, because if during one part of the operation heat has been thus expended, when the substance is brought back to its original condition this heat is again liberated or rejected.

Now when the state of gas or air is changed by the action of heat in any way whatever we can analyse the operation into three distinct sets of processes, viz.,

1st. Heating at constant pressure, the volume being changed;

2nd. Heating at constant volume, the pressure being changed; and

3rd. One or more cyclical processes.

Now during the latter processes no heat is spent in

internal work, and during the two former the heat thus spent is as we have seen $= K_v(\tau_2 - \tau_1)$. Hence the proposition is universally true that when the state of a gas is changed by the action of heat, the quantity of heat spent in doing internal work depends only on the difference of temperatures of the two states, and is equal to the specific heat at constant volume multiplied by this difference of temperatures.

From the above fundamental laws we are enabled to reason on all the questions which may arise regarding the thermodynamics of gases. All that we require to know is, how much heat is expended in doing external, and how much in doing internal work. The total heat expended is equal to the sum of these two quantities. When we possess this information we can deduce all that it is requisite to know regarding the pressure and temperature at every stage of the process. Conversely if we know the pressure and temperature we can calculate the external and internal work done, and the expenditure of heat. The internal work is, as has been proved above, always given by the expression $K_v(\tau_2 - \tau_1)$. The external work is different in different cases. For instance, if during the changes of volume and pressure sufficient heat be supplied to keep the temperature uniform, we get a certain quantity of external work. If on the contrary no heat be supplied we get quite another quantity, and if more than enough or less than enough heat be supplied to keep the temperature uniform, we get still different quantities of external work done in each case.

The quantity of external work done is perhaps best calculated and exhibited by means of diagrams. We have seen (see page 44) how the varying pressure and volume of a portion of gas can be represented by the ordinates of a line GD, fig. 11. We also saw (see page 63) how work done could be represented by the area of a rectangle. An extension of these methods will now be explained.

Let *ac*, *c*O, fig. 18, represent the initial, and *bd*, *d*O the final pressures and volumes of a portion of gas. Let the

intermediate co-ordinates of the curve *ab* represent the intermediate pressures and volumes while the gas is expanding. Draw a line *ef*, indefinitely near and parallel to the line *ac*. While the volume of the gas has been increasing from O*c* to O*f*, the pressure has been falling from *ac* to *ef*. Now the external work done is represented by the increase in volume *cf*, multiplied by the pressure. The pressure in this case is not uniform, but decreases as the ordinates of the curved line *ae*. We must therefore multiply the increase of volume by the average pressure. It is difficult to find the average pressure when the line *ae* is curved; but if we take *ef* as being very near indeed to *ac*, we may regard *ae* as being to

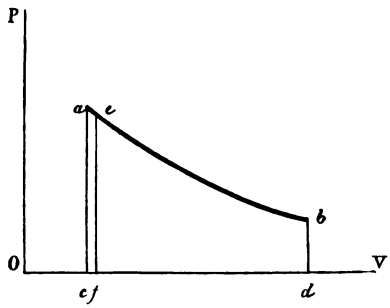

Fig. 18

all intents and purposes a straight line, and in this case the average pressure will be represented by the mean between the two lines *ac* and *ef*, viz. $\frac{ac+ef}{2}$. The external work done is therefore represented by the expression $cf \times \frac{ac+ef}{2}$. But this expression also represents the area of the strip *aefc*, therefore the external work done while the volume of the gas is increasing from O*c* to O*f* is represented by the area *aefc*. We can divide the whole figure *abdc* up into a series of such strips, and the above reasoning would hold good for

each of them. Now the sum of the areas of these strips equals the area of the figure *abdc*; therefore when the volume of a portion of gas increases from O*c* to O*d*, the pressure at the same time varying as the vertical ordinates of the line *ab*, the external work done during the process is represented by the area inclosed by the base line *cd* representing the increase of volume, the vertical lines *ac* and *bd* representing the initial and final pressures, and the line *ab*, which represents the way in which the pressure varies.

The line *ab* may be anything that we please. For instance, if during the expansion of the gas enough heat were supplied to it to keep the pressure uniform throughout, it would be a straight line drawn through *a*, parallel to ov. If enough heat were supplied to keep the temperature uniform, the line would, as has been proved, be an isothermal, which for gases is a common rectangular hyperbola denoted by the equation $pv =$ constant.

If no heat were supplied and none allowed to escape, the line would be an adiabatic, the equation for which will be shown to be $pv^{\gamma} =$ constant. The symbol γ which is of very constant occurrence denotes the ratio $\frac{K_p}{K_v}$.

Most of the lines occurring in the theory of the heat engine are denoted by the equation $pv^n =$ constant, where the index n varies according to the supply of heat. For instance, the two preceding cases are special instances of this equation in the first of which $n=1$, and in the second $n=\gamma$.

The area of the figure *abdc* depends, of course, upon the special form of the line *ab*, and can be readily calculated for each case by those who are familiar with the processes of analytical geometry.

We can now examine into the most important cases that arise.

1. Heat expended in changing the state of a gas when the temperature remains constant throughout the change. The total heat expended = the internal work done + the

external work done. The internal work in this case is nil, because the temperature does not change, and consequently the expression $K_v (\tau_2 - \tau_1)$ vanishes.

The external work is obtained by calculating the area of the diagram *abdc* (see fig. 18). We assume that the initial pressure and volume are represented respectively by the line *ac*, drawn to scale so as to represent pounds on the square foot, and O*c* drawn to represent cubic feet. Similarly the lines *bd* and O*d* represent the final pressure and volume to the same scale. As the temperature is uniform the line *ab* is a rectangular hyperbola, having for its asymptots OV and OP;

and the area[1] of the figure $abdc = ac \times Oc \times \log_e \frac{Od}{Oc}$.

Also, as by the principle of the hyperbola $ac \times Oc = bd \times Od$

$$\therefore \text{ the area is also } = bd \times Od \times \log_e \frac{Od}{Oc}.$$

It will be noted that the logarithms used are hyperbolic. A table of the hyperbolic logarithms of the most useful numbers will be found at the end of the book (see page 498).

[1] The area is calculated in the following manner:

Let *a b*, fig. 18, be a curve of the equation.

$$p v = \text{constant}.$$

The area *a b d c*, is the sum of a number of small strips such as *a f*. By making these strips indefinitely narrow they may each be represented by the expression $p \times dv$, where p represents the height, and dv the indefinitely small width.

Let $ac = p_1$; $Oc = v_1$ and $bd = p_2$; $Od = v_2$.

$$\text{Then the area} = \int_{v_1}^{v_2} p.dv.$$

$$\text{Now as } p v = p_1 v_1$$

$$\therefore p = \frac{p_1 v_1}{v};$$

and substituting

$$\text{Area} = p_1 v_1 \int_{v_1}^{v_2} \frac{dv}{v}$$

$$= p_1 v_1 (\log_e v_2 - \log_e v_1)$$

$$= p_1 v_1 \log_e \frac{v_2}{v_1} = p_1 v_1 \log_e r.$$

The ratio of the final volume O*d* to the initial volume O*c* is called the ratio of expansion, and is generally denoted by the symbol r. Calling the initial and final volumes v_1 and v_2, and the initial and final pressures p_1 and p_2 respectively, the expression for the area becomes $p_1v_1 \log_e r$, or $p_2v_2 \log_e r$ foot-pounds. Also if τ be the absolute temperature of the gas, then, as we have seen, $p_1v_1=p_2v_2=c\tau$,

$$\therefore \text{ the area} = c\tau \log_e r \text{ foot-pounds.}$$

This quantity is, therefore, the expenditure of heat in foot-pounds when a gas expands isothermally from volume v_1 to v_2.

2. Let the curve *ab*, instead of being an hyperbola of the equation pv=constant, be a curve of the form

$$pv^n = \text{constant},$$

where n may have any value we like to assign to it except unity. In this case the area [1] of the figure *abdc*

$$= \frac{p_1v_1 - p_2v_2}{n-1}$$

[1] The area is calculated in the following manner :

Let *a b*, fig. 18, be a curve of the equation

$$p v^n = \text{constant}.$$

To find the arc *a b d c*

Let $a c = p_1$; $O c = v_1$ and $b d = p_2$; $O d = v_2$;

also let p be any pressure ordinate, and v the corresponding volume.

$$\text{Then the area} = \int_{v_1}^{v_2} p.dv.$$

$$\text{Now as } p v^n = p_1 v_1^n$$

$$\therefore p = \frac{p_1 v_1^n}{v^n};$$

and substituting

$$\text{Area} = p_1 v_1^n \int_{v_1}^{v_2} \frac{dv}{v^n}$$

$$= p_1 v_1^n \frac{v_2^{1-n} - v_1^{1-n}}{1-n}$$

$$= \frac{p_1 v_1 - p_1 v_1^n v_2^{1-n}}{n-1}$$

$$= \frac{p_1 v_1 - p_2 v_2}{n-1}.$$

Let τ_1 be the initial, and τ_2 the final absolute temperatures. The expression for the area becomes then

$$\frac{c\tau_1 - c\tau_2}{n-1} = \frac{c}{n-1}(\tau_1 - \tau_2).$$

As before, the heat expended=the internal work+the external work

$$= K_v(\tau_2 - \tau_1) + \frac{c}{n-1}(\tau_1 - \tau_2).$$

Also, as has been proved before, $c = K_p - K_v$;

$$\therefore \frac{c}{n-1} = \frac{K_p - K_v}{n-1} \text{ and } \frac{c}{n-1}(\tau_1 - \tau_2) = \frac{K_v - K_p}{n-1}(\tau_2 - \tau_1);$$

$\therefore$ substituting this value of $\frac{c}{n-1}(\tau_1 - \tau_2)$ in the above equation, we get heat expended

$$= (\tau_2 - \tau_1)\left(K_v + \frac{K_v - K_p}{n-1}\right) = (\tau_2 - \tau_1)\left(\frac{nK_v - K_p}{n-1}\right).$$

3. Let no heat be communicated to or taken from the gas during the expansion. In other words, let the line *ab* be an adiabatic curve.

The last expression for heat expended, viz. $(\tau_2 - \tau_1)\left(\frac{nK_v - K_p}{n-1}\right)$ must, when no heat is expended, equal zero. Hence one or other of the terms within the brackets must equal zero. We know, however, that $(\tau_2 - \tau_1)$ cannot equal zero; because during the expansion the temperature falls, and therefore τ_2 is less than τ_1. If, therefore, we make the other term, viz. $\frac{nK_v - K_p}{n-1} = 0$, we get

$$nK_v - K_p = 0 \quad \therefore n = \frac{K_p}{K_v} = \gamma.$$

The equation pv^n=constant, becomes therefore pv = constant for the case of adiabatic expansion.

As no heat is supplied to the gas, the external work

must be done at the expense of the heat already existing in the gas, consequently the temperature falls, and the internal work done is of a negative character

The temperature at the end of the expansion may be found in the following manner, using the same symbols as before. We have $c\tau_1 = p_1 v_1$, and $c\tau_2 = p_2 v_2$.

$$\text{Now } p_1 v_1^{\gamma} = p_2 v_2^{\gamma} \qquad \therefore p_2 v_2 (v_2^{\gamma-1}) = p_1 v_1 (v_1^{\gamma-1})$$

$$\therefore p_2 v_2 = p_1 v_1 \left(\frac{v_1}{v_2}\right)^{\gamma-1}$$

$$\therefore c\tau_2 = c\tau_1 \left(\frac{v_1}{v_2}\right)^{\gamma-1} \quad \therefore \tau_2 = \tau_1 \left(\frac{1}{r}\right)^{\gamma-1}$$

which, as $\gamma = 1{\cdot}408$ becomes $\tau_2 = \tau_1 \left(\frac{1}{r}\right)^{\cdot 408}$

an expression which, when the initial pressure and volume and the ratio of expansion are given, enables us to find the final temperature.

Supposing in all the above examples that the gas were compressed back to its original condition, the varying pressures and volumes could be represented graphically, just as in the case of expansion. If the conditions of compression were the same as those of expansion, the same curve would represent each operation. For instance, if the temperature were maintained constant the curve would be an hyperbola. If no heat were added or abstracted, the compression curve would be an adiabatic line, and the temperature would rise as the compression continued. If the conditions of the compression were different to those of the expansion, the curve would also be different.

The Ideally Perfect Heat Engine.

We are now in a position to examine into the theory and conditions of working of the ideal heat engine referred to at the beginning of the chapter. This engine requires to be made of materials which do not exist in practice; the only

object in discussing it is therefore to separate the action of the heat on the gas from the accidents of its surroundings, so that we may be enabled to ascribe to the surroundings in actual engines their proper influence. The efficiency of the action of this engine does not depend in any way upon the mechanism by which its motion may be converted, but only on the manner in which it receives and rejects heat. We will therefore, for the sake of simplicity, suppose it to consist of

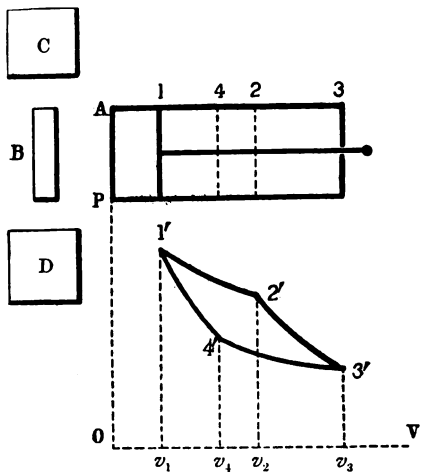

Fig. 19.

a working cylinder, connected by means of a piston and connecting rods with a crank.

Let AP, fig. 19, be the working cylinder which contains between the end AP and the piston when at its initial position 1 a certain quantity, say one pound, of gas at the temperature τ_1. The space in front of the piston is supposed to be a perfect vacuum, so that the engine is single-acting. The sides of the cylinder are supposed to be made of an ideal substance which can neither give heat to nor

receive it from the gas. The bottom of the cylinder is, on the contrary, supposed to be made of a substance which though it has no capacity for heat itself, is nevertheless a perfect conductor of heat. C is a source of heat having the absolute temperature τ_1. D is similarly a body having a temperature τ_2 less than τ_1 which is used for the reception of the heat rejected by the gas. B is a cover made of the same material as the sides of the cylinder which, when applied to the bottom AP, renders it perfectly non-conducting like the rest of the cylinder.

Let the piston commence to move forwards from the position 1. To prevent the temperature of the gas from falling, apply the source of heat C to the end of the cylinder. As this end is a perfect conductor, heat will flow into the gas and maintain the temperature constant. Let the co-ordinates of the point 1′ with reference to the lines OV, OP denote the initial volume and pressure of the gas. So long as the body τ_1 is kept in contact with the cylinder end, the gas will expand isothermally, and the variations in its pressure and volume will be denoted by the co-ordinates of the hyperbolic curve 1′2′. When the piston has reached the point 2, the exact position of which will be presently determined, withdraw the body τ_1 and apply the non-conducting cover B. The gas will now continue to expand adiabatically, as represented by the curve 2′3′. During this part of the expansion the temperature will fall, and the point 2′ must be so chosen that the temperature will, when the piston has reached the end of its stroke, have fallen to that of the cold body, viz. τ_2. The piston must now be caused to return in the opposite direction. As the engine is single-acting, this can only be done by the application of forces external to the engine. The piston when returning will compress the gas and do work upon it. To prevent the temperature from rising apply the cold body D to the end of the cylinder. The compression will then take place isothermally, at the temperature τ_2, along the hyperbolic

curve 3′4′. When the piston has reached the point 4—the position of which will be presently determined—the cold body must be withdrawn, and the non-conducting cover reapplied to the end of the cylinder. The compression of the gas must then be continued, and as no heat can escape it will take place adiabatically along the line 4′1′. During this part of the compression the temperature will rise, and, if the point 4 has been rightly chosen, it will reach τ_1 when the piston has returned to its original position.

During the first operation the gas has been receiving heat, and doing external work, which is represented by the area of the figure 1′ 2′ v_2 v_1. During the second operation the gas has received no heat from without, but, at the expense of the heat which it already possessed, it has done external work, represented by the area 2′ 3′ v_3 v_2. During the third operation, work has been done upon the gas, measured by the area 3′ 4′ v_4 v_3, and the gas has rejected heat into the body D; and during the fourth and last operation work has been done upon the gas represented by the area 4′ 1′ v_1 v_4, with the result of restoring it to the original pressure, volume, and temperature. We see therefore that the work done by the gas exceeds the work done upon the gas, by the difference between the sum of the two first, and the sum of the two last-mentioned areas. This difference is equal to the area 1′ 2′ 3′ 4′, which therefore represents the effective work done by the engine.

Calculation of the efficiency of perfect heat engines.—We must now calculate the heat expended, and the work done. During the first operation, the heat supplied to the gas is all expended in doing external work: for no internal work is done, as the temperature of the gas is not raised. The heat supplied therefore in foot-pounds is equal to the area 1′ 2′ v_2 $v_1 = p'$ v' $\log_e r = c\tau_1 \log_e r$, where r is as before the ratio $\frac{v_2}{v_1}$. During the second operation no heat is supplied to the gas. During the third operation, the gas

rejects heat equal in amount to the work done upon the gas $=c\tau_2 \log_e r$, where r is the ratio $\frac{v_3}{v_4}$ which will be presently proved$=\frac{v_2}{v_1}$. During the fourth operation the gas rejects no heat. The total heat supplied therefore $=c\tau_1 \log_e r$, and the total heat rejected $=c\tau_2 \log_e r$.

The work done can be calculated in two ways. We may either compute the area 1′ 2′ 3′ 4′, or we may make use of the principle of the cycle of operations. For since the gas returns to its original condition no heat is spent in doing internal work upon the gas and the heat expended must therefore equal the external work done, plus the heat rejected.

Consequently the external work

$$=c\tau_1 \log_e r - c\tau_2 \log_e r = (\tau_1 - \tau_2)\, c \log_e r.$$

The efficiency of the engine is the ratio of the work done to the heat expended

$$=\frac{(\tau_1-\tau_2)\quad c \log_e r}{\tau_1\, c \log_e r.} = \frac{\tau_1-\tau_2}{\tau_1}$$

That is to say the efficiency of the engine is the ratio of the difference of temperatures of the sources of heat and of cold to the temperature of the source of heat ; the temperatures being reckoned in absolute measure.

The efficiency of the engine can only become equal to unity, i.e. the engine can only turn the whole of the heat supplied to it into mechanical work, when the temperature $\tau_2=0$; that is to say, when the cold body has the absolute zero of temperature ; a result which is of course unattainable.

On the other hand, the nearer to unity the fraction $\frac{\tau_1-\tau_2}{\tau_1}$ becomes, the greater is the efficiency of the engine.

This result can only be attained by making $\tau_1-\tau_2$ as nearly as possible equal to τ_1. To do this we must make τ_1 as large and τ_2 as small as possible.

In practical engines, the limits of temperature which we can make use of are soon reached, and consequently the efficiency of such engines is necessarily low.

In order to fix the points 2 and 4 with precision, we must remember that the isothermal expansion and compression must be stopped soon enough to allow the temperature to fall to τ_2 and rise to τ_1 respectively during the subsequent adiabatic expansion and compression. We have seen (see page 85) that when a gas expands adiabatically, the temperatures and the ratio of expansion are connected together by the equation $\tau_2=\tau_1\left(\frac{1}{r}\right)^{\gamma-1}$.

Consequently $r^{\gamma-1}=\frac{\tau_1}{\tau_2}$ and $r=\left(\frac{\tau_1}{\tau_2}\right)^{\frac{1}{\gamma-1}}$

That is to say, in fig. 19, $Ov_3 \div Ov_2=\left(\frac{\tau_1}{\tau_2}\right)^{\frac{1}{\gamma-1}}$, which fixes the point 2.

Similarly during the adiabatic compression from τ_2 to τ_1 we have $Ov_4 \div Ov_1=\left(\frac{\tau_1}{\tau_2}\right)^{\frac{1}{\gamma-1}}$, which fixes the point 4. Hence we see that $\frac{Ov_1}{Ov_4}=\frac{Ov_2}{Ov_3}$; or the ratio of adiabatic compression equals the ratio of adiabatic expansion. It may likewise easily be proved that the ratio of the isothermal compression equals the ratio of isothermal expansion.

We can now prove that no other engine can have a greater efficiency than the one which has just been described. We must, however, first state what is meant by the reversed action of the engine. Let the piston, starting from the position 1, move forwards, the conducting cover B being applied to the cylinder end. The gas will expand adiabatically along the curve $1'4'$. As soon as the temperature has fallen to τ_2 apply the cold body D so as to keep the temperature constant, and allow the gas to expand still

further along the isothermal curve $4'3'$. When the point 3 is reached reverse the piston, by means of external forces, and compress the gas adiabatically along the line $3'2'$. As soon as the temperature has risen to τ_1 apply the hot body C and continue the compression isothermally till the piston reaches its original position, and the gas its original state. The result will be, that we shall have done the exact reverse of all that was done when the engine was worked in the usual way. Instead of the engine having done any work, external work has been done upon the gas represented by the area $1'2'3'4'$. Instead of heat having been taken from the hot and rejected into the cold body, it has been taken from the cold and rejected into the hot body. The quantity of heat taken from the cold body is $c\tau_2 \log_e r$ and the quantity rejected into the hot body is $c\tau_1 \log_e r$, the work done upon the gas being the difference of these two quantities.

If now any engine can be devised having a greater efficiency than the one described, let it be employed to drive this latter in the reversed manner. Then engine No. 2 takes heat from the hot body and rejects heat into the cold body, while engine No. 1 does exactly the reverse, taking heat from the cold body and rejecting it into the hot one. Let the power of each engine be the same. In the case of engine No. 1 this work is done upon the gas, and in the case of No. 2 it is done by the gas, so that upon the whole the engine does no external work, and the combination is self-acting, and can, if we neglect friction, go on running for ever.

Let R_1 be the heat which No. 1 takes from the cold body, and H_1 be the heat which it rejects into the hot body. Let H_2 and R_2 be the quantities which engine No. 2 takes from the hot body and rejects into the cold body. Now, the heat taken from the cold body by No 1, plus the work done on the gas, equals the heat given to the hot body; therefore $H_1 - R_1$ is the work done on the gas in

the case of No. 1, and $H_2 - R_2$ is the work done by No. 2. But, since the power of each engine is the same,

$$\therefore H_1 - R_1 = H_2 - R_2.$$

Now, the efficiency is the ratio of the work done to the heat supplied $= \frac{H_1 - R_1}{H_1}$ for engine No. 1, and $\frac{H_2 - R_2}{H_2}$ for No. 2. According to the hypothesis the efficiency of No. 2 is the greater, therefore $\frac{H_2 - R_2}{H_2} > \frac{H_1 - R_1}{H_1}$. In order that this may be so, the denominator of the first fraction must be less than that of the second, for the numerators are equal, therefore H_2 is less than H_1.

Now, H_2 is the quantity of heat which No. 2 takes from the hot body; and H_1 is the quantity which No. 1 rejects into it, and as H_2 is less than H_1 upon the whole the hot body receives heat; and similarly upon the whole the cold body parts with heat, and if the engine were kept at work long enough the whole of the heat in the cold body could be taken out of it and conveyed to the hot body, that is to say, heat could be transferred from a cold to a hot body by a *self-acting* contrivance, which is the exact opposite of all experience; consequently we must conclude that engine No. 2 has not got a greater efficiency than No. 1, and that no arrangement we can imagine can have a greater efficiency than No. 1.

The Second Law of Thermodynamics.—The statement of the principle from which the above conclusion is drawn is called the *Second Law of Thermodynamics.* It may be expressed as follows:

Heat cannot pass from a cold to a hot body by a self-acting process, unaided by external agency.

The peculiarity of engine No. 1 is its reversibility, and this is due to its receiving heat always at the temperature of the hot body, and rejecting it at that of the cold body. If this condition did not obtain, we could not work the engine

in the reverse sense. Hence we conclude, that for a heat engine to develope a maximum of work out of the heat supplied to the gas or other working substance it must receive all its heat at the temperature of the hot body, and reject it all at the temperature of the cold body. If these conditions are complied with, the maximum of work obtainable is got by multiplying the heat supplied to the engine, measured in foot-pounds by the ratio $\frac{\tau_1 - \tau_2}{\tau_1}$.

It may at first appear anomalous to the student that the whole of the heat supplied to the gas cannot be converted into work. Perhaps the best way to overcome this difficulty is, if he can conceive of any arrangement by which he hopes to get more work out of an engine, to calculate out the various steps in the process, and compare the heat expended with the work done.

Take, for instance, a pound of air inclosed in a cylinder as before, and at the pressure and temperature τ of the atmosphere. Heat the air, keeping the volume constant till it attains any desired pressure p_1, and corresponding temperature τ_1; next expand it adiabatically, till the original temperature of the atmosphere is reached, the corresponding pressure, p_2, being attained. During this part of the process the exact equivalent of the heat expended on the gas is converted into mechanical work. For the heat so expended is $K_v(\tau_1 - \tau)$, and the work done is (see page 84) $\frac{c}{\gamma - 1}(\tau_1 - \tau) = K_v(\tau_1 - \tau)$. But in order to bring the gas back to its original pressure, volume, and temperature, so as to be able to make another revolution of the engine, we must either compress the air at the constant temperature τ_1; or else we must temporarily open the end of the cylinder to the outer air, and force back the piston against the constant pressure of the atmosphere to its original position. In the first case, to compress the air at constant temperature τ_1, we must do work upon it represented by

$cT \log_e \frac{v^2}{v^1}$ (see page 83). In the second case we do work equal to the pressure of the atmosphere = 2116 lbs. per square foot, multiplied by the space through which it has to be overcome, viz. $v^2 - v^1$ feet. In either case the work so done upon the air must be subtracted from the work done by the air during the first half of the stroke, so that in neither instance can we realise the full equivalent of the heat expended. It may in fact be proved by a calculation similar to that on pages 88, 89 that in this case the efficiency is less than the maximum, viz. $\frac{T_1 - T}{T_1}$.

To take another case. If a portion of gas be expanded at constant temperature, it is known that the heat absorbed equals the external work done. It might therefore be supposed that a ready means was hereby presented of converting

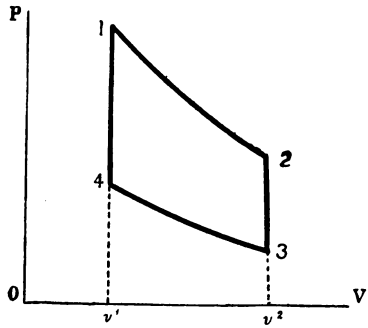

Fig. 20.

the whole of the heat supplied to an engine into work. Let the point 1, fig. 20, represent the initial condition of the gas as regards pressure and volume. Let it expand at constant temperature till the pressure and volume are indicated by the point 2. During this part of the operation external

work would be done represented by the area $12v^2v^1$ $= c\tau_1 \log_e \frac{v_2}{v_1}$ = the heat supplied to the gas during expansion. If we were now to restore the gas to its original pressure, volume, and temperature, we could do so by compressing it isothermally; but in this case the compression curve would be identical with the expansion curve, and consequently in compressing the gas we should have to do the same work upon it that we got out of it during the expansion, and the resulting work done would be nil. In order to get work out of the gas, we must therefore, when the point 2 is reached, make it reject heat. Let this be done at constant volume. The pressure will fall, and this part of the operation is represented on the diagram by the vertical straight line 2 3. When the pressure has fallen to the point 3, the temperature has also fallen to τ_2, and the heat rejected during the fall is $K_v(\tau_1 - \tau_2)$. The gas may now be compressed isothermally at the temperature τ_2, till the end of the stroke is reached, and the work done upon it equals $c\tau_2 \log_e \frac{v_2}{v_1}$. We thus arrive at the point 4, where we see that the pressure and temperature are less than at the initial position. In order, therefore, to complete the cycle of operations, we must now heat the gas at constant volume till its original temperature and pressure are restored, and in so doing heat equal to $K_v(\tau_1 - \tau_2)$ will have been expended. We shall, therefore, have the following quantities :—

$$\text{Heat expended} = c\tau_1 \log_e \frac{v_2}{v_1} + K_v(\tau_1 - \tau_2).$$

$$\text{Heat rejected} = c\tau_2 \log_e \frac{v_2}{v_1} + K_v(\tau_1 - \tau_2)$$

Work done = difference of two above quantities

$$= c(\tau_1 - \tau_2) \log_e \frac{v_2}{v_1}.$$

Efficiency = ratio of work done to heat expended

$$= \frac{c(\tau_1 - \tau_2)\ \log_e \frac{v_2}{v_1}}{c\tau_1 \log_e \frac{v_2}{v_1} + K_v(\tau_1 - \tau_2)} = \frac{\tau_1 - \tau_2}{\tau_1} \cdot \frac{1}{1 + \frac{K_v(\tau_1 - \tau_2)}{c \log_e \frac{v_2}{v_1} \cdot \tau_1}}$$

Now the first factor in the above result, viz. $\frac{\tau_1 - \tau_2}{\tau_1}$, represents the maximum efficiency attainable under any circumstances, and the second factor is a fraction, the denominator of which is greater than unity; therefore the efficiency in this case is less than the maximum efficiency.

It has been already pointed out (see page 89) that in order to approximate to perfect efficiency the fraction $\frac{\tau_1 - \tau_2}{\tau_1}$ must be made as large as possible, unity being its limit. This can only be done by increasing τ_1 and diminishing τ_2 as far as practicable. In other words the efficiency of a perfect engine will become greater in proportion as the temperature of the source of heat is increased, and that of the source of cold diminished. In practice we are extremely limited in the power of choosing both temperatures for two reasons: firstly, because the range of temperatures in nature is limited, for commercial purposes, by those of the products of combustion of coal on the one hand and ice on the other; and, secondly, because the highest temperatures thus available are far more than sufficient to destroy the substances of which engines are constructed.

If, by special materials and appliances, a heat engine could be made to use air having the higher limit of 1000°, and the lower limit of 493° absolute, equal respectively to 539° and 32° Fahrenheit, the theoretical efficiency of such an engine would be given by the fraction $\frac{1000 - 493}{1000} = \frac{507}{1000}$ or nearly $\frac{1}{2}$; that is to say, for every unit of heat supplied to such an engine, only half a unit could possibly be converted into mechanical work.

Laws connecting the Pressure, Volume, and Temperature of Dry Steam.

It will now be possible to apply the principles of the foregoing pages to the case of steam. As was explained in the last portions of Chapter II., dry saturated steam is a very imperfect gas, and the laws connecting its pressure, temperature, and volume cannot be expressed with the same simplicity as those relating to a perfect gas. In fact, the formulæ hitherto devised are purely empirical. The general results which were stated in Chapter II. will now, for convenience in calculation, be expressed in algebraical language.

Rankine's and Zeuner's laws connecting the pressure and volume of dry steam.—The product of the pressure and volume of dry saturated steam, instead of being expressed by the very simple law, pv=constant, is given according to Rankine by the equation

$$pv^{\frac{17}{16}}=\text{constant}.$$

According to Zeuner the index $\frac{17}{16}=1{\cdot}0625$ should be $1{\cdot}0646$. If the pressure be expressed in pounds per square inch and the volume in cubic feet, the constant has the value 475, and adopting Zeuner's index the equation becomes

$$pv^{1{\cdot}0646}=475.$$

The above formula relates to one pound weight of steam. The volume occupied by one pound of steam is called the *specific volume.* The density of the steam of course diminishes as the specific volume increases, and can be calculated by solving the above equation for v, which can be done with the help of a table of common logarithms, thus, $\log p+1{\cdot}0646\log v=\log 475=2{\cdot}6766936$

$$\therefore \log v=\frac{2{\cdot}6766936-\log p}{1{\cdot}0646}=2{\cdot}516-{\cdot}939\log p.$$

Connection between the pressure and temperature of dry steam.—The connection between the pressure and tempera-

ture of steam is also much more complicated than the corresponding relation for gases. It will be remembered (see page 75) that for common air the relation was expressed by the formula $pv = 53{\cdot}2.\ \tau$. In the case of steam there is no formula of general application, and the temperature must be taken for each pressure from the Table, page 489 *et seqq.*

Specific heat of water and steam.—The specific heat for water and steam is equally complicated. In the case of air, we have seen (see page 76) that the specific heat at constant pressure is a fixed quantity, $K_p = 183{\cdot}35$ foot-pounds, and the specific heat at constant volume K_v is also a constant quantity $= 130{\cdot}25$ foot-pounds. Also the heat expended in effecting any change of state in the air can be easily calculated, when we know these specific heats, and the quantity of external work done. In the case of water, however, the specific heat is constantly increasing from 772 foot-pounds at the point of maximum density, 39°F., and is about 802·88 foot-pounds when the temperature is 400°. While the water is being changed into steam, large quantities of heat are supplied to it which produce no effect upon the thermometer, and which consequently cannot be measured by any reference to the specific heat of the substance at that stage. When the water has become dry saturated steam, any further heat supplied to it does certainly raise the temperature, but it also changes the state from the saturated to the superheated condition.

When once thoroughly superheated, the properties of steam resemble those of a perfect gas, and may be reasoned on accordingly. The equation connecting its pressure, volume, and absolute temperature in this state is $pv = 85{\cdot}5\ \tau$, and differs from the corresponding equation for air only in the value of the numerical constant, which for air was proved to be 53·2. The heat required to raise one pound of superheated steam at constant pressure through one degree is given as 370·56 foot-pounds, and at constant volume as 285·03 foot-pounds, while the ratio of the first of these quantities to the last is $\gamma = 1{\cdot}3$.

Total heat required to convert water into steam.—In consequence of all these complications, we cannot deal with the quantities of heat required to effect changes of state in water and steam with the same ease and simplicity as in the case of gases and air. The total heat required to change water of a given temperature into steam at a given constant pressure is (see page 62) divisible into 3 parts, viz.

1st. The quantity required to heat the water from the given temperature to the natural temperature of formation of the steam ;

2nd. The quantity required to change the substance from the liquid to the gaseous state ;

3rd. The quantity required to do external work, that is, to overcome the external pressure through the space represented by the difference between the volumes of the steam and the water from which it was formed.

The two latter quantities taken together are usually called the *latent heat* of evaporation. It is, however, necessary to bear in mind that this latent heat consists of two elements.

The total heat required for any particular case may be extracted from the tables, the original temperature of the water being supposed to be 32° ; or it may be calculated very approximately by the following empirical rule :—

The total heat required to change one pound of water of 32° into steam of atmospheric pressure is known by experiment to be 885,200 foot-pounds, and for every degree of increase of temperature of the steam about 235·46 foot-pounds have to be added. Thus if T be the temperature of the steam, the total heat required is

$$885{,}200 + 235{\cdot}46\ (T - 212°).$$

If the water be warmer than 32° to start with, less heat will be required. If the specific heat of water were constantly 772 foot-pounds, we should have no trouble in calculating the quantity to be subtracted, and we may without sensible

error regard the specific heat as constant for the usual temperatures of water. Thus if t be the temperature of the water, it would take $772\ (t-32°)$ foot-pounds to heat the water from 32° to $t°$; this quantity must therefore be subtracted in the above formula, so that we get total heat required

$$=885{,}200+235{\cdot}46\ (T-212°)-772(t-32°).$$

The quantity of heat required to do external work in the above is not difficult to calculate. Let v be the volume occupied by the steam when formed in cubic feet. v' the space occupied originally by the water. Then the external work done is equal to the increased space occupied by the steam above that occupied by the water, viz. $(v-v')$, multiplied by the constant pressure of formation of the steam, viz. p pounds per square foot; or, external work done$=p(v-v')$ foot-pounds. Now v' is the space occupied by a pound of water, viz. ·016 cubic foot, and is so small that it may in general be neglected. Hence we may write :—

External work$=pv$ foot-pounds.

The product pv may be calculated from the equation given above (see page 97), viz. $pv^{1\cdot0646}=475$.

Another equation has been devised by Zeuner, and is commonly used, on account of its convenience, to express the external work empirically. Let h be the quantity of heat in foot-pounds required to heat the water from 32° to T the temperature of the steam; then,

External work $= 15{,}450+846T-h$ foot-pounds.

If we know the total heat required to raise the water from 32° to the temperature of the steam, and then evaporate it, and if we also know by calculation the external work done, we can deduce the quantity of heat spent in doing internal or latent work during evaporation, for,

The total heat$=h+$ internal work$+$external work.

$$\therefore\ 885{,}200+235{\cdot}46\ (T-212°)$$
$$=h+\text{internal work}+(15{,}450+846T-h)$$
$$\therefore\ \text{internal work}=819{,}832-611\ T \text{ foot-pounds.}$$

Expenditure of Heat in a Steam Engine.

We can now examine a question of great practical importance, viz. what quantity of heat must be supplied to a steam engine, in order to get a certain amount of work out of it. Steam engines regarded as heat engines may be divided into two principal classes, viz. 1. Those which work with full pressure of steam throughout the entire stroke, and 2. Those which use the full pressure of the steam during a portion only of the stroke, and then cut off all connection between the cylinder and the boiler, allowing the steam which is isolated in the cylinder to expand to the end of the stroke. The former are called non-expansive, the latter expansive engines. We will first consider the case of a non-expansive engine, and will suppose the steam supplied to it to be in the dry saturated condition, and free from moisture. It is necessary to mention the subject of moisture, because in the generality of cases the steam which enters a cylinder is not pure, but carries over with it from the boiler a large per-centage of suspended moisture.

How to represent heat by an equivalent pressure on the piston.—We have seen, in the case of the external work done by an engine, that the heat expended in doing this work can be represented by the pressure in pounds on the piston multiplied by the space in feet through which the piston moves.

The heat expended in raising the temperature of the water and in doing any internal work can be represented in a similar manner, by a pressure on the piston multiplied by the space through which it moves. Thus referring to fig. 13, page 63, it will be remembered that the total heat expended in converting a pound of water into steam was represented by the area of the three rectangles BA, AC, and CE, of which CE represented the heat spent in raising the temperature of the water, AC the heat spent in doing internal work while changing the water into steam, and BA the heat

spent in doing external work while the volume of the steam was increasing at constant pressure.

Now, we may take the line OA as representing the stroke of a steam engine, working with full steam throughout. The pressure, due to the resistance to the motion of the piston (see pages 62 and 63), is represented by the vertical line OB, and the external work done during one stroke of the piston is represented by the area of the rectangle BA. Now, the internal work done when changing the water into steam is represented by the area of the rectangle AC, and as both these rectangles have the same base, OA, their areas are to each other in the same ratios as the vertical lines, OB and OC; and just as OB represents the pressure due to external work, so OC may be said to represent an ideal pressure due to internal work, and CD an ideal pressure due to the heating of the water, and the sum of the three lines, viz. BD, represents a pressure due to the total expenditure of heat.

Now, if the area of the piston be one square foot, then the volume occupied by the steam at the end of the stroke or v=OA, and if H is the total heat expended expressed in foot-pounds, viz. the area BE, then the line BD, representing the pressure due to the heat expended, multiplied by v, equals the area BE, or denoting the line BD by P, we have

$$P \times v = H, \text{ or } P = \frac{H}{v}.^{1}$$

That is to say, the ideal pressure, due to the heat expended, equals the heat expended expressed in foot-pounds, divided by the volume in cubic feet occupied by the steam.

If v equals unity, then the work done by the piston equals the pressure on it, equals p; that is to say, when the piston travels through one cubic foot, the external work done equals

[1] Properly speaking v should be diminished by the volume of the original pound of water, but as this is very small the correction has been omitted in this and the succeeding calculations.

p, and the total heat expended equals P; or expressed generally, the work done per cubic foot swept through by the piston $=p$, and the heat expended $=$P.

Let us take the case of a condensing engine, working with dry saturated steam of 60 pounds pressure on the square inch, and find out now much useful work it can do per pound weight of steam used, and how much heat has to be expended in order to do the work.

At first sight it might be thought that the useful work done equals the pressure on the piston multiplied by the space through which it moves, and this would be the case if there were a perfect vacuum at the back of the piston. It is, however, impossible to realise a perfect vacuum with a condenser, and consequently the back of the piston experiences a pressure varying according to the perfection of the vacuum, and acting in the contrary direction to the steam pressure. This back pressure in a condensing engine usually varies from 2 to 4 pounds per square inch. In the case of a non condensing engine, the back of the piston is pressed upon by the whole weight of the atmosphere, or 14·7 pounds per square inch, as well as by the residual pressure of the exhaust steam, which cannot escape quickly enough through the narrow exhaust passages, so that with these engines the back pressure varies from 16 to 19 pounds per square inch.

In the present example the back pressure is taken as three pounds, and the effective forward pressure of the steam is $60-3=57$ pounds per square inch. Now the specific volume of steam of 60 lbs. is (see page 493) 7·037 cubic feet, and the effective external work or pv consequently equals $7{\cdot}037\times57\times144=57758{\cdot}4$ foot-pounds. This then is the useful work which one pound of steam can realise when worked non-expansively. Also as one horse-power per hour is 33000×60 foot-pounds, an engine working under the above conditions would require $\frac{33000\times60}{57758}=34{\cdot}28$ pounds weight of steam per horse-power per hour.

Now, the heat expended in order to attain this result is the whole heat of formation of the pound of steam at the temperature corresponding to 60 pounds pressure, viz. 293°. If the water were taken originally at 32°, this quantity of heat would be 904,106 foot-pounds (see Table I.), but if the water were taken originally at the temperature of the condenser, which would be about 104°, we should have to subtract from the above quantity the heat requisite in order to raise the water from 32° to 104°. This latter quantity expressed in foot-pounds would be approximately $(104°-32°)772=55,584$, and consequently the total heat expended would be $904,106-55,584=848,522$ foot-pounds.

Thus we see that, in order to obtain 57,758 foot-pounds of external work we have to expend 848,522 foot-pounds of heat, and the consequent efficiency of the engine would only be $\frac{57758}{848522}=\cdot068$; a result which shows how wasteful such a form of steam engine is. The above result, bad though it be, is far more favourable than anything that would take place in practice; for it must be remembered that we have not taken account of any losses due to radiation, conduction, leakage, &c., and we have supposed the steam to be formed without any waste of fuel; whereas we know that even in the best boilers this waste is very considerable.

At the end of the stroke the whole of the heat in the steam, which is equal to the heat of formation minus the external work done, is rejected into the condenser, part of it heating the injection water and the remainder is wasted.

If we desire to express the above quantities, per cubic foot swept through by the piston, instead of per pound of steam used, we can do so very simply. For the heat expended equals the work done plus the heat rejected. Now, if H be the total heat of formation of the steam from the temperature of the water, and v its specific volume in cubic

feet, then $\frac{H}{v}$ equals the heat of formation of one cubic foot of the steam. Also the external work done per cubic foot equals the pressure of the steam per square foot, equals p, and the heat rejected is the heat left in the cubic foot of steam after it has done its work $= \frac{H}{v} - p$. Now, part of the work done is spent in overcoming the resistance of the back pressure. If p_b be the back pressure per square foot, the work so spent per cubic foot swept through by the piston is also equal to p_b, and as this work is done upon the condensing steam it reappears as heat in the condenser, and must consequently be reckoned as so much heat rejected. Therefore the heat rejected, instead of being $\frac{H}{v} - p$, will be $\frac{H}{v} - p + p_b$.

Expansion of Steam.

It is evident from the foregoing that the heat rejected is very nearly equal to the total heat supplied. The only way of increasing the efficiency of a steam engine is to utilise some of this wasted heat. This object can be attained by cutting off all communication between the cylinder and the boiler when the former is partly filled, and then allowing the steam to expand during the remainder of the stroke. The amount of the expansion may be varied, according as the steam is cut off earlier or later during the stroke. There is theoretically no reason why the expansion should not be carried on till the final pressure of the steam equals the back pressure; but there are practical reasons, which will be explained hereafter, which render such high degrees of expansion inexpedient.

The great economical advantage of using steam expansively will be seen at once from the diagram, fig. 21. Let the steam be, as before, of 60 pounds pressure per square inch above zero, and the original temperature of the water 104°.

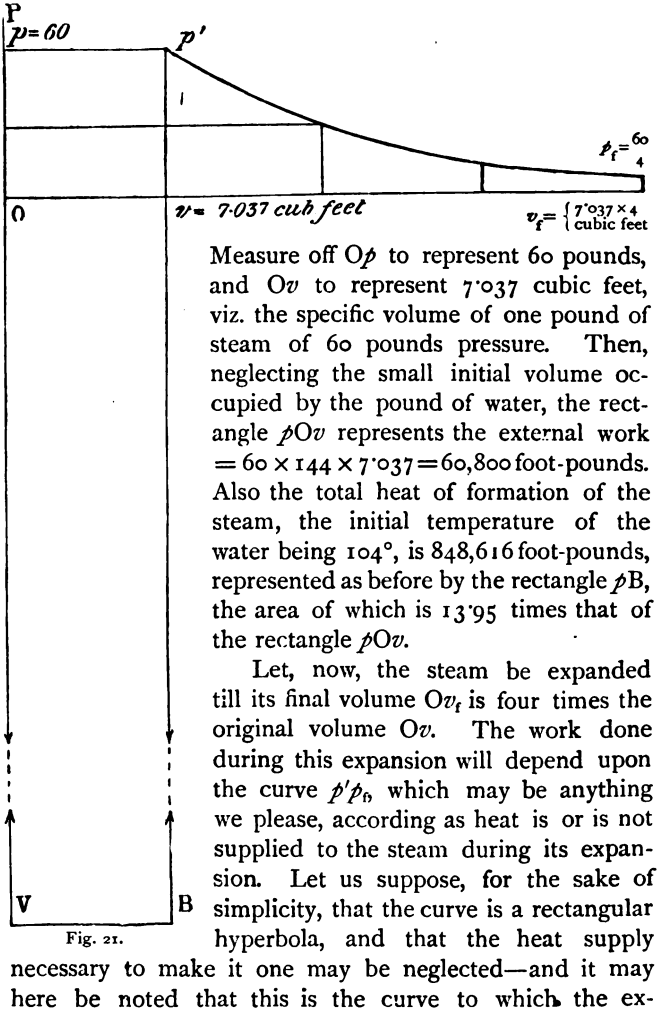

Fig. 21.

Measure off Op to represent 60 pounds, and Ov to represent 7·037 cubic feet, viz. the specific volume of one pound of steam of 60 pounds pressure. Then, neglecting the small initial volume occupied by the pound of water, the rectangle pOv represents the external work $= 60 \times 144 \times 7{\cdot}037 = 60{,}800$ foot-pounds. Also the total heat of formation of the steam, the initial temperature of the water being 104°, is 848,616 foot-pounds, represented as before by the rectangle pB, the area of which is 13·95 times that of the rectangle pOv.

Let, now, the steam be expanded till its final volume Ov_f is four times the original volume Ov. The work done during this expansion will depend upon the curve $p'p_f$, which may be anything we please, according as heat is or is not supplied to the steam during its expansion. Let us suppose, for the sake of simplicity, that the curve is a rectangular hyperbola, and that the heat supply necessary to make it one may be neglected—and it may here be noted that this is the curve to which the expansion line of steam most nearly approximates, under the ordinary circumstances of a well-constructed steam engine.

The area of the portion $p'vv_fp_f$ of the diagram is then $pv \log_e r$ or $p_fv_f \log_e r$; for since the curve is a rectangular hyperbola $p \times v = p_f \times v_f$. Also the total area pOv_fp_fp', representing the total external work done, is $pv + pv \log_e r = pv(1 + \log_e r)$. In the present instance $r = 4$, and $\log_e r = 1{\cdot}3863$, and $pv = 60{,}800$ foot-pounds, $\therefore pv(1 + \log_e r) = 60{,}800 \times 2{\cdot}3863 = 145{,}087$ foot-pounds. In order to obtain the useful work done, we must subtract from the above amount the work expended in overcoming the back pressure, say of 3 lbs. per square inch. As the back pressure is overcome through a space of $4 \times 7{\cdot}037$ feet, the work done $= 3 \times 144 \times 4 \times 7{\cdot}037 = 12{,}160$ foot-pounds, and the useful work therefore $= 132{,}927$ foot-pounds, against 57758·4 in the previous example, when there was no expansion. Now the expenditure of heat was shown to be 848,616 foot-pounds, and the efficiency is therefore $\frac{132{,}927}{848{,}616} = {\cdot}156$, as against ·068 in the case of the non-expansive engine. The ratio of the external work done to heat expended is represented graphically by the ratio of the area of the figure $pp'p_fv_fO$ to the area of the rectangle pB.

To find expenditure of heat when condition of steam at end of stroke is given.—The above result is only true if the expansion curve be a rectangular hyperbola. If, however, some other condition had been given, such as, that throughout, and at the end of the expansion, the steam should be dry and saturated, we should have had a different result; for we know that the expansion curve of dry saturated steam is not $pv =$ constant but $pv^{1{\cdot}0646} =$ constant.

In practical experiments it is found easier to ascertain the quantity and state of the steam at the end of the stroke, rather than at the point of cut off; we shall therefore next show how to find the expenditure of heat, when the condition of the steam at the end of the stroke is given, and the work done is known.

In the first instance, suppose that the original pressure of

the pound of steam is p, the final pressure p_f, and that the steam is dry and saturated at the end of the stroke. Now, the heat rejected is the amount of heat in the steam at the end of the expansion, together with the work done upon the exhausting steam, by the piston overcoming the back pressure p_b. The heat in the steam at the end of the expansion is the total heat of formation of dry saturated steam of the pressure p_f, minus the amount due to doing external work$=p_f v'_f$; for, of course, the heat spent in doing external work disappears from the steam, having been transmuted into the work done.

$$\text{Consequently heat rejected} = H_f - p_f v'_f + p_b v'_f,$$

where $p_b v'_f$ is the work done in overcoming the back pressure p_b, through the space v'_f, and H_f is the total heat of formation of dry saturated steam of the pressure p_f.

Now, the heat expended equals the heat rejected, plus the external work done by the steam during admission and expansion. If p_m be the mean or average pressure throughout the stroke, then $(p_m - p_b)v'_f$ is the external work done, and consequently

$$\text{Heat expended} = H_f - p_f v'_f + p_b v'_f + (p_m - p_b)v'_f$$
$$= H_f + p_m v'_f - p_f v'_f$$

If we wish to express these results per cubic foot swept through by the piston, we have only to divide by v'_f, the number of feet occupied by the steam at the end of the stroke, and we get

$$\text{Heat expended} = \frac{H_f}{v'_f} + p_m - p_f.$$

Steam Jackets.—We must now ascertain whether the heat contained in the steam, as supplied by the boiler, is as much as the above quantity $\frac{H_f}{v'_f} + p_m - p_f$, for if not, one of two things must happen, viz. either more heat must be supplied to the steam while it is in the cylinder from some

external source, or else at the end of the stroke it will not be dry and saturated, but a certain proportion will be condensed into water.

Let H_i be the total heat contained in a pound of the steam, in its initial condition, as supplied by the boiler, then, as v_f equals the contents of the cylinder, or the number of cubic feet swept through by the piston in one stroke, therefore $\frac{H_i}{v_f}$ is the expenditure of heat per cubic foot swept through, provided that no heat is obtained from any other source than from the boiler. But the heat actually expended per cubic foot swept through is not $\frac{H_i}{v_f}$ but $\frac{H_f}{v_f} + p_m - p_f$. Subtracting, therefore, the first from the last of these quantities, we get a difference $= p_m - p_f + \frac{H_f - H_i}{v_f}$. Now, the numerical value of the two quantities H_i and H may be taken from Table I., and as H_i is always greater than H_f the quantity $\frac{H_f - H_i}{v_f}$ will always be negative, and for every particular case the difference $p_m - p_f + \frac{H_f - H_i}{v_f}$ will be found to be a positive quantity ; therefore, the heat actually wanted for the steam in order that it may remain dry and saturated is greater than the quantity present in the steam as it is supplied by the boiler.

The difference must therefore be supplied to the steam while in the cylinder, and this is usually effected by surrounding the latter with a casing always kept full of boiler steam or hot air. The temperature of the steam in the jacket is evidently greater than the mean temperature of the steam in the cylinder, and consequently heat will flow from the former to the latter, and will either check or wholly prevent condensation, according to the quantity of heat thus supplied.

The question, whether or no it is desirable to prevent

condensation during expansion, is a rather complicated one, and will be discussed in Chapter XI.

In the foregoing we were only concerned with proving that if the condition be given that the steam must be dry and saturated at the end of the expansion, in order to fulfil this condition, heat must be supplied to the steam from a hot casing, which is generally called a steam jacket.

Rankine's empirical formulæ for the expenditure of heat in a steam engine.—From the above formula for the expenditure of heat, viz. : $\frac{H_f}{v_f}+p_m-p_f$ it would be easy to construct a numerical formula involving only the mean and final pressures, and the temperature of the steam and feed water and certain constants. It has, however, been found by Rankine that the results are equally well given by a very simple empirical formula which for condensing engines is :

$$\text{Heat expended} = p_m + 15p_f;$$

and for non-condensing engines :

$$\text{Heat expended} = p_m + 14p_f:$$

the results being expressed in foot-pounds per cubic foot swept through by the piston.

Theory of the Perfect Heat Engine applied to the Case of Steam.

We can now proceed to apply the principles laid down with regard to perfect heat engines to the case where steam is employed instead of a gas. The amount of steam and fuel necessary for a perfect steam engine under given circumstances will first be considered. The nature of the diagram indicating the varying states of the steam in such an engine will then be examined, and finally the question will be discussed how far actual steam engines of the best

construction comply with, and how far they depart from, the conditions of maximum efficiency.

The efficiency of a perfect heat engine has been shown (see page 89) to be $\frac{\tau_1 - \tau_2}{\tau_1}$, where τ_1 and τ_2 are the absolute temperatures of the sources of heat and cold. Hence, in such an engine, if H be the quantity of heat supplied, and W the exterior work done, we obtain the relation

$$\text{H.}\ \frac{\tau_1 - \tau_2}{\tau_1} = \text{W or H} = \text{W.}\frac{\tau_1}{\tau_1 - \tau_2}.$$

Hence if we require to know the least amount of heat necessary in order to obtain one horse-power per hour when the limits of temperature within which the engine works are known, we have $\text{W} = 33{,}000 \times 60 = 1{,}980{,}000$ foot-pounds,

$$\text{and H} = 1{,}980{,}000 \times \frac{\tau_1}{\tau_1 - \tau_2} \text{ foot-pounds.}$$

In a steam engine the limits of temperature ought to be the temperature of the hot gases in the furnace of the boiler on the one hand, and the temperature of the condenser on the other, or in the case of non-condensing engines, the lower limit is the temperature due to the pressure of the atmosphere, i.e. $212° + 461°$ absolute. We possess at present, however, no means of utilising the temperature of the furnace gases, and consequently the higher limit in a steam engine must be taken to be the temperature of the steam in the boiler.

Let us consider the case of a perfect engine working with steam of 60 pounds pressure, as before; the temperature of the condenser being $104° + 461°$ absolute. The absolute temperature of steam of 60 pounds pressure is $293° + 461° = 754°$. The quantity of work obtained per pound of steam is the total heat contained in the steam, multiplied by the fraction $\frac{754 - 565}{754} = \frac{1}{4}$ very nearly. Now, a perfect

engine, as will be seen presently, always uses the same water over and over again, and always evaporates it from the temperature of the steam; consequently the heat supplied to the water in order to turn it into steam of 293° is less than the quantity given in Table I., by the amount necessary to heat the water from 32° to 293°; that is to say, the quantity of heat in question is 904,106 – 202,444 =701,662 foot-pounds. Therefore the work obtainable per pound of steam is $701,622 \times \frac{1}{4} = 175,413$ foot-pounds.

In order to obtain from this engine one horse-power per hour we should require to expend therefore $\frac{1,980,000}{175,413}$ =11·3 pounds of steam. If we wish to find out at what expenditure of fuel this power is attained, we must know what heat can be developed by the combustion of a given weight of fuel. This subject will be fully dealt with in the chapter on boilers; but at present it may be stated that one pound of good average coal properly burnt should give 12,000,000 foot-pounds of heat. Now as one pound of steam of 60 pounds pressure requires for its formation, from water of 293°, 701,662 foot-pounds, the pound of coal should theoretically be able to evaporate $\frac{12,000,000}{701,662} = 17$ pounds of water, and consequently we ought to require $\frac{11·3}{17} = ·66$ pound of coal per horse-power per hour.

The actual amount of water which a pound of fuel can evaporate in a good boiler is, however, much less than the above; in fact, as will be seen hereafter, it seldom exceeds eleven, and is more often from seven to eight pounds. If, for the sake of simplicity, we suppose that 11·3 pounds of water are evaporated by a pound of coal, then, in the case of the engine under discussion, we should require to burn one pound of fuel per horse-power per hour. In the best constructed modern steam engines working with steam of the pressure under discussion, viz. sixty pounds absolute,

or about forty five pounds above the atmosphere, the amount of fuel burnt per horse-power per hour is far greater than one pound. It is, in fact, never less than two pounds, while in engines of inferior construction, from eight to ten pounds, and even larger quantities, are consumed. We see, therefore, plainly, that, in addition to the loss of heat which takes place in the boiler, there are other sources of waste. It becomes, then, necessary to compare the working of an actual engine with that of the theoretical engine, step by step, in order to ascertain all the causes of inefficiency.

Causes of loss of efficiency in steam engines.—In accordance with the principles laid down, the water should receive all its heat at the fixed higher temperature; in other words, it should be turned into steam at constant pressure. The steam should then be allowed to expand adiabatically, till the temperature falls to that of the condenser. It should next reject heat into the condenser at this fixed temperature, though none of the steam itself is supposed to enter the condenser. Finally, at a given point, the cooled steam, or mixture of steam and water as it would probably be, should be compressed till it returns to its first condition of water, of the original temperature of the steam.

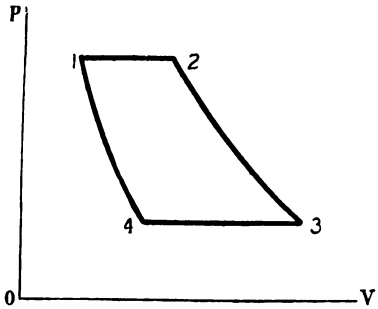

Fig. 22.

These changes are indicated by fig. 22.

Let the point 1 represent the volume and pressure of one pound of water, the pressure being that at which the steam is formed. In strict theory, the steam ought to be formed at the temperature of the furnace gases. This, however, as has been before stated, is practically impossible, and we will

suppose for the moment that the water receives its heat at the constant temperature, due to the pressure at which the steam is formed. During evaporation the pressure remains constant and the volume increases, till the whole of the water is converted into steam. This state of things is indicated by the point 2. The steam is now allowed to expand adiabatically along the curve 2 3 till the temperature has fallen to that of the source of cold. The volume is then reduced at the constant pressure, corresponding to the temperature of the source of cold, till the point 4 is reached, when the mixture of steam and water is compressed adiabatically along the line 4 1, till the whole is re-converted into water of the original temperature, pressure, and volume. The area of this diagram, as before, represents the external work done during the cycle of operations, and it may either be computed analytically, if we know the equations of the adiabatic curves, or, calculated more simply on the principle that the work done equals the heat supplied, multiplied by the efficiency of the engine; in other words it equals the heat necessary to turn a pound of water of the temperature τ_1 into steam of τ_1, multiplied by $\frac{\tau_1 - \tau_2}{\tau_1}$.

Now, in an actual steam-engine, the series of operations which takes place differs more or less at every step from that which has been just described. In the first place, the water, instead of receiving all its heat at the higher temperature τ_1, is introduced into the boiler as feed water at a much lower temperature. During the process of evaporation, the condition of receiving heat at constant pressure is fulfilled, so long as the pressure in the boiler is kept constantly the same, which it never can be when the steam is worked expansively.

During the expansion, the condition that heat shall not be supplied to, or abstracted from, the steam is not fulfilled, because the metals of which cylinders are constructed render the fulfilment of this condition impossible. Cylinders

are of three sorts ; the first sort is made of metal directly exposed to the outer air. In this case, the metal being a good conductor is rapidly heated by the steam, and parts with its heat to surrounding bodies by radiation and conduction, thus causing the steam to be cooled during its passage through the cylinder, so that the expansion line falls below the proper adiabatic curve.

The second class of cylinder is clothed with some non-conducting substance, so as to prevent the escape of heat to outside bodies. For the sake of simplicity, we will suppose the substance to be a perfect non-conductor. When first the steam enters such a cylinder it finds the metal cool, and parts with some of its heat ; after a few strokes, however, the cylinder gets warmed, and if the temperature of the steam remained uniform throughout the entire stroke, no further loss would ensue from this cause. But the temperature of the steam is only uniform while the line 1, 2 is being described ; after that point, and during expansion, the temperature drops from τ_1 to τ_2 ; consequently, when the steam enters, it finds the sides and end of the cylinder cooled down, they having just been in contact with steam of the temperature τ_2. Part of the heat of the steam, therefore, is spent in re-heating the metal of the cylinder. This causes part of the steam to condense, if it be originally in the dry and saturated condition. When, however, the expansion begins, the temperature of the steam rapidly drops below the temperature of the walls of the cylinder. These latter consequently give up part of their heat again to the steam, and partially re-evaporate the condensed portions. This re-evaporation is facilitated by the circumstance that a portion of the condensed steam has the temperature τ_1, and consequently, when the expansion commences and the pressure falls, it is too hot to remain any longer in the condition of water, and its surplus heat helps in re-evaporating it. Thus, though no heat is lost to external objects during the stroke, when the cylinder is perfectly clothed,

still, during one period, heat is taken from, and during another period given back to the steam, by the cylinder, and consequently, the curve of expansion is not, strictly speaking, adiabatic. The exact effect of this peculiar action on the shape of the expansion curve is difficult to ascertain, because the rapidity with which the metal of the cylinder can take up and give off heat is not accurately known. The subject will, however, be further examined in Chapter XI.

The third description of cylinder is surrounded by a jacket or casing filled with steam from the boiler, and which is itself covered with non-conducting substances so as to prevent the escape of heat to the outside. It is evident that in this case the temperature in the jacket is higher than the average temperature in the cylinder, and consequently heat will flow from the former to the latter throughout a great part of the stroke, and will thus tend to raise the curve of expansion above the adiabatic line. The effect of the action of the jacket upon the working of the steam engine will be more particularly considered in Chapter XI.

During the third operation in an actual steam engine, viz. the rejection of heat, the condition of maximum efficiency is not fulfilled, for the heat is not all rejected at the temperature of the condenser. If the expansion were carried so far that the temperature of the steam were reduced to the temperature of the condenser, this condition could be fulfilled, but in practice it is not found possible to carry the expansion so far, and consequently, when condensation commences, there is a sudden drop from the temperature due to the terminal pressure of the steam, to that of the condenser. This is illustrated by the diagram fig. 23, where the point 3 represents the pressure of the steam at the end of the expansion, and the vertical ordinate of the point 4 represents the back pressure due to the temperature of the condenser. When the steam commences to reject its heat, the temperature suddenly falls from that due to the pressure of the point 3 to that due to point 4. In strict theory the

expansion should have been continued to the point 3′, where the curve intersects the horizontal line of back, or condenser pressure.

During the remainder of the period of heat-rejection the condition of maximum efficiency is fulfilled very approximately. At this part of the process, however, another evil arises, for the metal of the cylinder was heated up to a certain temperature during the admission and expansion of the steam; when, however, the steam is being condensed, its temperature is much lower, and consequently the cylinder parts with some of its heat to the condensing steam, thus retarding condensation, and cooling the cylinder down ;

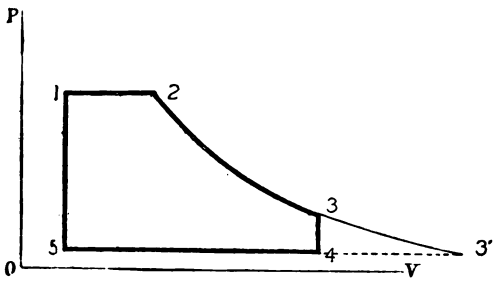

Fig. 23.

so that, as has been before stated, some of the fresh steam on entering is condensed. If the cylinder be provided with a steam jacket the first of these evils may be increased, for, during half the time such an engine is running, the steam jacket is employed in wasting heat on the condensing steam.

The fourth condition of maximum efficiency, viz. that at a certain point the rejection of heat should be stopped, and the mixture of steam and water in the cylinder should be compressed into water of the original pressure and temperature, which water should be used over again in the boiler, is not fulfilled at all in the ordinary steam engine. If the engine be of the condensing type, the condensation is

carried out completely, and all the heat in the steam is spent in warming up from 20 to 30 times its own weight of water employed in the condensation to a temperature about one third of that of the water in the boiler. Of this water, only one twentieth to one thirtieth can be used over again to feed the boiler, and must, when in the boiler, be suddenly raised from the temperature of the condenser to that of the steam, thus infringing the first condition of efficiency.

If, on the other hand, the engine be a non-condenser, the steam all escapes into the open air, and is there condensed, and the boiler is fed with cold water, unless some special provision is made for heating the latter with waste steam, or furnace gases, which arrangement has, of course, nothing to do with the engine, properly so called.

We thus see that the actual engine differs at every stage of its working from the theoretically perfect heat engine, and these differences are multiplied and rendered more complicated by numerous other circumstances which will presently be referred to. For instance, it has been taken for granted, in all that has gone before, that the engine receives dry saturated steam from the boiler. Now, as a matter of fact, boilers do not usually deliver dry steam, but send over large quantities of hot water with the steam into the cylinders. When this takes place, the calculations for heat expended and work done have to be materially modified; for it is evident that a large quantity of heat has been spent in warming this water up, from the temperature of the feed to that of the steam; which heat is wholly or in greater part wasted, as no work can be done by this water unless it evaporates in the cylinder. Under the most favourable circumstances the water can only be partially evaporated, viz. when a jacket supplies heat to the steam in the cylinder, and when by expansion the pressure of the steam is so far lowered that the water is too hot to remain water at the lower pressure, and consequently expends its surplus heat in partially evaporating itself. This subject of wet steam is

chiefly of interest in so far as it affects the subject of jacketing; and will consequently be referred to again in Chapter XI.

Again, in all that has gone before, it has been assumed that the action of the valves which admitted the steam from the boiler to the cylinder, and from the cylinder to the condenser, was perfect; that is to say, that they opened and closed fully and quickly, precisely at the proper moment, and in no way by their slowness of motion or imperfect design strangled the steam on its passage to or from the cylinder. In actual steam engines the valves not infrequently fall short of this ideal perfection.

The results usually attained in practice fall short of what they should do owing to the three following sets of causes :—

1. The boiler is imperfect, inasmuch as it wastes heat, and delivers water along with the steam to the cylinder.

2. The engine, considered merely as a heat engine, is imperfect for the following reasons :—

a. The limits of temperature within which it is possible to work it in practice are narrow, so that the numerical value of the fraction $\frac{\tau_1 - \tau_2}{\tau_1}$ is very small.

b. The series of operations does not comply with the conditions of maximum efficiency, for the heat is neither received nor rejected at constant temperatures, and the metal of which the cylinder is made is capable of absorbing, transmitting, and radiating heat, so that adiabatic expansion is impossible.

3. The engine, considered as a piece of mechanism, is defective for the following reasons:—

a. Work is lost in friction of the different moving parts.

b. The passages conveying the steam from the boiler to the cylinder, and from the latter to the open air or condenser, always impede somewhat the motion of the steam, thus diminishing the useful pressure on the piston and increasing the back pressure.

c. It is impossible to avoid leaving, and is even necessary to allow a certain space between the piston, when at its extreme positions and the face of the cylinder cover, which space, together with the cubic contents of the steam port, is called *clearance.* Now, it is evident that this clearance space has to be filled with fresh steam at every stroke, which does no work except when expanding, and consequently causes a loss of efficiency.

The losses due to the imperfections of the boiler and of the mechanism will be duly considered in the chapters devoted to these subjects. The losses due to the defects of the engine proper considered as a heat engine have been considered so far as space and the scope of this work will allow in the present chapter. Some of them will, however, be referred to again in Chapter XI., which deals principally with the refinements of the engine, contrived to neutralise the deficiencies.

Summary of the Laws and Formulæ of Thermodynamics.

It may be useful and convenient to sum up here the principal laws and formulæ of the science of heat, as explained in this and the preceding chapter.

First Law of Thermodynamics.—Heat and mechanical energy are mutually convertible. A unit of heat requires for its production, and produces by its disappearance, a fixed amount of mechanical energy.

British unit of heat.—The unit of heat used in this country is the quantity of heat required to raise one pound of water of the temperature 39·3° through one degree Fahrenheit.

Mechanical equivalent of heat.—One British thermal unit is equivalent to 772 foot-pounds of mechanical work.

Second Law of Thermodynamics.—Heat cannot pass from a cold to a hot body by a self-acting process unaided by external agency.

Boyle's law applied to gases.—The product of the pressure and volume of a portion of gas is a constant quantity so long as the temperature remains constant.

For air at 32° the constant quantity is 26,214 foot-pounds. Hence the expression of the law for air is :

$$pv = 26{,}214 \text{ foot-pounds.}$$

Law of Charles and Gay Lussac applied to gases.—When the pressure is constant all gases expand alike for the same increase of temperature. The amount of the expansion between 32° and 212° is ·3654 of the original volume; and for each degree between 32° and 212° it equals $\frac{\cdot 3654}{180} = \cdot 00203$.

Similarly, when the volume remains constant the pressure varies in the above proportion.

Combination of the two foregoing laws.—The product of the pressure and volume of a portion of gas is proportional to the absolute temperature. Thus :

$$\frac{p_1 v_1}{\tau_1} = \frac{pv}{\tau} \text{ and } \therefore p_1 v_1 \tau = pv\tau_1.$$

N.B. The absolute temperature is equal to the ordinary temperature on Fahrenheit's scale plus 461°.

Hence, remembering that the absolute temperature of 32° is 493, and that the value of pv for 32° is 26,214, we get the very important law

$$pv = 53{\cdot}2\,\tau.$$

The specific heat of gas at constant pressure is the same at all temperatures.

The mechanical equivalent of the heat required to raise one pound of air, one degree, at constant pressure is :

$$K_p = \cdot 2375 \text{ thermal unit} = 183{\cdot}35 \text{ foot-pounds.}$$

If a gas expand without doing external work, its temperature is unchanged.

The mechanical equivalent of the heat required to raise one pound of air, one degree, at constant volume is :

$$K_v = \cdot 1686 \text{ thermal unit} = 130\cdot 2 \text{ foot-pounds.}$$

The ratio of the two above numbers is :

$$\gamma = \frac{183\cdot 25}{130.2} = 1\cdot 408.$$

Expenditure of heat during isothermal expansion :

For isothermal expansion Boyle's law applies.

$$\therefore pv = \text{constant,}$$

and the external work done during the expansion

$$= pv \log_e r = c\,\tau \log_e r \text{ foot-pounds.}$$

As the temperature of the gas does not alter during the expansion, there is no internal work done, and, consequently, the above expression represents the total heat supply.

Expenditure of heat when expansion takes place according to the formula :

$$pv^n = \text{constant.}$$

The external work done during expansion

$$= \frac{p_1v_1 - p_2v_2}{n-1} = \frac{c}{n-1}(\tau_1 - \tau_2) \text{ foot-pounds.}$$

The internal work done in changing the temperature from τ_1 to τ_2

$$= K_v(\tau_2 - \tau_1)$$

Therefore, the total heat expended is the sum of the two above quantities :

$$= (\tau_2 - \tau_1)\left(\frac{nK_v - K_p}{n-1}\right)$$

Expenditure of heat during adiabatic expansion :

The results in this case are got by substituting $\gamma = 1\cdot 408$ for n in the above formula. Hence

heat expended in doing external work during expansion

$$= \frac{c}{\cdot 408}(\tau_1 - \tau_2) \text{ foot-pounds ;}$$

the internal work

$$= K_v (\tau_2 - \tau_1)$$

and the total heat supplied

$$(\tau_2 - \tau_1) \left(\frac{1{\cdot}408\ K_v - K_p}{{\cdot}408} \right)$$

The final temperature in adiabatic expansion is

$$\tau_2 = \tau_1 \left(\frac{1}{r} \right)^{{\cdot}408}$$

The efficiency of a perfect heat engine is the ratio of the difference of the absolute temperatures of the sources of heat and cold, to the absolute temperature of the source of heat

$$= \frac{\tau_1 - \tau_2}{\tau_1}$$

Law connecting the pressure and volume of dry saturated steam.

$$p v^{1{\cdot}0646} = 475$$

where the pressure is expressed in pounds per square inch and the volume in cubic feet.

$$\therefore \log. v = 2{\cdot}516 - {\cdot}939 \log. p.$$

The specific heat of water is constantly varying. It has the value unity = 772 foot-pounds only at the temperature 39·3°. At 400° the specific heat = 802·88 foot-pounds.

Superheated steam.

The law connecting the pressure, volume, and absolute temperature of superheated steam is

$$p v = 85{\cdot}5\ \tau$$

as against $p v = 53{\cdot}2\ \tau$ in the case of air.

The mechanical equivalent of the heat required to raise one pound of superheated steam, one degree, is

At constant pressure, 370·56 foot-pounds.
At constant volume, 285·03 foot-pounds.

The ratio of the two numbers, or $\gamma = 1{\cdot}3$.

Total heat required to change one pound of water of 32° *into steam of* $t°$ = 885,200 + 235·46 ($t°$ − 212°) approximately.

If the water were originally hotter than 32°, say $t_1°$.

Total heat required = 885, 200 + 235·46 ($t°$ − 212°) − 772 ($t_1°$ − 32°) approximately.

Of the above the quantity required to do external work

$$= p v \text{ foot-pounds.}$$

where $p v$ is calculated from the equation $p v^{1·0625} = 475$.

Zeuner's empirical equation for the heat expended in doing external work.

External work = 15,450 + 846 $t - h$ foot-pounds.

where h is the quantity of heat required to raise the water from 32° to the temperature of the steam.

Expenditure of heat per pound of steam expressed by an equivalent pressure.

$$P = \frac{H}{v}$$

where P is the equivalent pressure required. H is the heat of formation of one pound of steam in foot pounds, and v the corresponding volume of the steam.

Expenditure of heat per cubic foot swept through by the piston.

$$\text{Heat expended} = \frac{H}{v},$$

$$\text{and heat rejected} = \frac{H}{v} - p + p_b$$

where p = pressure of the steam per square foot, and p_b = the back pressure per square foot.

Expansive working of steam.

To find expenditure of heat when steam at end of stroke is dry and saturated.

Let H_f be the total heat of formation of one pound of steam having the pressure p_f per square foot at some point just before the end of the stroke and the corresponding

volume v_f. Let p_m be the mean pressure, and let the other symbols have the same meanings as before.

$$\text{Heat rejected} = H_f - p_f v_f + p_b v_f.$$

$$\text{Heat expended} = H_f - p_f v_f + p_b v_f + (p_m - p_b) v_f = H_f + p_m v_f - p_f v_f.$$

Heat expended per cubic foot swept through by the piston

$$= \frac{H_f}{v_f} + p_m - p_f.$$

Rankine's empirical formula for the expenditure of heat in a steam engine.

$$\text{Heat expended} = p_m + 15\, p_f \text{ for condensing engines.}$$
$$= p_m + 14\, p_f \text{ for non-condensing engines.}$$

CHAPTER IV.

CONNECTION BETWEEN THE SIZE OF AN ENGINE, THE EVAPORATIVE POWER OF THE BOILER, AND THE EXTERNAL WORK WHICH CAN BE DEVELOPED.

Navier's modification of Boyle's law applied to steam—Work done during expansion of steam calculated by Navier's formula—De Pambour's theory of the double-acting steam engine—Its two fundamental principles and their mathematical expression—Analysis of the resistance to the motion of the piston of a steam engine—Back pressure—Engine friction—Load ; horse power—Examples of the connection between the size of cylinder, the piston-speed, the rate of expansion, the evaporation in the boiler, and the power developed by the engine—Locomotive engines—Analysis of the resistance to be overcome by the pistons of locomotives—Back pressure—Engine friction—Resistance to uniform motion of engine, tender, and train—Resistance due to gradient—Resistance due to inertia of weights to be moved—Atmospheric resistance—Application of De Pambour's theory to locomotives—Examples.

IN the preceding chapters the physical properties of steam and gases, and the laws which regulate their changes of state, have been briefly considered. The purpose of the present chapter is more practical, it being proposed to show what mechanical work can be accomplished by an engine of a given size working at a given rate of expansion, when attached to a boiler capable of yielding a given quantity steam. Or, *vice versâ*, what quantity of steam must be evaporated in order that the engine may under the given circumstances perform the given quantity of work.

It would be very easy to devise suitable formulæ to solve these questions if the law of expansion of dry saturated steam in a cylinder could be expressed with the same simplicity as Boyle's law for the expansion of gases. To facilitate the calculation of the questions we will in this chapter make

use of an empirical modification of Boyle's law, discovered by Navier, and which is more suitable for the purposes of calculation than Rankine's law.

If we were to make use of Boyle's law, and to start with the constant obtained by multiplying the pressure by the corresponding volume of a pound of steam of atmospheric pressure, the calculated volumes would all be too small for pressures above the atmosphere, when compared with the volumes as given by actual experiment. To avoid this error, Navier increased the value of the constant in Boyle's law, and added a small constant to the number showing the pressure. Thus, instead of $pv = c$, Navier adopted a formula of the form

$$v = \frac{C}{p+k}$$

where C is a larger number than c, and k is the constant added to the pressure.

The numbers which give the best results are, for steam between the pressures of 20 lbs. and 180 lbs. absolute C = 28,200 and $k = 4$; and for steam below 20 lbs. C = 31,000 and $k = 4$. Hence the formula becomes in the two cases

$$v = \frac{28000}{p+4} \text{ and } v = \frac{31000}{p+4}.$$

The relative volume of steam is its volume compared with that of the water from which it is formed. The Tables give the absolute volumes of steam formed from one pound of water. Now as the volume of one pound of water is $\frac{1}{62{\cdot}42}$ cubic feet, we have only got to multiply the volumes as given in the Tables by 62·42 in order to get the relative volumes. According to this formula, the ratio of the volumes V_{P_1} to V_P at two different pressures P_1 and P would no longer be equal to the ratio of the pressures P to P_1 but to the ratio $P+k$ to P_1+k. Thus,

$$\frac{V_{P_1}}{V_P} = \frac{P+k}{P_1+k}$$

Calculation of the work done during the expansion of steam by Navier's formula.

Let P be the pressure of the steam on entering the cylinder in pounds per square inch. After it has raised the piston through the height l, let it be cut off and allowed to expand to the end of the stroke L. At any point x between l and L the pressure p may be got from the proportion

$$p+k : \mathrm{P}+k :: l : x$$

$$\therefore p=(\mathrm{P}+k)\frac{l}{x}-k$$

and multiplying each side of the equation by the elementary space dx and integrating between the limits $x=l$ and $x=\mathrm{L}$, we have

$$\int_l^{\mathrm{L}} p\,dx=(\mathrm{P}+k)\,l\log_e\frac{\mathrm{L}}{l}-k\,(\mathrm{L}-l).$$

The quantity on the right-hand side of the equation is the work done per square inch of piston during the expansion of the steam. If the area of the piston be A square feet, then the work done by the steam up to the time it was cut off is equal to the total pressure on the piston, or 144AP multiplied by the space through which it moves = 144APl. Add to this the total work done during expansion

$$=144\mathrm{A}\left[(\mathrm{P}+k)\,l\log_e\frac{\mathrm{L}}{l}-k\,(\mathrm{L}-l)\right],$$

we get

$$144\mathrm{A}\left[l\,(1+\log_e\frac{\mathrm{L}}{l})\,(\mathrm{P}+k)-k\mathrm{L}\right].$$

Let the resistance which can be just overcome by the steam be at the rate of R pounds per square inch, then we must have,

$$144\mathrm{A}\left[l(1+\log_e\frac{\mathrm{L}}{l})(\mathrm{P}+k)-k\mathrm{L}\right]=144\mathrm{ARL}$$

$$\therefore 1+\log_e\frac{\mathrm{L}}{l}=\frac{\mathrm{L}}{l}\times\frac{\mathrm{R}+k}{\mathrm{P}+k}.$$

In the above the fraction $\frac{L}{l}$ = the rate of expansion usually denoted by the letter E and the fraction $\frac{R+k}{P+k}$ is, according to Navier's formula, the ratio of the relative volumes of the steam at the pressures P and R respectively. If we represent these relative volumes by the symbols V_p and V_R we get the equation :

$$1+\log_e E=E\frac{V_p}{V_R}$$

by means of which all questions relating to the work done by steam when used expansively in a cylinder can be calculated.

For example, take an engine having a cylinder 30 inches diameter, and 60 inches stroke, working with steam having a pressure of 60 lbs. per square inch, cutting off at a quarter stroke. Find the work done per stroke.

We have $(1+\log_e 4)=4\times\frac{V_{60}}{V_R}$.

Now, $\log_e 4=1{\cdot}3862$, and $V_{60}=440$, from table of volumes ;

$$\therefore V_R=\frac{4\times 440}{2{\cdot}3862}=\frac{1760}{2{\cdot}3862}$$
$$=737{\cdot}5$$

which number corresponds, according to the table, to the relative volume for the pressure, 34·5 lbs. per square inch.

This, then, is the value of all the resistances to the motion of the piston, of whatever nature, reduced to pounds per square inch ; and the work accomplished in overcoming them, per stroke of the piston,

$$=34{\cdot}5\times\text{area of piston}\times 5\text{ feet}$$
$$=35\times 707\times 5$$
$$=121{,}957{\cdot}5\text{ foot-pounds.}$$

Similarly, we might have used the formula in order to determine at what pressure steam should enter the cylinder, in order that it might overcome a resistance of 34·5 lbs. per square inch, if cut off at a quarter-stroke.

De Pambour's Theory of the Double-Acting Steam Engine.[1]

De Pambour was the first to form a theory of the mechanical action of the steam engine, connecting the size of the cylinder, the evaporative power of the boiler, the rate of expansion, and the work done. His theory depends on the two following principles:—

1. When the engine is running at uniform speed, the work done by the steam on the piston is equal to the work due to overcoming the resistance to the motion of the piston.

2. The steam which is evaporated in the boiler is equal to that used in the cylinder.

The first of these propositions is evident to anyone acquainted with the elements of mechanics. If the average pressure on the piston were greater than the average resistance, the motion of the piston would be accelerated, and the engine would, consequently, not be running at a uniform rate of speed. If, on the contrary, the resistance predominated, the motion of the piston would be retarded.

As to the second principle, it is now known not to be strictly true. At first sight it would appear that, with the exception of preventible leakages through safety valves and imperfect joints, the steam generated in the boiler must all be used in the cylinder, but we know from Chapter III. that when expansion takes place in a cylinder, part of the

[1] The term double acting, which has not been previously explained, is applied to those engines in which steam acts alternately on either side of the piston instead of on one side only, as is the case with a few engines in use in Cornwall and elsewhere, for pumping water from mines.

steam is condensed back into water, and, as we shall afterwards see, if this condensation is prevented in the cylinder by the use of a steam-jacket, the condensation takes place in the jacket, instead of in the cylinder, and in neither case can the steam be said to be used in the cylinder, in the manner contemplated by De Pambour, who, in his theory, supposed that the whole of the steam was used in producing mechanical effects. In spite of this defect, the theory may be accepted as sufficiently accurate for many practical purposes.

Mathematical expression of De Pambour's first principle.

Let P be the pressure of the steam when admitted to the cylinder.

„ p be the pressure of the steam when the piston has moved over the space x, after the steam has been cut off.

„ L be the length of the stroke in feet.

„ l be the distance traversed before the steam is cut off.

„ C be the clearance, i.e. the space between the initial position of the piston and the bottom of the cylinder.[1]

Before expansion commences, the work done per square inch of area of piston is equal to the pressure P multiplied by l, the space traversed $= Pl$. After expansion commences, the pressure must be calculated by Navier's formula. Thus, at the moment expansion commences, the space occupied by the steam is $l+C$. At any point x, the space occupied is $x+c$, and we have, as before, the following proportion:

$$l+C : x+C :: p+k : P+k$$

from which we find the pressure p at the position x is

[1] The clearance includes also the contents of the admission passage for the steam, reduced to a corresponding length of cylinder.

$$p=\frac{l+C}{x+C}\times(P+k)-k$$

Multiplying by dx to obtain the work done during the passage of the piston over the small space dx, and integrating between the limits l and L, we obtain for the total work done during expansion the expression

$$\int_l^L p\,dx=\log_e\frac{L+C}{l+C}(P+k)(l+C)-k(L-l).$$

If to this we add the work done before the steam was cut off, viz. Pl, we obtain the total work done during the stroke

$$=\left(\log_e\frac{L+C}{l+C}+\frac{l}{l+C}\right)(P+k)(l+C)-kL.$$

Now, according to De Pambour's first principle, this quantity of work equals the total resistance which the piston encounters, multiplied by the length of the stroke. The resistance is composed of different elements, some of which vary in amount at different parts of the stroke; but, calling the average resistance R pounds per square inch of piston area, and RL=work done per stroke in overcoming the resistance; we have,

$$\log_e\frac{L+C}{l+C}+\frac{l}{l+C}=\frac{(R+k)\ L}{(P+k)\ (l+C)}$$

Now, by Navier's law $\frac{R+k}{P+k}$ is the ratio of the relative volumes of the steam at the pressures P and R, or $\frac{V_P}{V_R}$. Also $\frac{L}{l+C}$ is a ratio which it is convenient to represent by the separate symbol E. Hence

$$\log_e\frac{L+C}{l+C}+\frac{l}{l+C}=\frac{V_P}{V_R}E$$

is the mathematical expression of the first principle.

The mathematical expression of the second principle is as follows. Let F be the number of cubic feet of water evaporated per minute in the boiler, then the steam formed from this water at the pressure $P=FV_p$ cubic feet. This quantity, according to De Pambour, is equal to the quantity consumed in the cylinder. Now the amount of steam used in the cylinder per minute depends on the piston speed, i.e. on the number of strokes made per minute and on the point of cut-off, viz. $l+C$.

If v be the piston-speed per minute,

$$\text{then } \frac{v}{L} = \text{the number of strokes per minute,}$$

$$\text{and } \frac{v}{L} \times a\,(l+C)$$

the number of cubic feet of steam used per minute, where a=area of piston.

$$\text{Therefore, } va \times \frac{I}{E} = FV_p, \text{ or } va = E.FV_p,$$

but we have already seen that

$$E.V_p = \left(\log_e \frac{L+C}{l+C} + \frac{l}{l+C}\right).V_R,$$

and multiplying both sides by F we have,

$$E.F.V_p = \left(\log_e \frac{L+C}{l+C} + \frac{l}{l+C}\right)F.V_R.$$

$$\text{Therefore, } va = E.F.V_p = \left(\log_e \frac{L+C}{l+C} + \frac{l}{l+C}\right)F.V_R.$$

The resistance, R, to the motion of the piston is made up of the back pressure, the friction of the engine, and the load. The back pressure in non-condensing engines is made up of the pressure of the atmosphere plus an additional quantity due to the resistance of the steam ports to the exhaust steam, and to the compression of the steam remaining in the cylinder after the exhaust is closed by the valve.

The pressure of the atmosphere may be taken on the average as 14·7 lbs. to the square inch; the remainder of the back pressure varies in amount according to the size of the exhaust passages, and the point of the return stroke where the exhaust is closed. It may be roughly taken at 3 lbs. per square inch; making the total back pressure 17·7 lbs. In the case of condensing engines the back pressure depends on the perfection of the vacuum maintained in the condenser, and also on the resistance of the exhaust passages, and the point where these latter are closed by the valve. Its amount, as a rule, in engines in fairly good condition, does not exceed 4 lbs. per square inch.

The friction of the engine is made up of two parts, viz. one due to the friction of the unloaded engine, and the other due to the load. The friction of the unloaded engine depends on a variety of conditions, such as the diameter of cylinder relative to the power given out by the engine, the length of stroke, the relative length of the connecting rod to the stroke, the nature of the valves whether balanced or not, and, lastly, the general condition of the engine as to workmanship, repair, and lubrication. It is consequently impossible to assign a general value to this quantity. De Pambour took it as being 1 lb. per square inch for engines of average size and in fair condition, and 0·5 lb. per square inch for large engines in good condition. The additional friction due to the load on the engine is extremely difficult to calculate. As a general rule it may be said to increase directly with the load, but this statement is by no means universally true, for the load may in many cases be driven from the main axle in such a way as to diminish the friction on the main bearings, and experiments with the friction brake have been made which show that in some cases the loss due to friction remains nearly constant, while the load on the brake is increased.

The addition to the friction assumed by De Pambour in his calculations was one seventh of the load.

The load multiplied by the distance moved by the piston in a given time is the useful work done by the engine during that time. The load here referred to is not the actual weight lifted, or pressure overcome by the engine at the point of application of the weight or pressure, but is supposed to be 'reduced' to the motion of the piston in the ratio of the length of the stroke to the actual distance the real load is overcome during a stroke. Thus, supposing an engine having a cylinder of 10 inches diameter, or 75·5 square inches area, and 2 feet length of stroke to be employed in turning a winding drum by its direct action, the drum having a diameter of 3 feet; and suppose further that the drum is used to lift a weight of 10 cwt. vertically upwards; we have in this case the actual load of 1,120 lbs., which for each stroke of the engine is lifted up a height equal to half the circumference of the drum or 4·71 feet, but the load 'reduced' to the piston is

$$\frac{1120 \times 4{\cdot}71}{2} = 2643{\cdot}2 \text{ lbs.}$$

and the pressure due to the load per square inch of piston area is

$$\frac{2643{\cdot}2}{75{\cdot}5} = 35 \text{ lbs.}$$

If we call the pressure per square inch due to the load $=w$, and assign to the other elements of the resistance their values as given above, we have for non-condensing engines

$$R = w + \frac{w}{7} + 1 + 17{\cdot}7 = \frac{8w}{7} + 18{\cdot}7.$$

and for condensing engines

$$R = \frac{8}{7}w + 1 + 4 = \frac{8}{7}w + 5.$$

The horse-power of the engine is found by calculating the number of foot-pounds of work done in the cylinder per minute, and dividing the result by 33,000, which latter number is the number of foot-pounds per minute equal to

one horse-power. In calculating the power of the engine, the work done in overcoming the back pressure, which is for the most part pure waste, is left out of the account. The useful power of the engine is simply the power required to overcome the resistance due to the load, and is equivalent to the power found as above, minus that required to overcome the friction of the engine. Thus, if w be the pressure per square inch due to the load, then $144a.w$ = total pressure on piston due to load, and if v be the velocity of the piston in feet per minute, then $144a.v.w$ = work done per minute in overcoming resistance of load, and $\frac{144a.v.w}{33{,}000}$ = useful horse-power exerted.

EXAMPLE.

The diameter of cylinder of a non-condensing engine is 15 inches, the stroke is 30 inches; the number of revolutions per minute = 70; the steam is cut off at one third of the stroke, and the boiler is able to evaporate ·4 cubic ft. of water per hour: find out the useful power given out by the engine.

We have area of cylinder in square feet = 1·227. Piston speed in feet per minute $= \frac{30 \times 2 \times 70}{12} = 350$.

We have already established the relation,

$$va = \left(\log_e \frac{L+C}{l+C} + \frac{l}{l+C}\right).F.V_R, \text{ we require to find R.}$$

The quantity within the brackets has for a cut-off = $\frac{1}{3}$ stroke, and clearance = 5 per cent. of stroke, the value 1·87. Also F = ·4 cubic feet. Hence we have

$$V_R = \frac{350 \times 1{\cdot}227}{1{\cdot}87 \times {\cdot}4} = 573,$$

which number, by the table of Volumes, corresponds with the pressure of 45·5 lbs. per square inch.

This pressure is the total resistance to the piston per square inch, which, as the engine is non-condensing $= \frac{8w}{7} + 18{\cdot}7$ lbs. per square inch.

$$\therefore R = \frac{8w}{7} + 18{\cdot}7 = 45{\cdot}5$$

$$\therefore w = \frac{28{\cdot}8 \times 7}{8} = 23{\cdot}4 \text{ lbs.}$$

And the useful power

$$= \frac{23{\cdot}4 \times 144.a.v}{33{,}000} = 43{\cdot}6 \text{ H P.}$$

Find the initial pressure of steam in the cylinder.

We have,

$$\log_e \frac{L+C}{l+C} + \frac{l}{l+C} = \frac{V_P}{V_R}.E.$$

$$\therefore 1{\cdot}87 = \frac{V_P}{573} \times 2{\cdot}63,$$

$$\therefore V_P = \frac{1{\cdot}87 \times 573}{2{\cdot}63} = 407{\cdot}4,$$

which number corresponds very nearly with the relative volume for the pressure 65 lbs. per square inch.

Find the power of the engine with the above evaporation and cut-off provided a condenser were used.

We should obtain as before $V_R = 573$, and $R = 45{\cdot}5$ lbs. per square inch, but in this case $\frac{8}{7}w + 5 = R = 45{\cdot}5$,

$$\therefore w = \frac{40{\cdot}5 \times 7}{8} = 35{\cdot}5 \text{ lbs. per square inch nearly,}$$

and the useful horse-power exerted

$$= \frac{144.a.v \times 35{\cdot}5}{33{,}000} = 66{\cdot}3 \text{ H. P.}$$

Find the power, and the initial pressure in the cylinder, if the cut-off were at half the stroke, all the other data as above.

Using the same equation as before, we have

$$va = 429{\cdot}45 = \left(\log_e \frac{L+C}{l+C} + \frac{l}{l+C}\right) \times {\cdot}4 \times V_R.$$

The quantity within the bracket has in this case the value 1·55

$$\therefore V_R = \frac{429{\cdot}45}{1{\cdot}55 \times {\cdot}4} = 692{\cdot}5$$

$$\therefore R = 37{\cdot}5 \text{ lbs. per square inch} = \frac{8}{7}w + 18{\cdot}7,$$

$$\therefore w = 16{\cdot}4 \text{ lbs.}$$

and the useful power exerted

$$= \frac{144\ a.v \times 17{\cdot}3}{33{,}000} = 30{\cdot}5 \text{ H.P.}$$

This last example shows the advantage gained by working expansively, *when the evaporation remains constant*, the powers exerted at cuts-off $\frac{1}{4}$ and $\frac{1}{3}$ being respectively 43·6 and 30·5.

Let everything remain the same as in the last example except the piston speed, which is reduced to 300, we have

$$368 = \cdot 62\ V_R,$$
$$\therefore V_R = 593 \therefore R = 43 \cdot 5 \text{ lbs. per square inch,}$$
$$\therefore w = 21 \cdot 7 \text{ lbs. per square inch,}$$

and the useful horse-power

$$= \frac{144.a.300 \times 21 \cdot 7}{33{,}000} = 34 \cdot 23 \text{ H.P.}$$

This example shows that by reducing the piston-speed, all the other conditions remaining constant, we can increase the power exerted, because the slower speed, with a *fixed* evaporation, enables a higher pressure to be maintained in the boiler, and consequently a higher average pressure in the cylinder. Hence the conclusion might be drawn that, the slower the piston-speed with a fixed rate of evaporation, the more advantageously the engine could be worked, and this conclusion is true in so far as slow piston-speed favours the attainment of high pressure, though, as will be explained subsequently, slow speeds cause a loss of power in promoting the condensation of the steam in the cylinder, a circumstance of which De Pambour's theory takes no account. The problem how to work the engine to most advantage with a given boiler-power is not the one which most frequently presents itself to the engineer; on the contrary, it is more often his business to proportion the boiler power to the engine, rather than to adjust the rate of expansion of the engine so as best to suit the boiler.

The problem of finding the evaporation, i.e. the necessary supply of steam, when the size of cylinder, rate of expansion, speed of piston and power of the engine are given, can also be easily solved by means of the equation made use of in solving the above examples. Also, by the help of this same formula, and the empirical formula for the relative volume

of steam (see page 127), the diameter of the cylinder may be calculated when the power of the engine, the piston-speed, the cut-off, and the evaporation are known.

EXAMPLE.

The useful power exerted by an engine is 43·6 H.P.; the rate of evaporation is ·4 cubic foot per minute; the piston-speed, 350 per minute, the cut-off takes place at one third the stroke: find the diameter of the cylinder. We have as before,

$$va = \left(\log_e \frac{L+C}{l \times C} + \frac{l}{l+C}\right).F.V_R$$

$$\therefore 350\,a = 1{\cdot}87 \times {\cdot}4 \times V_R;$$

$$\text{but } V_R = V_{\frac{8w+18{\cdot}7}{7}},$$

$$\text{and, according to Navier, } V_R = \frac{28000}{R+4};$$

substituting the value of V_R, and reducing, and remembering that $\frac{144.a.v.w}{33{,}000}$ = the useful horse-power = 43·6,

we obtain a = 1·20 square feet,

or d = 14·98 inches,

a result which agrees very well with the previous example when the approximate nature of the formula for V_R is taken into account.

LOCOMOTIVE ENGINES.

In the case of locomotives the equations hitherto used apply in principle, but the expression for R, the resistance, requires modification. To understand the nature of the resistance a short account must be given of the method of action of locomotives. Fig. 24 is an outline diagram of such an engine. A is the boiler, a description of the details of which will be found commencing at page 371; B is one of the cylinders. It will be noticed that the connecting rod is attached to a crank formed on the axle of the main or driving wheels, one of which is seen at *c*; the wheels are keyed to their axles. Two of the other

wheels, which support the weight of the engine and boiler, are seen at c',c'. The frame to which the cylinders are bolted, and on which the boiler rests, and which also carries the bearings of the three axels, is shown at *e*, *e*. With the details of the construction of the locomotive we have now nothing to do ; we merely wish to arrive at the relationship between the resistance to be overcome by the pistons, the

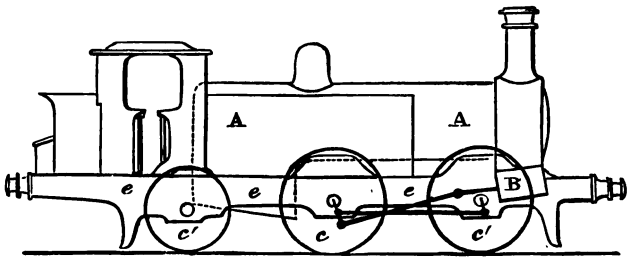

Fig. 24.

size of the cylinders, and the evaporative power of the boiler. The locomotive engine has not merely to propel itself along the rails, but also to pull after it a tender containing coals and water, as well as the train. When steam is admitted to the cylinders, and the pistons move backwards and forwards, the driving wheels, *c*, revolve. When this takes place, one of two things must happen—either the wheels will revolve, the whole engine remaining stationary, the wheels merely slipping on the rails ; or else the wheels will revolve and at the same time roll on the rails, thus propelling the whole engine, together with the tender and train attached to it, through space. The former of these two effects would take place if the surface of the rails and the outside rim of the wheels were perfectly smooth and hard, so that there was no friction between them ; or, even if friction subsisted between the wheel and rails, the former might merely slip on the latter, if the resistance to motion of the train were very great. The latter of the two effects would take place whenever the

friction between wheel and rails is sufficient to allow of the resistance to motion of the train being overcome.

The friction between the driving wheels and rail is called the adhesion, and is equal to the weight on the wheels multiplied by a coefficient which depends on the condition of the surface of the rail. In ordinary dry weather the coefficient equals 0·15, but in damp weather or when the rails are greasy it falls to 0·07.

The resistance which has to be overcome by the pistons is made up of several elements.

1st. There is the back pressure which, as a locomotive is a non-condensing engine, is equal to the atmospheric pressure plus an extra quantity due to the forcing of the exhaust steam through the blast pipe (see page 376), and to the compression caused by the closing of the exhaust port before the end of the stroke when the engine is working expansively (see page 338).

The excess of pressure due to the blast pipe when all the other conditions remain constant varies—

As the square of the speed of piston ;

As the pressure of the steam at the moment the exhaust begins, i.e. at the point of release, which point in the case of locomotives is implicitly connected with the rate of expansion (see page 338) ;

Inversely as the square of the area of the nozzle of the blast pipe.

The above is only applicable to the case of dry steam. When much water is present in the cylinder at the point of release, the back pressure may be increased up to as much as seventy per cent. in excess of what it would be with dry steam.

2nd. The resistance due to friction of the mechanism of the engine. This quantity, as in the case of stationary engines, in part depends upon the load, but it is usual in calculations affecting locomotives to make use of a formula which includes not only the friction of the mechanism but

also the resistance to rolling motion at a uniform speed of the whole weight of the engine and tender. This formula is given below.

3rd. The frictional resistance to uniform motion of the whole train including the engine and tender. This is usually expressed by giving the direct pull in pounds necessary in order to propel each ton's weight of the train along a level line at a given speed. The pull varies with the condition of the line, the state of the surface of the rails, the state of the rolling stock and the speed. For instance, with line and rolling stock in good condition, dry rails, and a speed of ten miles an hour, it would be necessary to exert a pull of about $8\frac{1}{2}$ lbs. per ton of weight of train, including engine and tender.

If M be the speed in miles per hour, and T the weight of the train in tons, exclusive of engine and tender, the resistance to uniform motion may be expressed by the formula

$$\{6 + \cdot 3(M - 10)\}T.$$

If T^1 be the weight of the engine and tender, the corresponding resistance is

$$\{12 + \cdot 3(M - 10)\}T^1,$$

which expression includes the frictional resistance of the mechanism of the engine referred to in the preceding paragraph.

4th. *Resistance due to gradient.*—If the train be moving up an incline, its whole weight has to be raised up a vertical height equal to the difference in levels between the foot and the summit of the incline. If the gradient be represented by the fraction $\frac{1}{a}$, then, by the laws of statics, the force in pounds necessary to lift a weight of T tons up such an incline, neglecting friction, is

$$\frac{T \times 2240}{a} \text{ lbs.}$$

This quantity has to be added to the frictional resistances, as set forth in the last paragraph, if the incline is ascending, and subtracted if descending.

5th. *Resistance due to the passage of the train through the air.*—This, of course, depends largely upon the force and direction of the wind. In calm air, it is proportional to the square of the velocity of the train. A strong side wind, by pressing the tires of the wheels against the rails, may increase the frictional resistance of the train by as much as twenty per cent. No formula has yet been devised which satisfactorily takes account of the resistance due to the ever-varying force and direction of the wind and speed of the train.

6th. When a train is being started from a state of rest, in addition to the frictional resistances to motion, the whole mass of the train has to be put in motion—that is to say, the inertia of the train has to be overcome. The resistance due to this cause is the principal one which has to be considered in the case of urban railways. It does not, however, enter into the computations affecting trains which have attained a uniform rate of motion, and it is these latter only which are dealt with in De Pambour's theory.

It will be noticed that many of these elements of resistance, such, for instance, as the back pressure and the force of the wind, depend, for their numerical value, on so many variable circumstances that it is impossible to express them accurately with simplicity. By substituting their values, when obtained, for R in the general equation for double-acting engines, viz.

$$va = \left(\log_e \frac{L+C}{l+C} + \frac{l}{l+C}\right) FV_R,$$

it would, of course, be possible to obtain an equation which would enable all problems, connecting the speed of the engine, the rate of evaporation, the dimensions of the

cylinders, the rate of expansion, the weight of engine, tender, and train, and the varying resistances, to be solved; but such an equation would be too complicated, and, when used for finding the speed, of too high dimensions for ordinary use. The analysis of the resistances given above will, however, be useful to students, as it will often facilitate the solution of individual problems.

CHAPTER V.

THE MECHANICS OF THE STEAM ENGINE.

Elementary principles of dynamics—Definitions of mass, weight, velocity, motion, force—Units employed in their measurement—The laws of motion and examples of their application—Work and energy—Motion of bodies in circles—Application to fly-wheels—Centrifugal force—Conversion of work done in the cylinder into work done on the crank—1st case, when pressure of steam is uniform, connecting rod supposed to be of infinite length and moving parts without weight—2nd case, when steam is allowed to expand, the other conditions remaining unchanged—Curve of effort on crank pin—3rd case, when the length of the connecting rod is taken into account—4th case, when the weights and velocities of the reciprocating parts are taken into account—Power absorbed in accelerating these parts—Power restored by their retardation—The consequent modification of the pressures shown by indicator diagrams necessary for calculating effort on crank pin—Effect of steam distribution on the action of the moving parts—Means of equalising the tangential effort on crank-pin—Fly-wheels—Theory of their action—Graphic diagrams illustrating their action.

BEFORE discussing the questions of applied mechanics which arise in the study of the moving parts of the steam engine, it will be useful for the sake of accuracy, to recapitulate briefly the elementary principles of dynamics, a previous acquaintance with the principles of the composition of resolution of forces and velocities, on the part of the reader being, however, assumed. We will start with the following definitions :—

1. Mass. This word denotes the quantity of matter contained in a body.

2. Weight is the attraction which the earth exercises on a mass.

3. Velocity is the speed at which a body moves, i.e. the space which it traverses in a given time.

4. Motion. This word is employed in dynamics, not

merely to denote movement on the part of a body, but also takes account of the mass of the body moved. Thus, if two bodies have each the same velocity, but the mass of one be double that of the other, then the *motion*, or *quantity of motion*, or *momentum*, as it is variously termed, of the body having the larger mass is double the motion of the smaller one. If the bodies had each the same mass, but the velocity of one were double that of the other, then the motion of the body having the greater velocity would be likewise double that of the other. When the velocity remains constant, the motion varies as the mass moved. When the mass is constant, the motion varies as the velocity. Therefore, generally, the motion varies as the mass × the velocity.

5. Force is any cause which produces, or tends to produce, motion in a body, or which changes, or tends to change, the motion of a body.

Units adopted in measuring Mass, Weight, Velocity, and Force.

The only means we have of measuring the masses of different bodies, i.e. the quantities of matter in them, is by weighing them; that is to say, by comparing the attractions of the earth on them relatively to some standard substance. Consequently, the measure of mass is dependent upon the unit chosen to measure weight.

The unit of weight adopted in this country is the weight in London of a certain piece of platinum, kept in the office of the Exchequer, and called a pound avoirdupois. The weight of this piece of platinum varies in different parts of the globe. Its weight depends on the attraction exercised by the earth upon the matter contained in it. This force of attraction, called gravitation, was discovered by Newton to depend on the distance between the centre of the earth and the object attracted. In consequence of the flattening

of the earth towards the poles, and its bulging out towards the equator, the surface of the earth is nearer to the centre in London than in more southern places, and consequently the weight of the standard piece of platinum is greater in London than it is at the equator.

The mass of a body is then measured by its weight at a given place. There are two units of mass made use of in dynamics. The so-called *gravitation unit of mass* is the quantity of matter contained in a body weighing 32·2 pounds.

The so-called *absolute unit of mass is* the quantity of matter in a body weighing one pound.

Let M denote the mass of a body measured by the gravitation unit, then its weight by the definition is $W=M\times 32\cdot 2$ lbs. The symbol g is used to denote the number 32·2. Hence we have

$$W=M.g \text{ or } M=\frac{W}{g}.$$

The velocity of a body is measured in different ways according as it is a linear velocity, i.e. due to a motion of translation of the body from one point to another; or an angular velocity, i.e. due to the rotation of the body round an axis.

Velocity is uniform when the body traverses equal spaces in equal times. When uniform, linear velocity is always measured by the number of feet of linear space, traversed in one second of time. Thus, for instance, we speak of a body having a velocity of two thousand feet a second.

Let v denote the velocity of a body moving uniformly. Let s denote total the number of feet which it passes over, and t denote the number of seconds occupied in describing the s feet.

$$\text{Then } v=\frac{s}{t} \text{ or } s=t.v.$$

Further, the centrifugal force (see page 159) which in all cases tends to diminish the weight is greater at the equator than at the poles.

Force is also, like mass, measured in two ways. According to the *gravitation* system, a force is measured by the weight which it can support. Thus a string is said to exercise a force of ten pounds when the tension in the string is sufficient to prevent a force of ten pounds from falling to the earth. The unit of force in gravitation measure is the force which can support a weight of one pound.

The second or *absolute* system of measuring force is more in harmony with the definition. By this system a force is measured by the velocity which it can impart to a given mass in a given time, when acting continuously on the mass for that time. Thus, for instance, the force of gravity is measured by the velocity which it can generate in a given mass when acting on it for a second of time. The unit of force in absolute measure is the force which can generate a velocity of one foot per second, when acting on a mass weighing a pound, during one second of time.

The Laws of Motion.

First Law.—Every body continues in a state of rest or of uniform motion in a straight line, unless compelled by impressed forces to change that state.

This law lays down that matter has of itself no power to change its own condition of rest or of uniform motion. In other words, it possesses what is called inertia. Consequently, when we note that a body is not moving with a uniform velocity, we know that it is being acted upon by external force.

Second Law.—Change of motion is proportional to the impressed force, and takes place in the straight line in which the force is impressed.

In the above statement the word motion has the meaning already explained, viz. Mass multiplied by velocity.

Thus if two equal forces act on two unequal masses, the quantity of motion generated in each case will be the same, but the greater mass will have the least velocity, and the

product of the mass multiplied by the velocity will be the same in each case. If the velocities of the two masses are to be equal, then the force acting on the greater mass must be greater than the other in the same ratio that the mass itself is greater.

This law enables us to compare the relative magnitudes of forces, for we have only to observe the velocities generated by the various forces in the same mass, when acting for the same time. The standard for comparison is the velocity generated in a mass by the force of gravity, i.e. by its own weight when acting for a second of time. This velocity is g or 32·2 feet per second; that is to say, if the force of gravity act on a free body for one second, it will at the end of the second have imparted to the body a velocity of 32·2 feet per second. Thus a force F acting on a body weighing 20 lbs. for a second generates a velocity of 50 feet per second; what is the magnitude of the force in gravitation measure?

The force of gravity, i.e. the weight of the body, or 20 lbs., would generate a velocity in the body of 32·2 feet per second. Therefore we have

$$F : 20 :: 50 : 32{\cdot}2.$$

$$\therefore F = \frac{50 \times 20}{32{\cdot}2} = 31 \text{ lbs.}$$

In the general case, if the velocity v be generated in one second by a force F in a body weighing W lbs., then

$$F : W :: v : g.$$

$$\therefore F = \frac{Wv}{g} \text{ lbs.}$$

The statement in the above law that the change of motion takes place in the direction in which the impressed force acts may be illustrated by the following example :—

A ball is projected from a rifle in a perfectly horizontal direction, AB (fig.25), from a height above the ground, AH. Directly it leaves the muzzle of the gun it is acted on by two

forces, viz. the momentum acquired while in the barrel, and which would in a given number of seconds carry it to, say, B; and the force of gravity which acting alone would in the same time carry it to, say, H. According to the law, each force generates motion in the ball in the direction in which each acts, and the consequence is, that at the end of the given time, the ball

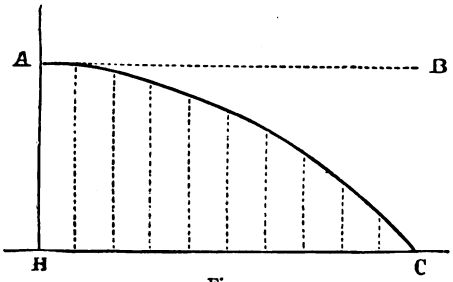

Fig. 25.

will have travelled as far forward as B, and as far downwards as H, and hence its resultant position will be C. The law in effect states that dynamical forces may be compounded and resolved in the same manner as statical forces.

The motion of bodies moving under the action of a constant force.—Let us take, for example, a body falling freely from a state of rest under the action of gravity. At the end of the first second, its velocity, having been zero to start with, will have increased up to 32·2 or g feet per second. As the force continues to act uniformly it will have produced precisely the same effect by the end of another second; that is to say, the velocity will have been gradually increased by another 32·2 feet per second. Hence the velocity at the end of the second second will be 64·4 or $2g$ feet per second. At the end of 3 seconds it will be $3g$ and so on, and generally at the end of t seconds $v=gt$.

Next, as to the space traversed at the end of t seconds. If it were travelling all the time with its final velocity gt we should, using the formula $s=vt$, have for the space

$s = gt.t = gt^2$. As, however, the velocity is constantly increasing, we can only take the average, which, in consequence of the perfectly uniform nature of the rate of increase, or acceleration as it is called, is very easy to calculate, and is in fact half the sum of the initial and final velocities, and in this particular case $= \frac{0+gt}{2} = \frac{gt}{2}$.

Hence, instead of $s=gt^2$ we have

$$s=\frac{gt^2}{2}=\tfrac{1}{2}tv,$$

where v is the final velocity. These formulæ connect the space, time, and velocity.

If we wish to connect the space traversed with the final velocity, without taking account of the time we can eliminate t by combining the formulæ,

$$s=\tfrac{1}{2}gt^2, \text{ and } v=gt.$$

$$\text{Hence } s=\tfrac{1}{2}g.\frac{v^2}{g^2}=\frac{v^2}{2g}$$

$$\text{Therefore } v^2=2g.s.$$

$$\text{or } s=\frac{v^2}{2g}.$$

Third Law.—To every action there is always an equal and contrary reaction.

Examples of the Application of the Laws of Motion.

Example (1).

A body falls from rest under the influence of gravity; what will be its velocity at the end of 10 seconds, what will be the total space traversed, and what the space traversed during the tenth second?

1. $v=gt=32{\cdot}2\times 10=322$ feet per second.
2. $s=\frac{1}{2}gt^2=16{\cdot}1\times 100=1{,}610$ feet.
3. Space described in nine seconds.

$s=\frac{1}{2}gt^2=16{\cdot}1\times 81=1304{\cdot}1$ feet.

$\therefore$ Space described in tenth second $= 1610-1304{\cdot}1$

$=305{\cdot}9$ feet.

EXAMPLE (2).

A rifle bullet is shot vertically upwards with a velocity of 1,000 feet per second; find the maximum height to which it would reach if it experienced no resistance from the air. How long a time will it take to reach this height? The height to which the bullet will reach is equal to the height down which a body must fall from rest in order to acquire a velocity of 1,000 feet per second. For, from the moment it commences to rise, its velocity is being diminished at the rate of 32·2 feet per second, till the initial velocity is all expended and the bullet come to rest at the top of its flight. If from this point it commenced to fall it would attain a velocity of 1,000 feet a second on reaching the starting point. Using, therefore, the formula

$$s = \frac{v^2}{2g} = \frac{1{,}000{,}000}{2 \times 32{\cdot}2},$$

we have $s = 15{,}527$ feet.

To find the time occupied. As a velocity of 32·2 feet is generated in each second, a velocity of 1,000 feet per second will be generated in $\frac{1{,}000}{32{\cdot}2}$

$= 31{\cdot}05$ seconds.

EXAMPLE (3).

A weight of 12 lbs. rests on a perfectly smooth surface and is connected by a string passing over a smooth pulley with a weight of 6 lbs. hanging vertically downwards. What velocity per second will the weights have at the end of the first second from rest?

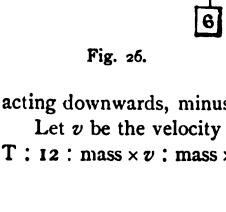

Fig. 26.

The tension, T, in the string is uniform throughout the string. Also, the weight 12 is caused to move by the tension of the string, while the weight 6 is caused to move by its own weight acting downwards, minus the tension in the string acting upwards.

Let v be the velocity generated in the mass of 12 lbs. by T; then T : 12 : mass $\times v$: mass $\times g$

$$\therefore \frac{T}{12} = \frac{v}{g}.$$

As the velocity of the mass of 6 lbs. is the same $= v$, and as this velocity is generated by the force $6 - T$, we have

$$6 - T : 6 : \text{mass} \times v : \text{mass} \times g$$

$$\therefore \frac{6-T}{6} = \frac{v}{g},$$

$$\therefore \frac{T}{12} = \frac{6-T}{6} \therefore 6T = 72 - 12T,$$

$$\therefore 18T = 72 \quad \therefore T = 4 \text{ lbs.}$$

To find the velocity generated at the end of one second.

Gravity, or the weight 12, acting on the mass of 12 lbs. for one second, would generate a velocity of 32·2 feet per second, therefore the tension $T = 4$ lbs. will generate a velocity

$$= \frac{4 \times 32{\cdot}2}{12} = 10{\cdot}73 \text{ feet per second.}$$

EXAMPLE (4).

A mass of 200 lbs. is moved from rest by a constant force F, and passes over a space of 60 feet in the first second; what is the measure of the force?

As the space passed over from rest is 60 feet, the velocity at the end of the second is $2 \times 60 = 120$ feet per second. The force of gravity, or 200 lbs., would in the same time generate a velocity of 32·2.

$$\therefore F : 200 :: 120 : 32{\cdot}2.$$

$$\therefore F = \frac{200 \times 120}{32{\cdot}2} = 745{\cdot}3 \text{ lbs.}$$

The following example illustrates the application of the laws of motion to the moving parts of a steam engine.

EXAMPLE (5).

The piston, piston rod, and connecting rod of an engine weigh 400 lbs., the stroke $1\frac{2}{3}$ ft., and the number of revolutions 200 per minute. During each revolution this mass starts from a state of rest; its velocity is gradually increased up to a maximum, and from this point it diminishes till it comes to rest again at the end of the stroke. During the first part of the stroke a certain proportion of the total steam pressure is required to generate this velocity in the moving parts, and whatever pressure remains over is all that is available for transmission to the crank. During the latter part of the stroke, after the

point of maximum velocity is reached, the speed of the moving parts has to be reduced to nothing by the end of the stroke, and while this reduction is taking place the moving parts press upon the crank-pin with more or less severity depending on their weight, their maximum velocity, and the space in which the velocity is reduced from the maximum to zero. Consequently, while during the first part of the stroke the crank-pin has only a portion of the steam pressure transmitted to it, during the latter portion, on the contrary, it is subjected not only to the full steam pressure acting on the piston, but also to the extra pressure due to the pulling up of the moving parts.

In the present case the length of stroke and speed of rotation of the crank are such, that at the commencement of the stroke the piston moves through ·000183 ft. in the thousandth part of a second; what is the total pressure, P, required to move the mass of 400 lbs. from rest over this space in a given time?

The velocity at the end of the time = 2 × ·000183 ft. per $\frac{1}{1000}''$,

$\therefore$ the velocity per second at the end of one second

$$= 2 \times \cdot 000183 \times 1000^2 = 365 \cdot 4.$$

Now, gravity, or the weight of the moving parts, i.e. 400 lbs., is capable of generating in them a velocity of 32·2 feet per second.

$$\therefore 400 : 32 \cdot 2 :: P : 365 \cdot 4.$$

$$\therefore P = \frac{400 \times 365 \cdot 4}{32 \cdot 2} = 4539 \text{ lbs.}$$

The diameter of the cylinder is 10 inches, its area $\therefore$ = 78·5 sq. inches, and the pressure of steam at the commencement of the stroke necessary to impart the required velocity to the moving parts

$$= \frac{4539}{78 \cdot 5} = 57 \cdot 8 \text{ lbs. per square inch.}$$

At the extreme end of the stroke the motion of the moving parts is arrested at the same rate as it is imparted at the commencement,[1] and consequently a pressure at the rate of 57·8 lbs. per square inch of piston area is transmitted to the crank in addition to whatever pressure of steam may happen to be acting on the piston at that moment.

Energy.—The term energy, or capacity for doing work, has been already explained (see page 25). The matter is

[1] For the sake of simplicity the influence of the length of the connecting rod relatively to the length of the crank in retarding the motion of the piston, &c., is here neglected.

now referred to again for the purpose of showing the effect on the working of steam engines, of the energy stored up in masses in motion. A body in motion possesses energy ; for, if the motion be, for instance, vertically upwards, it will carry the body up to a certain height before it is brought to rest, i.e. it will overcome the attraction of the earth through a certain space. The height to which the body will rise is, as explained in Ex. (2), p. 152, equal to the height down which the body must fall in order to acquire the same velocity.

In questions concerning the steam engine we are chiefly concerned with the energy of bodies in motion. Very frequently work is said to be stored up in a body in motion, or in a raised weight. What is really meant is that energy or the capacity of doing work is stored.

Take as an example the case of a cannon-ball weighing 100 lbs. and having a velocity of 2,000 feet a second ; what is its capacity for doing work ? The velocity of 2,000 feet a second would be acquired by falling down a height S, calculated by the formula

$$S=\frac{v^2}{2g}=\frac{4,000,000}{64\cdot4}=\ 62,111 \text{ ft.}$$

Therefore the velocity of 2,000 ft. per second is capable of raising the body to a height of 62,111 feet, and the work which would be done=the height multiplied by the weight

$$=\frac{v^2}{2g}\times w=62,111\times 100 \text{ foot-pounds.}$$

Vice versâ, in order to impart this velocity to the cannon-ball, 6,211,100 foot-pounds of work would have to be done upon it before it left the bore of the gun. If the bore of the latter were six inches in diameter, and ten feet long from the front of the powder-cartridge to the muzzle, what would be the average pressure of the powder gases per square inch? As 6,211,100 foot-pounds of work have to be done on the shot while it traverses the space of ten feet, the total average pressure on the shot must be

$\frac{6,211,100}{10} = 621,110$ lbs. Also as the diameter of the bore is six inches, its area is 28·27 square inches, and the average pressure per square inch $= \frac{621,110}{28{\cdot}27} = 21,970$ lbs., or a little less than ten tons.

Similarly take the case of the steam engine given in Ex. (5) p. 153. The moving parts which weigh 400 lbs. attain a maximum velocity towards the middle of the stroke, which is reduced to nothing at the end of the stroke. Required to find the work which the moving parts are capable of doing after having attained their maximum velocity, the length of stroke being $1\frac{2}{3}$ feet and the number of revolutions 200 per minute. The path described by the crank-pin in each rev. $= 1\frac{2}{3}\pi$ ft. $= 5{\cdot}236$ ft. and the velocity of the crank-pin per second

$$= \frac{5{\cdot}236 \times 200}{60} = 17{\cdot}5 \text{ ft.}$$

The energy stored up in the moving mass at this velocity is obtained from the formula $\frac{wv^2}{2g}$.

$$= \frac{400 \times 17{\cdot}5 \times 17{\cdot}5}{64{\cdot}4} = 1902 \text{ foot-pounds.}$$

This energy is given out while the piston is traversing half the stroke,[1] i.e. ten inches, and is consequently equivalent to a pressure of $\frac{1902 \times 12}{10} = 2282{\cdot}4$ lbs., acting through this space. As the area of the piston is 78·54 square inches the energy stored up in the moving parts is equivalent to an average pressure of $\frac{2282{\cdot}4}{78{\cdot}54} = 29{\cdot}06$ lbs. per square inch during the latter half of the stroke.

[1] This statement is only true when the connecting rod is infinitely long. It is also true for finite connecting rods if taken to apply to the mean of the forward and back strokes.

Motion of bodies in circles.—In all the cases hitherto considered, the motion has been in a straight line, but in dealing with the mechanics of the steam engine cases of great importance occur in which the motion takes place in a circular path. Such for instance is the motion of the fly-wheel, which is a wheel having a heavy rim. It is generally keyed to the crank axle of the engine, and is used for modifying the effects of any irregularity either in the driving power or in the resistance to be overcome. When, for instance, the driving power is in excess of the resistance to be overcome, the surplus is expended in increasing the velocity of the fly-wheel; and, *vice versâ*, when the resistance is in excess of the driving power, the energy stored up in the fly-wheel is expended in helping to overcome the resistance, during which operation its velocity is lowered.

The consideration of the motion of bodies in circles is somewhat complicated by the fact that different parts of the bodies may be at different distances from the centres of the circles in which they are moving, and as the velocities necessarily vary directly with the distance from the centre, so also do the quantities of motion. Take, for instance, such a body as a fly-wheel represented by fig. 27. It is composed of a rim, a set of arms, and a central boss. The velocity of the rim is many times greater than that of the boss, and again the velocity of the exterior portion, *a*, of the rim is greater than that of the interior portion, *b*; consequently it is usual in calculations respecting fly-wheels, to consider

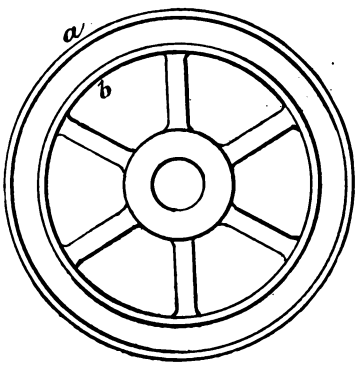

Fig 27

the whole of the weight as concentrated at a certain distance from the centre, where its effect will be the same as the sum of the effects of the various portions of the real wheel, each acting at its own distance from the centre. It is very often a complicated calculation to determine this distance with accuracy, but for all practical purposes we shall be sufficiently correct if we take the mean radius of the rim as the distance at which the whole of the weight is supposed to be concentrated.

The laws of motion, as already stated and illustrated, apply equally when the direction of the motion is in a circle. Thus, for instance, if a weight w move round a centre with a velocity v, the energy stored up in it$=\frac{wv^2}{2g}$. For a given number, N, of revolutions per second the velocity v varies with the length of the radius r, and equals $2\pi r$N. Substituting this expression for v in the above equation we have

$$\text{Energy}=\frac{w4\pi^2r^2\text{N}^2}{2g},$$

and consequently the energy varies as the square of the radius, that is of distance of the weight moved from the centre. A fly-wheel, therefore, of a given weight, the mean radius of the rim of which is five feet in length, is rather more than twice as efficient as a reservoir of energy as if its mean radius were 3·5 feet.

EXAMPLE (6).

How much energy is stored in a fly-wheel of 5,000 lbs. weight, the mean radius of the rim of which is 4 feet, and the number of revolutions 60 per minute? N.B.—The whole of the weight is, for simplicity, supposed to be concentrated at the end of the mean radius.

The mean velocity per second, $v=\frac{2.\pi.4.60}{60}=25{\cdot}13$ feet.

$$\text{The energy}=\frac{w.v^2}{2g}=\frac{5{,}000\times 631{\cdot}5}{64{\cdot}4}$$
$$=49{,}029 \text{ foot-pounds.}$$

EXAMPLE (7).

A fly-wheel weighs 5,000 lbs. and the mean rim moves with a maximum velocity of 35 feet per second. On account of the inequality of the force transmitted to the crank, the fly-wheel has, during a portion of the stroke, to expend 9,000 foot-pounds of energy; what will its velocity be after having done so?

The maximum energy $= \frac{5{,}000 \times 35 \times 35}{64{\cdot}4} = 95{,}108$ foot-pounds.

After expending 9,000 foot-pounds, the energy remaining

$$= 95{,}108 - 9{,}000 = 86{,}108 \text{ foot-pounds.}$$

$$\therefore \frac{w.v^2}{2g} = 86{,}108.$$

$$\therefore v^2 = \frac{86{,}108 \times 2g}{5000} = 1109.$$

$$\therefore v = 33{\cdot}3 \text{ feet per second,}$$

being a loss of 1·7 foot per second from the maximum velocity, which is equivalent to a variation of 4·8 per cent. from the maximum or of 2·4 per cent. from the mean velocity.

Centrifugal force.—By the first law of motion a body will continue to move in *a straight line* unless compelled to do otherwise by impressed forces. When a body moves in a circle it is, however, changing its direction from instant to instant, and consequently must be continuously under the influence of some force. Suppose this force were removed, the body would no longer move in the circle, but would fly off in a straight line at a tangent to the circle from the point at which the force was removed. This is true of any and every point on the circumference, from which it is evident that the direction of the force which compels the body to move in the circle is always at right angles to the tangent at any point, and consequently always points to the centre. This force, which keeps a body moving in a circle, is, on account of its direction, always called the centripetal force. The resistance which the mass of the body opposes to being moved towards the centre, and which by the third law of motion is

equal to the centripetal force, is called centrifugal force. This force may be measured as follows.

Let a body be supposed to start from the point *a*, fig. 28, and move in the circle represented, with the uniform velocity *v* feet per second. If the centripetal force F were removed, the body would during a very short time *t* move in a straight line over the space *ab*. By the second law of motion the effect of the centripetal force would therefore be to cause the body to move over the space *bc* during the time *t*.

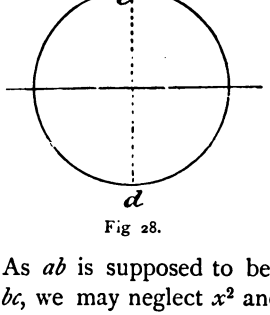

Fig 28.

By a well-known proposition in Euclid $bc \times bd = ab^2$. Calling $bc=x$ we have $x\ (2r+x)=ab^2$. As *ab* is supposed to be very small, and consequently also *bc*, we may neglect x^2 and put $ab=ac$.

$$\therefore 2rx=ac^2 \therefore x=\frac{ac^2}{2r}.$$

Also, since the motion in the circle is uniform, and since *ac* is the space moved in the time *t*, we have

$$ac=tv \therefore x=\frac{t^2v^2}{2r}.$$

Calling the weight of the body *w*, and *f* the velocity which the centripetal force F can generate in *w* in one second, we have

$$\mathrm{F} : w :: f : g$$

$$\therefore \mathrm{F}=\frac{wf}{g}.$$

We have next to express *f* in terms of *v* and *r*. Now *x* is the space *bc* which the body would move over from rest under the influence of the centripetal force in time *t* secs.

Therefore the velocity at the end of time $t'' = 2x$ per t''

$$= \frac{2x}{t} \text{ per sec.}$$

Therefore the velocity which would be acquired at the end of one second is

$$f = \frac{2x}{t} \div t = \frac{2x}{t^2},$$

and substituting the value of x as given above, we have

$$f = \frac{t^2 v^2}{r t^2} = \frac{v^2}{r}.$$

$$\therefore \mathrm{F} = \frac{w v^2}{g r}.$$

This important expression which is constantly made use of gives the centripetal force in terms of the weight of the body, its velocity, and the radius of the circle in which it moves.

If the velocity is given in revolutions per second, n, we have

$$v = 2\pi r n,$$

and the above formula becomes

$$\mathrm{F} = \frac{w}{g} \times \frac{4\pi^2 r^2 n^2}{r}$$

$$= w n^2 r \times 1{\cdot}226.$$

If the revolutions are given as so many per minute, N, we have

$$n = \frac{\mathrm{N}}{60}$$

$$\therefore \mathrm{F} = w \left(\frac{\mathrm{N}}{60}\right)^2 r \times 1{\cdot}226.$$

$$= w \mathrm{N}^2 r \times 0{\cdot}00034.$$

Conversion of the Pressure of Steam on the Piston into Rotative Effect on the Crank Axle.

One of the most important applications of mechanical science to questions relating to the steam engine is, to ascertain the exact effect which the pressure of the steam on the piston has in causing the crank to rotate. In dealing with this question there are several points to consider :—

First of all, in the great majority of cases the pressure of the steam varies considerably at different parts of the stroke.

Secondly, this variable pressure is transmitted to the crank-pin through a connecting rod, which is constantly changing its angle of inclination to the axis of the cylinder, as it swings between its extreme positions on either side of this axis.

Thirdly, the varying pressure transmitted through the connecting rod meets the crank at an angle which is constantly changing. The pressure may be resolved at the crank-pin into two components, one in the direction of the crank, and the other at right angles to it, i.e. tangential to the circle described by the crank-pin. Of these the latter alone produces any turning effect on the crank, the former producing merely pressure on the bearing. The tangential component, or *turning effort on the crank*, as it may be called, varies in value continually, for it depends not only on the net pressure of the steam on the piston, but also on the varying angles of inclination of the connecting rod, and the crank.

Fourthly, the effective turning effort on the crank depends not only on the above-mentioned variables, but also on the weights and velocities of the reciprocating parts, viz. the piston, and piston and connecting rods; for, as we have seen before, p. 153, Ex. 5, a considerable proportion of the steam pressure may, during a portion of the stroke, be

absorbed in merely imparting motion to the reciprocating parts, and may consequently never reach the crank-pin at all; while on the other hand these parts as they come to rest may impart a considerable pressure to the crank-pin quite independently of the pressure due to the steam on the piston.

The problem will be investigated in the first instance freed from all possible complications. The pressure of the steam will be supposed to be uniform throughout the stroke. The connecting rod will be taken to be of infinite length, in other words it will be supposed to act always parallel to the axis of the cylinder. Lastly, the moving parts will be imagined to be without weight, or their velocity may be supposed to be so small that no appreciable part of the steam pressure is absorbed in imparting motion to them.

In the next instance the pressure of the steam will be supposed to vary during the stroke; then the angular vibration of the connecting rod will be taken into account, and finally the effects of the weights and velocities of the reciprocating parts will be considered. In every case graphical methods will be employed, in preference to analytical, to investigate the problems.

In the diagram, fig. 29, let the circle ABC represent the path of the crank-pin. Let AC represent the direction of the axis of the cylinder. Let the pressure of the steam on the piston throughout the stroke be P lbs. per square inch, and let the scale of the diagram be such that the length of the radius OA represents P lbs. The reason for so doing will soon become apparent. First assume that the crank lies in the position AO. The pressure transmitted through the crank at this moment acts radially through the centre O, and has no effect whatever in turning the crank. The same is true when the crank occupies the position OC hence the two positions OA, OC are called the dead centres. Next suppose the crank to occupy the position OB, at right angles to the dead centres. As the connecting rod is

supposed to be of infinite length it acts in the direction B′B parallel to AC, and consequently the whole of the force

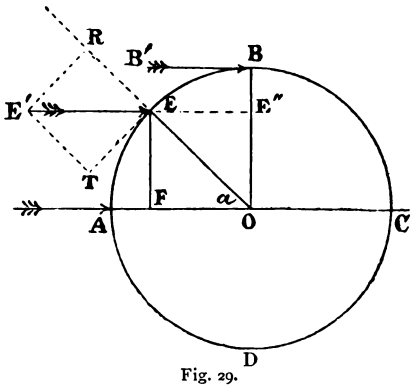

Fig. 29.

transmitted has the effect of turning the crank. The arm of the lever with which the force acts is BO, viz. the radius of the crank, and the turning moment per square inch area of piston=P × BO. The same is true for the position D diametrically opposite to B. Hence we see that while the crank is at A and C the steam pressure has no effect whatever in turning it, at B and D, on the contrary, its whole effect is in turning. At any other point in any of the four quadrants the force of the steam is partly expended in turning the crank, and is partly transmitted through the crank as mere pressure on the main bearing at O.

Take, for instance, the point E. At this position, the force acts with a leverage measured by the length of the perpendicular let fall from the point O on the direction of EE′, viz. E″O=EF=EO sin α, and the turning moment consequently

$$=P \times EO \sin \alpha.$$

The tangential force which acts at the end of the crank, and tends to turn it round, as distinguished from the

turning moment is got by dividing the above quantity by the length of the crank arm. Calling this force P_T, we have

$$P_T = \frac{P \times EO \sin \alpha}{EO} = P \times \sin \alpha.$$

This expression is equally true for any point on the circumference of the circle. Hence, we see that the tangential pressure on the crank, when the connecting rod is infinitely long, is equal to the pressure on the piston multiplied by the sine of the angle of inclination of the crank to the axis of the cylinder.

The same result may be got by resolving the force P at the point E, into two components, viz. one acting radially, ER, and the other tangentially, ET. Of these, ER merely produces pressure on the main bearing, while ET alone tends to turn the crank.

Now, ET=EE′ sin EE′T.

Also, EE′=P=EO, the scale of the figure being such that EO represents P.

And the angle EE′T=α, because E′T is parallel to EO and the angles ETE′ and EFO are both right angles. Therefore the two triangles are equal, and ET=EF=P sin α.

Thus we see that, though the pressure on the piston may be perfectly uniform throughout the stroke, the turning effort on the crank is very variable, and begins by being zero at the dead centre, increases to a maximum when the crank is at right angles to the axis of the cylinder, again decreases to zero by the time the other dead centre is reached, and so on during the return stroke, or second half of the revolution.

It may be here noted that it was this fact, that the tangential pressure on the crank is always less than the pressure on the piston, except for two positions of the crank, which led old writers on the steam-engine into the blunder of asserting that there is a loss in the employment of the crank as a means for converting reciprocating into circular motion. We now know, by the definition of work, that there is no such loss; for, although the average tangential

pressure on the crank is much less than the pressure on the piston, on the other hand, the path traversed by the crank in a revolution is greater than that traversed by the piston in a double stroke, in the ratio of the circumference of a circle to its double diameter, i.e. $2 : \pi = 1 : 1{\cdot}57079$.

By the principle of work, the lesser average pressure on the crank, multiplied by the path described by the crank-pin, must equal the greater pressure on the piston multiplied by the space traversed by the latter.

Graphic representation of the tangential effort on the crank-pin.—The variable tangential pressure on the crank-pin

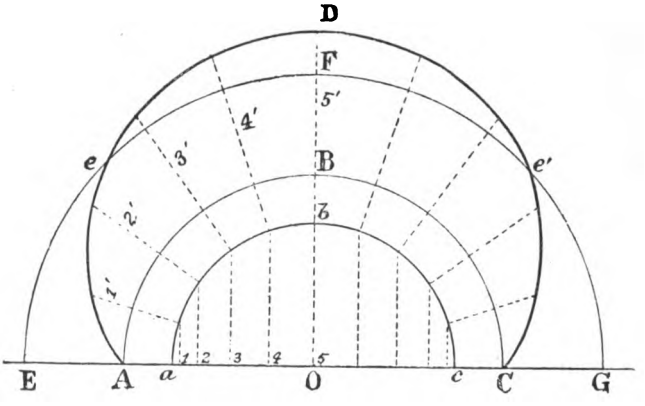

Fig. 30.

throughout a revolution can be very well shown, graphically, by means of a diagram. Let the semicircle ABC (fig. 30) represent the path described by the crank-pin during half a revolution. Draw O*a* to represent the uniform pressure on the piston to scale, and with centre O and radius O*a*, draw the inner semicircle *abc*. Divide the circumference of this semicircle into 10 equal divisions, for the sake of convenience, and draw radial lines through each point of division, intersecting the semicircle ABC. Then, at each position of the crank

represented by the points of division of the outer semicircle, the tangential force on the crank is equal to the pressure on the piston multiplied by the sine of the angle of the crank. As aO represents the pressure on the piston, the tangential forces are represented in magnitude by the perpendiculars 1, 2, 3, 4, 5, &c. let fall from the points of division of the inner circle on the line aO. On the prolongations of the radial lines beyond the outer circle set off the lines 1′, 2′, 3′, 4′, 5′, &c., equal, respectively, to 1, 2, 3, 4, 5. Join the extremities of these lines by the curved line ADC. Hence, ADC represents, graphically, the tangential pressure at every position of the crank; since, for any position, we have only to draw a radial line through the point in question, and the piece intersected between the outer circle and the curved line will represent the tangential force. If the tangential pressure were uniform all round the circle the curved line ADC would be a circle concentric with the path of the crank-pin. Its deviation from concentricity is the measure of its want of uniformity. The average tangential pressure on the crank-pin may be represented by drawing the circle EFG from the centre O, the line EA which represents this average pressure being obtained by the following proportion

$$\text{EA} : a\text{O or P} :: 2 : \pi.$$

When the engine is running at a *uniform rate of speed*, this average tangential pressure on the crank, is, of course, exactly equal to the resistance which the work to be done offers to the motion of the crank-pin; for, if the resistance were greater, the speed would be reduced, and if the resistance were less, the speed would be increased, and in neither case would the engine be running uniformly. Consequently, this average tangential pressure circle may equally well be called the Resistance Circle.

The diagram (fig. 30) only shows the tangential pressures for one half of the revolution, but the other half is, of course, a precisely similar figure, and need not, therefore, be

shown. By inspection of the diagram, we see that there are four points during a complete revolution where the actual tangential pressure exactly equals the average, viz. the points *e e'*, &c.; where the resistance circle intersects the curves ADC, AD'C. Between the points *e e'* and the corresponding points *e''e'''* the pressure is in excess of the resistance, while between *e'''e* and *e'e''*, the resistance is in excess of the pressure; consequently, during the two first intervals, the surplus work is poured into the fly-wheel, and during the last two intervals the deficiency in work is supplied by the energy of the fly-wheel being diminished. As the fly-wheel can only receive or restore energy by having its velocity increased or diminished, we see that the velocity of the crank-pin is not, strictly speaking, uniform, but it can be kept within any assigned limits of deviation from uniformity, by altering the weight of the fly-wheel.

The diagram shown on fig. 30 can also be drawn on a straight base instead of on the circumference ABC. This form of the diagram is more generally used in practice, because it is easier to test the accuracy of the work; but it is not so graphic to the eye as the circular form of fig. 30. To construct the diagram on a straight base, draw a straight line AC equal in length to the semicircumference of the circle described by the crank pin. Divide AC into

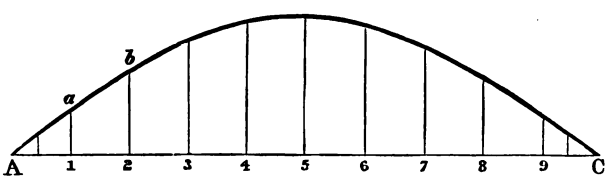

Fig. 30 A.

ten equal parts corresponding with the divisions of the semicircle ABC. From each of the points of division, 1, 2, 3, &c., erect a perpendicular, and mark off the lengths 1*a*, 1*b*, 1*c*, &c., equal to the lines 1', 2', 3', &c., in fig. 30.

Through the points *a b*, &c., draw the curve A*ab*C; then the ordinates of this curve will give the tangential pressures on the crank for any position of the latter.

It is evident that the area bounded between the straight line AC and the curve measures the work done upon the crank; for the ordinates represent the effective pressures on the crank pin, and the abscissæ the spaces through which they are exerted. Now the amount of work done upon the crank is, as has been shown above, equal to the work done upon the piston. Hence the area of fig. 30A should be exactly equal to the area of the indicator diagram. By measuring the area and comparing it with that of the indicator diagram, we have a ready check of the accuracy of the work.

If we wish to show the diagram of tangential pressure for the whole revolution, we have only to prolong AC to double its original length and construct on the prolonged portion another curve precisely similar to A*ab*C.

We will now take the case of an expansion diagram and show the effect which the want of uniformity in the steam pressure acting on the piston has upon the form of the diagram which shows the tangential effort on the crank-pin. We will further suppose as before that the connecting rod is infinitely long, and that the moving parts possess no weight, or are moving at a very slow velocity.

The steam diagram is shown at the upper part of fig. 31. The cut-off is supposed to take place at $\frac{3}{10}$ of the stroke. The engine is non-condensing. The first thing to ascertain is the net pressure of the steam which urges the piston forward. In order to find this it will in most cases be necessary to construct from the ordinary indicator diagram a new diagram showing the actual pressures after deducting the corresponding back pressures (see page 340). In the present instance this step will not be necessary, because to avoid complication an indicator diagram has been chosen in which the back pressure is uniform throughout the stroke

and the compression curve at the end of the return stroke is exactly similar to the exhaust curve at the commencement; consequently to obtain the net pressures on the piston we have only to measure the length of vertical ordinate bounded between the upper and lower boundary lines of the diagram.[1]

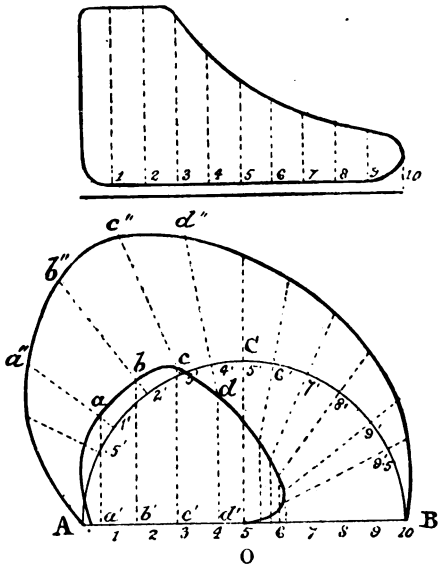

Fig. 31.

Divide the length of the diagram into ten equal parts, and from each point of division draw a vertical ordinate to the upper boundary of the diagram. Draw a line AB of the

[1] This is a case which would probably never occur in practice. In all ordinary cases two diagrams are required : viz. one from the top and the second from the bottom cover of the piston. The net pressures are then taken by deducting from the gross pressure as shown by one diagram the simultaneous back pressure as shown by the other (see page 340). This precaution is very frequently neglected, and has led to serious errors in the calculation of curves of twisting moments.

same length as the diagram to represent the diameter of the crank-pin circle. Divide the line AB into ten equal parts, and from each point 1 2 3 &c. erect a perpendicular 1,1′ 2,2′ 3,3′ &c., intersecting the circumference at the points 1′ 2′ 3′ &c. These lines are not actually drawn, so as to avoid complicating the diagram. Then at each position of the crank-pin 1′ 2′ 3′ &c., the direct pressure on the pin is represented by the corresponding ordinate taken from the indicator diagram. From the centre O draw radial lines O1′, O2′, O3′, &c. intersecting the circumference. It is only necessary to show the portions of these lines which are prolonged beyond the circumference ACB. On these lines measure off the parts O*a*, O*b*, O*c*, O*d*, &c., equal respectively to the ordinates of the steam diagram 1, 2, 3, 4, &c. Then, as in the first case, the actual *tangential* pressures on the crank-pin will be equal to these lines O*a*, O*b*, O*c*, O*d*, &c., multiplied by the sines of the angles which the crank makes with AB. In other words, the tangential pressures will be equal to the perpendiculars let fall from the points *a*, *b*, *c*, *d*, &c., on AB, i.e. *aa*′, *bb*′, *cc*′, *dd*′, &c. Produce the radial lines O*a*, O*b*, &c., and from the points 1′, 2′, 3′, 4′, &c. on the circumference set off the parts 1′*a*″, 2′*b*″, 3′*c*″, 4′*d*″, equal in length respectively to *aa*′, *bb*′, *cc*′, *dd*′, &c., then the curve drawn from A to B through the points *a*″, *b*″, *c*″, *d*″, &c., will be the diagram of tangential effort, or twisting moment on the crank-pin.

To ascertain the diameter of the circle of average tangential pressure, i.e. the resistance circle, we have only to compute the average steam pressure as shown by the indicator diagram and multiply the same by the fraction $\frac{2}{\pi}$ the product will give the radius of the resistance circle. The tangential effort on the return stroke is, of course, exactly similar to the curve A*a*″*b*″B, and is obtained in the same way.

The above method is purely graphical. In actual

practice it will probably be found more expeditious to construct a curve of twisting moments, the radial ordinates of which are found by multiplying the steam pressure for any given position of the crank by the leverage at which it works, and then setting off the moment thus obtained to scale. Thus, for the position of the crank 2′, fig. 31A, we have a pressure of 18 lbs. to the square inch, according to the scale on which the diagram is drawn when the piston occupies the position 2 corresponding with the position 2′ of the crank. Also the leverage at which it acts is 2′*b*. If the radius of the crank be one foot, then to the same scale 2′*b*=·799 ft., and the product 18 × ·799= 14·3 = the twisting moment. Draw a radial line through 2′ and set off 2′*b*′= 14·3 to any convenient scale. Proceed in a similar manner for all other positions of the crank, and the curve A*b*′B drawn through the extremities of the radial lines is the curve of twisting moments.

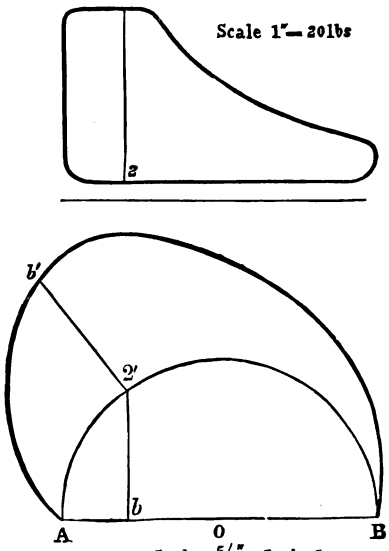

Fig. 31 A.

Influence of the connecting rod in modifying the curve of tangential effort on the crank-pin.—In actual steam engines the connecting rod is of course always of finite length, and consequently is always acting at an angle to the axis of the cylinder, except when the crank-pin is on the dead centres.

The size of the angle which the connecting-rod makes with the axis for any position of the crank-pin depends on the length of the rod compared to the length of the crank. The shorter the relative length of the rod, the greater the angle for any given position of the pin. To find the inclination of the connecting rod for any given position of the crank we have only to take the length of the rod as a radius, and

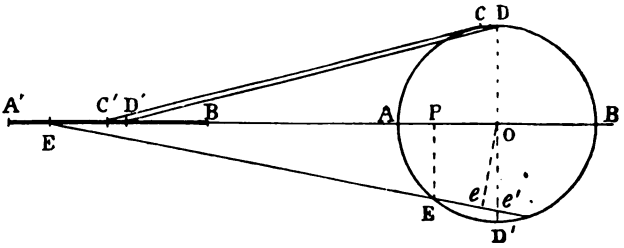

Fig. 32.

from the centre of the crank-pin to describe an arc intersecting the axis of the cylinder prolonged. The line joining the point of intersection with the centre of the crank-pin gives the angular position of the connecting rod and also the position of the piston for the given position of the crank-pin. Thus, let the length of the connecting rod be four times that of the crank, so that when the crank-pin is at A the piston rod end of the connecting rod will be at A′, and $AA' = 4AO$. When the crank has moved round a quarter of a revolution to D, with centre D and radius $= AA'$ describe an arc intersecting the line AO. The point of intersection will be D′, the distance AD′ being greater than the half-stroke of the piston, which latter equals A′C′. When the piston is at half-stroke, the crank will occupy the position OC.

The tangential effort on the crank due to the piston pressure may be calculated by estimating the effect of this pressure in the direction of the connecting rod, and then resolving this tangentially and radially to the crank circle ; or, it may be more conveniently computed geometrically by finding

the leverage with which the connecting rod acts on the crank-pin. The amount of the tangential pressure will, as before, be proportional to the length of the arm of the lever. Take, for instance, the position E of the crank-pin, the connecting rod assumes the position EE′, and the leverage with which it acts, instead of being EP, as would be the case were the rod of infinite length, is *e*O, found by producing EE and letting fall a perpendicular upon it from O. The product of the line *e*O into the pressure or tension in the connecting rod gives the twisting moment. Now the pressure or tension in the connecting rod is to the pressure on the piston as the line EE′ is to the line E′P. Also the triangle O*ee′* is similar to the triangle EE′P, and

$$\text{O}e : \text{O}e' :: \text{E}'\text{P} : \text{E}'\text{E}$$
$$\therefore \text{O}e' \times \text{E}'\text{P} = \text{O}e \times \text{E}'\text{E}.$$

That is to say, the pressure on the piston multiplied by O*e′* equals the force in the connecting rod multiplied by O*e*, which latter product is the twisting moment.

Hence to obtain the twisting moment we have only to prolong the line of the connecting rod till it intersects the position DD′ of the crank, and the product of this line into the pressure on the piston gives the moment required. By proceeding in this way fig. 31A may be modified so as to take account of the influence of the connecting rod. By an inspection of fig. 32 it is evident that the new arms of the moments are greater than those obtained with infinite connecting rods until the axis of the connecting rod intersects the point D, after which they are less till the end of the stroke is reached. During the return stroke the opposite effect takes place, the arms of the levers being shorter during the first portion, and longer during the latter part of the stroke, than for the corresponding positions when the connecting rod is of infinite length. The arm of the moment is a maximum when the axis of the connecting rod makes a tangent with the circle described by the crank-pin.

An inspection of the diagrams, figs. 30 to 31A, shows how far from uniform is the tangential effort on the crank-pin, and consequently how irregular is the driving power, in the case of single cylinder engines, even when, as in the first case illustrated, the steam pressure is uniform throughout the stroke, and the angularity of the connecting rod is neglected. There are various methods of diminishing this irregularity of driving power. One plan is to fit on to the crank-shaft a fly-wheel of adequate weight and dimensions to overcome the irregularity. The principle of the action of fly-wheels has already been explained (page 157). Another and very usual plan is to use two or more cylinders with the cranks forming angles with each other. These are usually so arranged that the tangential effort on one crank is a maximum when it is a minimum on the other crank. This subject will be again referred to, and examples will be illustrated, when all the disturbing causes which influence the forms of twisting moment diagrams have been explained, but in the meantime it will be a useful exercise for the student to prepare such diagrams for the two cases illustrated in figs. 30, 31 (pp. 166 and 170), assuming that in each case a second cylinder of equal size with the first, and working under precisely similar conditions, is added, and that the cranks are set at right angles to each other.

The method of proceeding is as follows:—A precisely similar diagram to the curve A$a''b''c''$ ..... B, fig. 31 (p. 170), must be constructed on the diameter COD which is at right angles to AOB. When complete, there will be four of these curves round the circle ACBD, viz. one for each stroke of each cylinder. The radial lines intercepted between the circumference and the curves for each position 01′, 02′, &c. of the crank, &c., must then be added together, and a resultant curve drawn through their extremities. This curve will be a diagram of the tangential effort for the two cylinders combined.

Influence of the weights and velocities of the reciprocating parts.—We must next consider the effects of the weights and velocities of the reciprocating parts on the form of the tangential effort diagram. In the first part of this chapter (p. 153) it was shown that a large portion or even the whole of the steam pressure during the first part of the stroke might be absorbed in generating the velocity in these parts; and consequently, that only a portion, or in some case not any of the steam pressure on the piston was available for transmission to the crank-pin; while, on the other hand, the effect of the velocities of the parts being reduced during the latter portion of the stroke would be to cause a greater pressure to appear on the crank-pin than is due to the steam pressure in the cylinder. It is evident, therefore, that to construct a tangential effort diagram for such cases we must first deduce from the indicator diagram a new diagram, by taking away from the steam pressure, during the first part of the stroke, such a portion of it as goes to accelerate the velocity of the moving parts, and *vice versâ*, adding to the steam pressure during the remainder of the stroke such a portion as represents the effect of their retardation.

If the steam pressure were free to act on the reciprocating masses in the same way that gravity acts on a falling body, or in the way that the pressure of powder-gases in a gun acts on a projectile, it would be easy to calculate the effects produced; but when the motion is controlled by a crank revolving uniformly, it is a little more difficult to calculate the increments of velocity imparted to the piston in successive intervals of time. The pressures required to impart these accelerations, as they are called, are of course proportional to the amounts of the increments.

We must again suppose for the sake of simplicity that the connecting rod is of infinite length. In such a case while the crank travels through successive equal angles corresponding to the positions A, A′, A,″ (fig. 33) the piston moves through successive spaces AA_1, A_1A_2, A_2A_3 &c. The

velocity of the piston at each point A_1 A_2 A_3 &c. may be calculated graphically as follows :—

Let the radius of the circle A B C represent to scale the linear velocity of the *crank-pin* in feet per second. At any point A″ for example, corresponding to the position A_2 of the piston, the velocity of the crank-pin may be resolved into two components, one horizontal and the other vertical. The horizontal component will be the velocity of the piston. From the point A″ draw a tangent A″T to the circle. Make A″T=the radius of the circle, then A″T represents the velocity of the crank-pin. Draw A″H horizontal and TH vertical, then will A″H and TH represent respectively the horizontal and vertical components of the velocity at the point A″, and A″H represents also the velocity of the piston when at a point in the stroke corresponding to the position A_2 in the line AC. In the same way the piston velocity may be obtained for any other point in the stroke. It may, however, be ascertained more simply as follows. The two triangles A″TH and $A''A_2O$ are in every respect equal to each other and A″H equals $A''A_2$. Now $A''A_2$ is the sine of the angle of the crank × A″O, therefore the velocity of the piston at any point A_2 is proportional to the sine of the angle of the crank for that position, and is in fact equal to the length of the perpendicular drawn from the given point, such as A_2, to meet the circumference of the circle, when the velocity of the crank-pin is represented by the length of the radius of the crank circle. Hence we see that the velocity of the piston at a series of successive positions A_1 A_2 A_3 &c. is represented by the vertical ordinates $A'A_1$ $A''A_2$ $A'''A_3$ &c.

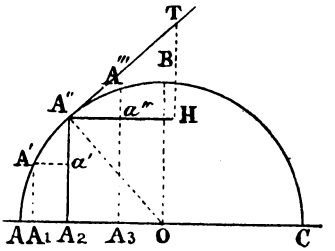

Fig. 33.

As the crank-pin is supposed to travel at a uniform velocity, the crank-pin circle represents time, just as does the hour circle of a watch, and equal divisions of this circle, such as AA′, A′A″, A″A‴, &c., represent equal divisions of time. Now, the difference between the velocity of the piston at the beginning and end of any such interval is the increment of velocity, or acceleration imparted to the piston during the interval. Thus the difference between the velocities at A′ and A″ equals A″*a*′, and similarly between A‴ and A″ equals A‴*a*″. Now, the force or pressure required to impart a velocity to a given mass in a given interval of time is proportional to the velocity imparted, and when this latter and the mass are known the force may be calculated (see p. 149). If we suppose the divisions AA′ &c. to be taken so small that the force acting throughout the interval may be considered as uniform and the acceleration imparted uniformly, then in this case any division such as A″A‴ may be considered a straight line, and the two triangles A″A‴*a*″ and A″A_2O are similar, because the angle OA″A‴ may be considered a right angle and *a*″A″A_2 is a right angle, and taking away the common angle the remainder *a*″A″A‴= the remainder OA″A_2. Also the angles at *a*″ and A_2 are right angles, therefore the third angles in each triangle are equal and the two triangles are similar, therefore $\frac{A'''\ a''}{A'''A''}=$ $\frac{A_2O}{A''O}$=cosine of angle of crank. Consequently the acceleration at any position of the crank equals the velocity of the crank-pin multiplied by the cosine of angle of crank, and the forces required to produce the accelerations are proportional to them.

The magnitudes of the forces may be found in the following manner. Suppose that the weight of the reciprocating parts is all concentrated round the crank-pin, the connecting rod being, as before, infinite. The weight is kept moving in the circular path by the action of centripetal force (see p. 159).

The centripetal force always acts radially, and at any point D, fig. 34, may be resolved horizontally and vertically. The horizontal component is the measure of the force which imparts motion to the reciprocating parts. The vertical component merely produces an upward or downward pressure on the bearings. At the dead centre A the force has no vertical component, and therefore the entire centripetal force produces acceleration of the reciprocating parts.

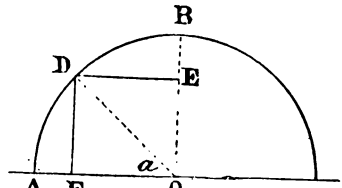

Fig. 34.

At B there is no horizontal component, and therefore there is at this point no acceleration, at any other point D, the horizontal component $=DE=FO=DO\cos a$. If the radius of the crank circle represent the centripetal force, then the horizontal component at any point=the centripetal force multiplied by the cosine of the angle of the crank.

The expression for the centripetal force in terms of the weight, the revolutions per minute, and the radius of the crank, is given on p. 161, and is $F=wN^2.r\times$ ·00034. This may be expressed in pounds per square inch of piston area by dividing by the area of the piston. Draw a circle A B C D (fig. 35) of which the length of the radius represents the centripetal pressure per square inch of piston area. Then the pressures per square inch of piston area required to accele-

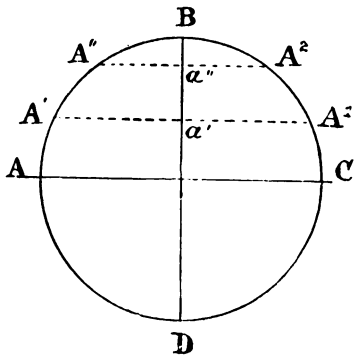

Fig. 35.

rate the reciprocating parts at any points corresponding to the positions A′A″ &c. of the crank will be represented by the lengths of the horizontal lines A′*a*′, A″*a*″ &c. While the crank is moving from A to B the pressures go to accelerate the reciprocating parts, and must therefore be subtracted from the indicator diagram pressures for the corresponding points, if we wish to arrive at the true turning effort on the crank. On the other hand, as the crank moves from B to C the opposite effect takes place, the motion of the reciprocating parts is being retarded, and imparts pressures to the crank-pin, which for any given positions A^2, A^1 &c. are

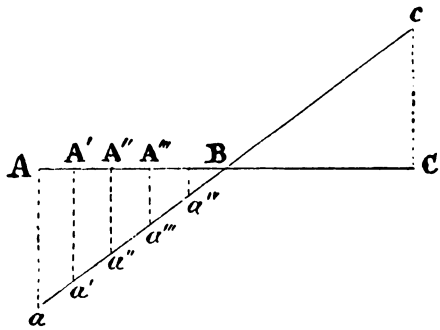

Fig. 36.

measured by the horizontal lines A^2a'', A^1a' &c. These pressures must therefore be added to the pressures shown by the indicator diagram for the corresponding points.

A simpler diagram for use with the indicator diagrams is made as follows: Let AC, fig. 36, represent the stroke of the piston or diameter of crank-pin circle, to the same scale as the indicator diagram is drawn. Calculate the centrifugal force per square inch of piston area from the known weights of the reciprocating parts, radius of the crank circle, and number of revolutions per minute, by the formula given, p. 161, and draw a perpendicular A*a* to the same scale as the pressure scale of the indicator diagram, to represent this

force. Erect an equal perpendicular from C. Join *ac*. Divide the line AC into ten equal parts corresponding to the ten divisions of an indicator diagram, then will the ordinates A*a*, A′*a*′, A″*a*″, &c. between A and B represent the pressures to be subtracted from those given by the indicator diagram, while the corresponding ordinates between B and *c* represent pressures to be added to those shown by the diagram.

Influence of the weights of the reciprocating parts in vertical engines.—If the engine were of the vertical type, we should also have to take account of the pressures required merely to overcome the force of gravity acting on the weights of the piston, &c. During the up stroke the weights act against, and during the down stroke in the same direction, as the motion of the piston. Hence we should have to modify fig. 36 as follows. Add to A*a* a portion *aa*′, fig. 37, representing the weight of the reciprocating parts per square inch of piston, to the same scale as A*a* represents the pressure required to accelerate the parts. At the other end of the stroke the action of the weights is to reduce the pressure restored by the retardation of the reciprocating parts to the crank pin. Set off therefore *cc*′=*aa*′ and join *a*′*c*′; then the ordinates of the line *a*′B′ represent the pressures to be subtracted from those given by the indicator diagram, while the ordinates of B′*c*′ represent pressures to be added to those shown by the diagram.

In the reverse stroke the action of the weights is in the same direction as the steam pressure, and aids the acceleration of the reciprocating parts at the commencement of the stroke, and increases the pressures on the crank-pin at the end. This is clearly shown by the ordinates of the line *a*′*c*′ in fig. 38. The influence of the direct weights of the reciprocating parts becomes of great practical importance in the case of the large low pressure cylinders of quick running compound engines in which the average steam pressures are low, and the weights often reach as much as 3·5 lbs. per square inch of piston.

It is impossible to exaggerate the benefit to be derived by testing the proposed indicator diagrams of any engine under design, in the manner described, before finally settling the weights of the moving parts, the pressure and distribu-

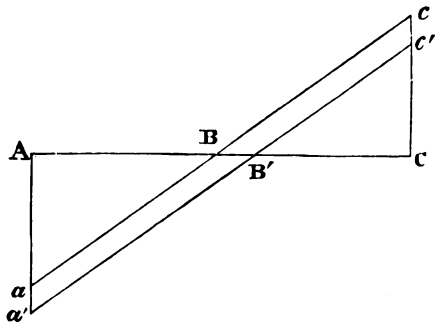

Fig. 37.

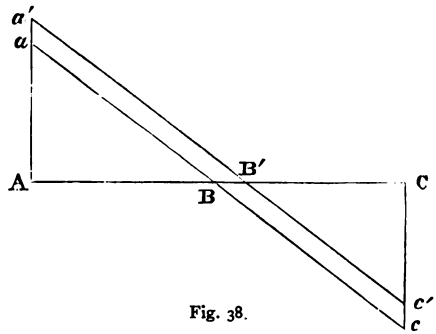

Fig. 38.

tion of the steam in a quick-running engine. This will be clearly shown from an example taken from actual practice, after we have considered the last remaining complicating circumstance, viz. the effect of the length of the connecting rod on the pressures required to accelerate and retard the reciprocating parts.

Effect of the connecting rod in modifying the influence of the reciprocating parts.—A finite connecting rod, instead of moving always parallel to the axis of the engine, vibrates from side to side, and is always inclined at an angle to the axis of the engine, except when the crank is on the dead centres. The result of this is that during the first quarter of a revolution the piston moves through more than half the stroke, and its average velocity is therefore greater than when the connecting rod is infinite, and *vice versâ* during the second quadrant the piston has to move through less than half stroke and its average velocity is therefore less. These changes are illustrated by fig. 39, in which the length of the connecting rod *ab* is three times the length of the

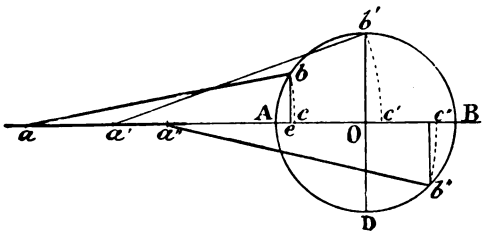

Fig. 39.

crank. While the crank has been moving from A to b' the piston has moved through a distance $Ac'=AO+Oc'$, where $Oc'=a'b'$ versin $Oa'b'$. While the crank moves from b' to B the piston moves through a distance$=c'B=OB-a'b'$ versin $Oa'b'$. During the return stroke the opposite effect takes place, while the crank moves from B to D, the piston moves through Bc', and while the crank moves from D to A, the piston moves through $c'A$. Generally speaking, if n be the ratio of the length of the connecting rod to that of the crank, then for any position of the crank, say b, in the first and second quadrants, the distance moved through by the piston$=Ae+cc=$versin angle of crank$+n$ versin angle of connecting rod. On the other hand for any position, say b'', in the second

or third quadrants the distance traversed by the piston= versin angle of crank $- n$ versin angle of connecting rod.

The consequence of the increased velocity during the first part of the forward stroke is that more of the steam pressure will be required to accelerate the moving parts than in the case of an infinite connecting rod. The amount of this pressure varies from point to point, and is proportional to the amount of the acceleration at each point. During the latter part of the stroke, the retardation is less sudden, and consequently the moving parts never exert as great a pressure on the crank pin in coming to rest as they would do were the connecting rod infinite. During the return stroke the converse takes place. During the first portion of this stroke the steam pressure required to produce acceleration is less, and during the latter portion the pressure exerted by the moving parts on the crank pin is greater, than in the case of an infinite connecting rod. As, however, the length of the connecting rod cannot alter the *area* of the diagram, fig. 36, but only its shape, we shall find that the period during which high pressure is being exerted is less, and that during which low pressure is being exerted is greater than in the case of the infinite rod.

To calculate the exact amount of these pressures for every point of the stroke would be rather a tedious process. It is, however, easy to ascertain the pressures for three positions of the piston, and a curve drawn through these points will enable us to measure the pressures expended in accelerating the reciprocating parts, or given out during their retardation. The relative velocities of the crank-pin and the piston may easily be ascertained geometrically in the following manner.

Let AB, fig. 40, be a portion of the path of the crank pin, and C*b* any position of the crank, and *ab* the corresponding position of the connecting rod. The velocities of the two ends of the connecting rod are in different directions, the cross-head end always moving in the fixed direction *a*C

with variable velocity, while the crank pin end always moves with fixed velocity at right angles to the momentary position of the crank arm. As the velocity of the crank pin is known, that of the cross-head, which is the same as that of the piston, can be ascertained. Produce *ab* till it cuts the perpendicular CB in *e*. From *b* draw *bf* perpendicular to C*b*. Let the velocity of the crank pin be represented by the radius C*b*. Make *bf*=C*b*. Then *bf* represents the

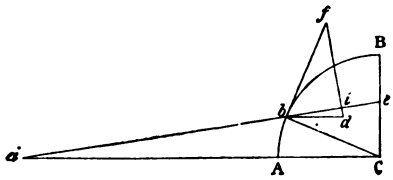

Fig. 40.

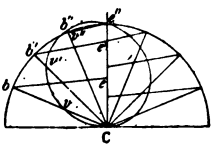

Fig. 41.

velocity of *b* both in magnitude and direction. From *f* let fall a perpendicular, *fi*, to the direction of the connecting rod, and produce the line *fi*. From *b* draw *bd* parallel to the direction of velocity of the point *a*. Then, since the connecting rod is of invariable length, the components of the velocities of each end resolved along the rod are equal. Now *bi* is this component for the velocity of *b*, therefore it is also the corresponding component for *a*, and therefore *bd* represents the magnitude as well as the direction of the velocity of *a*. Now, comparing the triangles C*be*, *bfd*, their angles are equal, because the sides of one triangle are perpendicular to those of the other; also the side *bf* equals the corresponding side C*b*, therefore the two triangles are equal, and therefore the line C*e*, cut off from CB by the prolongation of the line *ab*, represents the variable velocity of the point *a*, while C*b* represents the constant velocity of *b*.

We can now construct a very simple curve of piston velocities. For any positions of the crank, C*b*, C*b'*, &c. (fig. 41), set off C*v*, C*v'*, &c.=C*e*, C*e'*, &c. Through all the

points v, v', &c., draw the closed curve Cvv' . . e''C, then the portion of the radius intercepted, at any position, between the centre C and the circumference of this curve represents the velocity of the piston for that position. It will be noticed that a portion of this curve travels outside the crank pin circle, showing that the piston velocity during a part of the revolution is greater than that of the crank pin. The maximum velocity and corresponding position of the crank can be obtained from the diagram. In practice, when the connecting rod is three or more times as long as the crank, the position for maximum velocity corresponds very nearly with the position when the connecting rod makes a tangent with the crank pin circle. Now for this position,

$$\frac{\text{length of connecting rod}}{\text{length of crank}} = \text{tan angle of crank.}$$

For instance, when the connecting rod is 3, 4, 5, or 6 times the length of the crank we have

$$\tan \phi = 3,\ 4,\ 5, \text{ and } 6 \text{ respectively,}$$

which, by means of a table of natural tangents we find correspond with values of ϕ of 71° 34′, 75° 58′, 78° 42′, and 80° 32′ respectively.

Take the case of a piston rod four times the length of

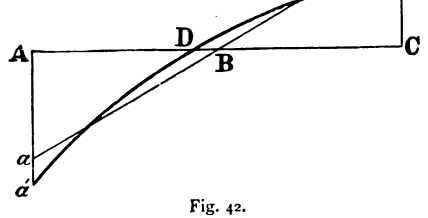

Fig. 42.

the crank. It is required to find three points on the curve $a'c'$, fig. 42 which corresponds with the straight line ac,

fig. 36, and the ordinates of which measure the pressures which have to be added to or subtracted from those of the steam diagram. The point D corresponds with the position which the piston occupies when the connecting rod makes a tangent with the crank circle—that is to say, when the crank is at the angle 75° 58′; for, at this point, as has been stated above, there is neither acceleration nor retardation. The points a' and c' of the curve may be found from the following considerations.

When the crank-pin gets into line with the axis of the cylinder—*i.e.* at the two dead centres, the weights of the reciprocating parts act with a full centrifugal effect on the crank. At the near dead centre they tend to pull the crank towards the cylinder, while at the far centre their action on the crank is reversed. The centripetal force in the crank at the near centre gives the measure and direction of the force absorbed in accelerating the reciprocating parts when the connecting rod is supposed infinite. When, however, the rod is of finite length another effect is produced; for on passing the near centre the crank-pin end of the connecting rod describes an infinitely small arc of a circle round the piston rod end as a centre, and a centrifugal effect is produced which increases that in the crank. On the other hand, when the far dead centre is being passed the centrifugal tendency in the connecting rod end diminishes that in the crank. The amount of the centrifugal force in the connecting rod circle may be derived from that in the crank, very simply, because the weights are common to both, also the velocities are the same, and nothing differs but the length of the radii. Now the radius of the connecting rod is four times that of the crank, therefore, as the centrifugal forces vary inversely as the radii when other things are equal, the centrifugal force in the connecting rod is one fourth of that in the crank. Add therefore to Aa, a piece $aa' = \frac{Aa}{4}$, and subtract from Cc a piece $cc' = \frac{Cc}{4}$ then will the

points a' c' be the initial and terminal points on the curve. The point D has been already found. It may be proved that the curve to be drawn through the three points a', D, c' is a parabola, but for all practical purposes a circular arc is sufficiently accurate. The ordinates of this curve between a' and D represent pressures which are expended in accelerating the moving parts, which pressures must therefore be subtracted from the pressures as shown by the indicator diagram ; on the other hand, the ordinates between D and c' represent pressures which are exerted by the moving parts on the crank pin when they are being brought to rest, and which must therefore be added to the pressures as shown by an indicator diagram. For the return stroke the same diagram may be used, starting from C as the commencement of the stroke. The ordinates of c'D represent the pressures absorbed in accelerating, and those of Da' the pressures restored during retardation.

If the engine be of the vertical type a correction must be applied to fig. 42, similar to that already explained in the case of fig. 37.

An inspection of fig. 42 shows what a powerful influence on the working of the engine may be exerted by the action of the reciprocating parts. This influence may, according to circumstances, be either good or bad. Thus, take the case of a quick running single cylinder expansion engine. The steam pressure in such an engine would be high at the commencement of the stroke and low at the end, but the power required to impart motion to the reciprocating parts absorbs pressure at the beginning of the stroke, and thus relieves the pressure that would otherwise come on the crank ; while at the end of the stroke the opposite effect takes place, and thus the inertia of the moving parts may tend to equalise the pressure throughout the stroke, and may consequently promote steady running. In this respect the action is similar to that of a fly-wheel. On the other hand, it may happen, if the speed at which the engine runs

is very high, or if the reciprocating parts are very heavy, that the whole of the steam pressure in the cylinder is insufficient to impart the requisite motion at the commencement of the stroke, and consequently the deficiency of force must be supplied by the fly-wheel, the result being that at the commencement of the forward stroke the connecting rod is actually being dragged by the crank-pin instead of turning the latter. As soon, however, as a certain amount of motion is imparted, the pressure of the steam begins to be felt on the crank-pin, and the strain on the rod changes from tension into compression, and a knock or jar is experienced at the joints which greatly tends to wear out the parts.

The following example, taken from the well-known Allen engine, has occasionally been adduced to illustrate this point. The essential particulars of this engine are given below :—

Diameter of cylinder	=1 foot.
Stroke	=24 inches.
Revolutions per minute	=200
Weight of reciprocating parts	=470 lbs.
Steam pressure	=60 lbs. per sq. inch.
Ratio of length of connecting rod to crank	=6·16 : 1.

ABC, fig. 43, shows an indicator diagram when steam is cut off at one twentieth of the stroke.

If the connecting rod were infinite, the pressure per square inch of piston area required to accelerate the moving parts at the commencement and end of the stroke would be obtained by the formula given on page 161, viz.:

$$F = \frac{w N^2 r \cdot 00034}{\pi \frac{d^2}{4}} = \frac{470 \times 40000 \times 1 \times \cdot 00034}{113 \cdot 1}$$

$$= 56 \cdot 5 \text{ lbs. per square inch.}$$

Measure from A to C, and from D to C′, 56·5 lbs. to scale, and join CC′. Then the ordinates of the triangle

ACE represent pressures to be subtracted from the corresponding steam pressures as deduced from the diagram,

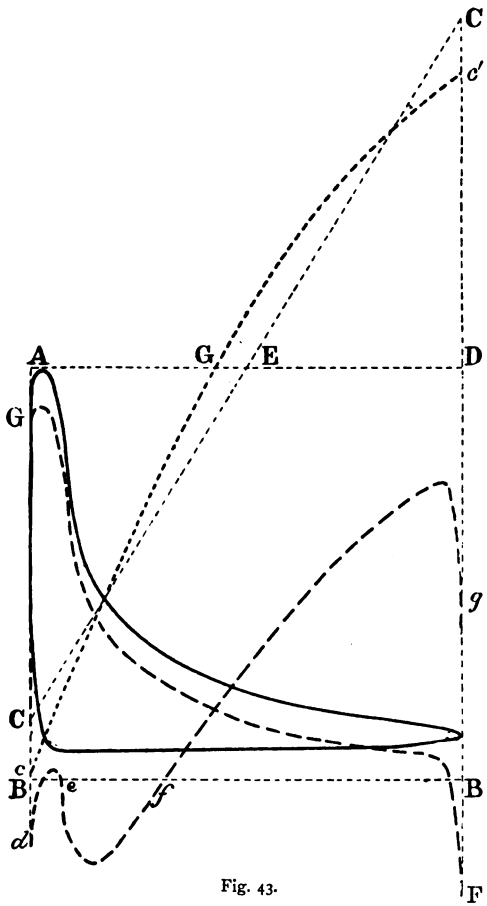

Fig. 43.

and the ordinates of the triangle DC'E represent pressures to be added to those of the diagram in order to arrive at

the true pressures transmitted through the connecting rod to the crank-pin, on the assumption that the length of the connecting rod is infinite. The effect of the connecting rod being 6·16 times the crank in length is to add $\frac{56·5}{6·16}$ =9·17 to AC and to subtract the same quantity from DC′. Mark off therefore *c*C and *c′*C′ each = 9·17 lbs. Ascertain the point G in the manner already described, and through *c*G*c′* describe a parabolic curve. The ordinates of this curve above and below the line AD represent pressures to be respectively added to and subtracted from those deduced from the indicator diagram. We must next find from the indicator diagram the true net pressures of steam on the piston. This is done by deducting from the pressures, as shown by the diagram, the simultaneous back pressures as shown by another diagram, taken from the other cylinder cover. In the absence of such a diagram, we may make use of the back pressure line of the diagram on fig. 43, remembering, however, that from the steam pressure at A must be deducted the back pressure at the exhaust end. We thus obtain the true curve of pressures shown by the line GF beneath the expansion line of the diagram, and terminating below BB at the point F. Having now deducted from the true steam pressures the amounts given by the ordinates of the curve *c*G*c′*, we obtain the curve *defg*, the ordinates of which represent the true pressures transmitted through the connecting rod. The pressures above the line BB′ are positive, while those below are negative, and show that the steam pressure on the piston, wherever the curve falls below the line, are insufficient even to accelerate the moving parts. Thus, for instance, we have seen that at the commencement of the stroke 56·5+9·17= 65·67 lbs. per square inch are required for this purpose, whereas only 60 lbs. are available, without even deducting for the back pressure. The deficiency has to be supplied by the fly wheel, and the consequence is that at the commence-

ment of the stroke the piston is actually being dragged forward by the crank-pin, instead of pushing the latter round, as it should do. When the curve *defg* rises above the line there is a small amount of pressure transmitted to the crank-pin, and consequently the strains in the piston and connecting rods are reversed from tension to compression, and a knock must occur if there is the least wear in the brasses. Between *e* and *f* the steam pressure is again insufficient, and the piston is again drawn round by the crank-pin, and at each of these points a knock will occur.

It will be noticed that the general effect of the action of the reciprocating parts is to completely reverse the pressures as deduced from the indicator diagram ; for whereas these are greatest at the commencement of the stroke and dwindle down to nearly nothing at the end, when the reciprocating parts are taken into account, the pressures are greatest at the end of the stroke and are actually negative at the beginning.

During the return stroke the state of things at the commencement will not be so bad because the pressure required is only $56{\cdot}5 - 9{\cdot}17 = 47{\cdot}33$ lbs., and there is sufficient steam power available for this purpose and to leave a balance over to transmit to the crank pin. On the other hand, at the end of the stroke the state of things will be very unfavourable for easy running because of the enormous accumulation of pressure.

In order to improve this engine, one of three plans might be adopted. The initial steam pressure might be increased : the piston speed might be diminished ; or, lastly, the weights of the moving parts might be reduced. A combination of these methods would, of course, also be effectual.

Effect of distribution of steam on the action of the reciprocating parts.[1]—Great care must be taken when designing the valve gear of high-speed engines to see that the distribution of the steam is properly effected; otherwise the ill effects of

[1] Students who have no knowledge of indicator diagrams should read this section after studying Chapter VIII.

a bad distribution may be greatly aggravated. Take, for example, the very common fault of a late exhaust, as represented by fig. 44 Assuming that the diagram of the return stroke is equally bad, the result will be that there will be a back pressure $a'b' = ab$ at the commencement of the stroke, and the effective steam pressure will only be Aa', which, if the piston speed is high, and the weight of the reciprocating parts considerable, may not be sufficient to impart the desired velocity to these latter. A late admission of steam will, of course, produce a similar result to a still greater degree.

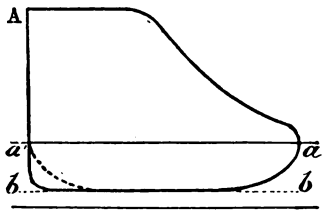

Fig. 44.

On the other hand a good compression curve, which, as we shall hereafter see, is an excellent feature in an indicator diagram, is productive of great good from the point of view of steady running, when its effect on the action of the reciprocating parts is taken into account. We have seen that the tendency with high-speed expansive engines is that the effective pressure on the crank-pin is transferred to the end of the stroke. Now it is very undesirable that the pressure on the pin should be very high at the *extreme* end of the stroke, as it causes heavy strains on many parts of the machinery; but the effect of a marked curve of compression is to cause a considerable back pressure at the end of the stroke, which pressure must of course be deducted from the pressure on the crank-pin, due to the combined effort of the steam and the retardation of the reciprocating parts. This action is of course much assisted if the exhaust opens early. In fact, in such cases it often happens that when a strong compression curve exists the back pressure on the piston is much in excess of the direct pressure.

This effect is clearly shown in fig. 43, and is also illustrated by fig. 45. The indicator diagram is clearly recognisable ; FGH, the curve showing the effect of the reciprocating parts, is indicated by the full line. The irregular dotted line AB is the resultant curve, giving the true nett pressures on the piston, and showing a negative pressure at the end of the stroke, on the assumption that a diagram taken from the other end of the cylinder would give a similar line of back pressure to that shown on fig. 45.

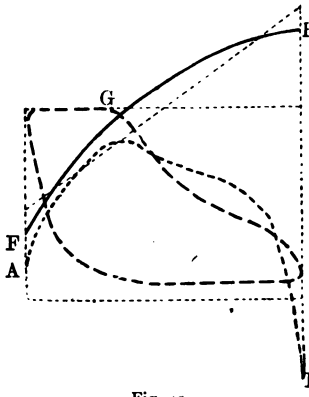

Fig. 45.

The student is now in possession of the means of constructing a curve of twisting moment, or tangential effort on the crank-pin, taking into account all the principal modifying circumstances. It will have been observed that the effect of the connecting rod is twofold. Firstly, by its varying inclination it alters the length of the tangential component of the pressure transmitted to the crank-pin (see p. 173) ; and secondly, it modifies the action of the reciprocating parts in the manner just explained.

The various steps to be taken to produce a complete curve of effort on the crank-pin are as follows :—

1st. To obtain a pair of indicator diagrams, viz. one from each end of the cylinder.

2nd. By taking account of the back pressure at each point in the stroke to deduce a pair of resultant diagrams showing the true pressures on the driving sides of the piston, as explained pages 190 and 340.

3rd. To find the effect of the weights and velocities of the reciprocating parts as modified by the length of the

connecting rod, and to correct the last found diagrams accordingly, as explained pages 183 to 192.

4th. To deduce from the last diagrams the circular diagrams of effort on the crank-pin, taking into account the obliquity of the connecting rod, as explained pages 172 to 175.

These various steps have been explained at such length that it will not be necessary to give a final example illustrating them. The student will have no difficulty in applying the principles to any example whatever.

In the case of two or more cylinders being coupled on to one crank shaft, the diagram of effort will have to be made for each cylinder separately, unless they are identical in dimensions and steam distribution, and a resultant diagram formed by adding their separate effects together; see page 175.

How to approximate to uniformity of effort on the crank-pin.—In all of the examples given the engines have had single cylinders, and though it is possible in such engines to attain to great uniformity of driving power throughout the greater portion of the revolution, even when very high rates of expansions are made use of, by running them at high speeds so as to utilise the action of the reciprocating parts, nevertheless, the difficulty exists that at the beginning of each stroke there is no rotative effort whatever on the crank. There are two methods of overcoming this defect.

First. Two or more cylinders may be made use of coupled on to the same crank shaft, but with the cranks set at angles to each other, in such a manner that one engine is producing its maximum rotative effort when the other is at the dead centre. Each of these cylinders may be identical in dimensions and in steam distribution, as is the case with locomotives and many types of land engines; or the engine may be compound, one cylinder being much larger than the other, and the steam which has been partially expanded in the small cylinder being used over again in the larger. This type of engine, which presents many advantages from the

point of view of fuel economy, will be more particularly described in Chapter XI. It has, on account of its economical properties, come into very general use for marine purposes.

Second. A fly wheel may be used which, by absorbing energy when the turning power is in excess of the resistance, gives it out again when this condition is reversed, and thus enables the engine to pass the dead centre.

The double cylinder type of engine is invariably used where there is much stopping and starting and reversing to be done, because it may be easily started from any position of the cranks. The single cylinder engine with a suitable fly wheel is used with advantage whenever there is much steady running to be done in one direction. This is the case with the majority of factory engines. The single cylinder type with a fly wheel, and a high rate of expansion, has even been used successfully for marine purposes.

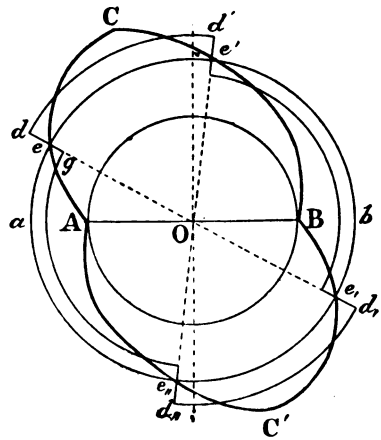

Fig. 46.

Graphic representation of the action of a fly wheel.—In cases where the resistance is uniform the power applied tangentially to the crank-pin in single cylinder engines is in

excess of the resistance during two portions of the revolution and is less than the resistance during the other two portions. This is illustrated in fig. 46, where AB is the crank circle, *ab* the circle of uniform resistance. The curve showing the twisting moment on the crank is denoted by ACB, BC'A. The tangential pressure is in excess of the resistance from *e* to *e'* and from e_1 to e_{11} and during the other two arcs the resistance is in excess. The average value of the excess of twisting moment is represented by *ed*, and of the resistance by *eg*.

The same thing may be illustrated also by a diagram constructed on a straight base similar to fig. 30A. The line ABA' is equal in length to the circumference of the crank-pin circle. ACB and BC'A' are the curves of twisting

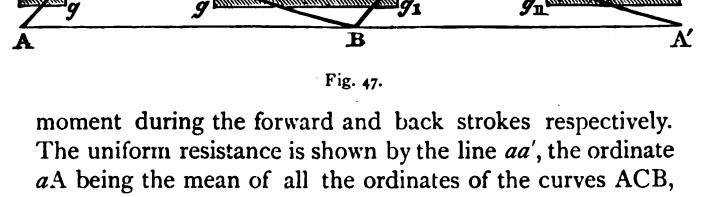

Fig. 47.

moment during the forward and back strokes respectively. The uniform resistance is shown by the line *aa'*, the ordinate *a*A being the mean of all the ordinates of the curves ACB, BC'A'. The excess of tangential pressure between *ee'* and e_1e_{11} above the resistance is shown by the ordinates of the curves *e*C*e'* and $e_1C'e_{11}$, and the excess of resistance above tangential pressure during the remainder of the stroke is shown by the ordinates of A*e*, *e'*B, Be_1, and e_{11}A'. At the four points *e*, *e'*, e_1, e_{11}, the resistance exactly equals the tangential pressure. The average values of the excess of tangential pressure is given by the lines *ed* and e_1d_1, the areas *edd'e'* and $e_1\,d_1\,d_{11}\,e_{11}$ being equal respectively to the areas *e*C*e'* and $e_1C'e_{11}$. Similarly the average values of the excess of resistance are given by the ordinates *eg* and *e'g'*. Also the

sum of the areas $edd'e'$ and $e_1 d_1 d_{11} e_{11}$ is exactly equal to the sum of the areas ag, $e'g_1$, and $g_{11}a'$.

The weight of fly wheel necessary in any given case depends on the following conditions :—

1. The mean diameter of the fly wheel rim.
2. The velocity at which this rim moves.
3. The variation from uniform speed which is to be permitted.
4. The fractions of the entire revolution during which the tangential force is above or below the uniform resistance.
5. The average amount of the excess of power of resistance.

When these conditions are known we can compute the weight of the fly wheel.

The value of elements 4 and 5 may be computed for any given case by constructing a diagram similar to fig. 47.

Let W=weight of the fly wheel in lbs.

V=the mean velocity of its rim in feet per second.

V_1 and V_2=the maximum and minimum velocities.

$\frac{1}{k}$=the fraction of the mean velocity allowed for variation, that is to say the difference between V_1 and $V_2 = V \times \frac{1}{k}$.

Then the energy stored up in the fly wheel when at its maximum velocity

$$= \frac{WV_1^2}{2g}$$

and when at its least velocity the energy remaining

$$= \frac{WV_2^2}{2g}$$

Therefore the work given out by the fly wheel while its velocity falls from V_1 to V_2

$$= \frac{W(V_1^2 - V_2^2)}{2g}$$

Now this work is measured by the product $ed \times ee' \times A$ the area of the piston (see fig. 47), ed being the average excess of tangential pressure in lbs. per square inch of piston.

$$\therefore ed \times ee' \times A = \frac{W(V_1^2 - V_2^2)}{2g}$$

$$= \frac{W(V_1 + V_2)(V_1 - V_2)}{2g} = \frac{W \times 2V \times \overline{V}k}{2g}$$

$$\therefore W = \frac{g \times k \times ed \times ee' \times A}{V^2}$$

Now V, the mean velocity of the fly wheel rim in feet per second, may be expressed as follows in terms of the diameter D, and the number of revolutions per minute R

$$V = \frac{\pi . D . R}{60}$$

Substituting this value of V in the above equation and reducing to tons, we have very nearly

$$W \text{ (tons)} = 5{\cdot}25 \frac{k \times ed \times ee' \times A}{D^2 R^2}$$

It will generally be found that the value of ed differs in the two strokes; we must therefore take the value which shows the greatest inequality in making the above calculation for the weight of the fly wheel, otherwise it will permit a greater fluctuation than is desired. The value of k varies according to the kind of work the engine has to do. When employed in driving dynamo machines or spinning machinery, or in any kind of work where very uniform motion is required, the variation from the mean velocity should not exceed from 1 to $1\frac{1}{4}$ per cent., that is to say the value of k would vary from 100 to 80. In other cases the variation may be as much as one twentieth, or 5 per cent. or $k = 20$. It will be seen from the foregoing formula that the weight of the fly wheel varies directly as k, that is to say the less the permissible variation from the mean

speed the greater the weight of the wheel. It also varies inversely as the square of the diameter and inversely as the square of the number of revolutions per minute.

The foregoing investigation applies to the case where the resistance is practically uniform. If the resistance fluctuates during each revolution, as for instance when the engine is driving a two-bladed screw propeller, a proper curve of resistance would have to be drawn in lieu of the line $a\,a'$, fig. 47. When the resistance is liable to sudden fluctuations, as for instance in the case of factories where a large number of machines are occasionally thrown on or off, the fly wheel by itself is powerless to produce even an approximation to steady running. To effect this in such cases is the duty of the governor, which controls the actual power developed by the engine, either by throttling the steam on the way from the boiler, so that its pressure is reduced considerably by the time it enters the cylinder ; or else, by varying the rate of expansion so that the engine develops more or less power in proportion to the work it has to do. For description of various governors see p. 239.

CHAPTER VI.

THE MECHANISM AND DETAILS OF STEAM ENGINES.

Cylinders with their fittings—Clearance—Steam passages—Valve boxes—Jacketing—Lubricators—Pistons—Piston packings—Piston rods—Cross heads and slide bars—Connecting rods—Cranks and eccentrics—Eccentric rods—The strains in crank shafts—Journals—Shaft bearings and pedestals—Axle boxes—Governors—Fly wheels.

IT is intended in this chapter to give an account of the separate parts which constitute the mechanism of the steam engine, excepting only the valves and valve-gear, which require separate treatment. The variety of form and arrangement of the parts of steam engines is so great, that it will only be possible to give a few representative examples. A more extended knowledge of the mechanism can only be gained by close observation of numerous engines, or working drawings.

The Cylinder.—The cylinder of a steam engine is the closed vessel in which the piston works backwards and forwards. It is so called because the interior is cylindrical in shape, though the form of the exterior is complicated by sundry additions. Examples of stationary engine cylinders are given in figs. 5 to 7, and of marine engine cylinders in figs. 194, 195, 197 to 200. Fig. 48 is a longitudinal section of a steam cylinder of a locomotive engine. It is made of cast iron, the interior being carefully bored so as to form a smooth and cylindrical surface for the passage of the piston. It consists of the following principal parts. The cylindrical body AA, which is cast in one piece;—the valve box BB, in the thickness of which are formed the two

steam passages SS and the exhaust passage E;—the two covers CC, which are flanged, and which are attached to the body of the cylinder by means of studs and nuts. The cover through which the piston rod works is provided with a stuffing box D and gland *e*, to prevent the steam escaping round the rod. This object is accomplished in the following manner. The space *aa* between the rod and the inner cylindrical surface of the stuffing-box is filled with plaited hemp saturated with tallow, or with one of the numerous patent packings now procurable. The gun-metal gland *e*, through which the piston rod passes, is forced up

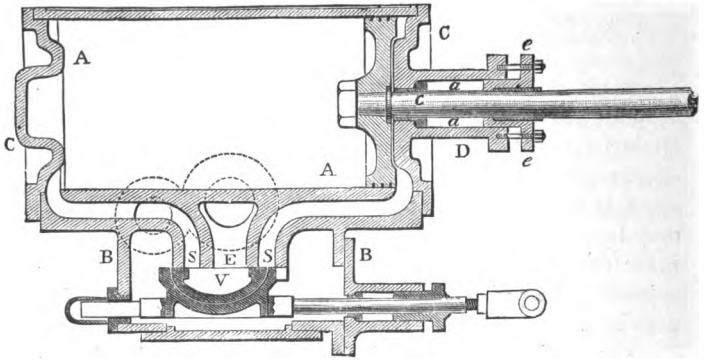

Fig. 48.

against the packing by means of the two nuts and screwed studs shown in the drawing. The result is that the packing can be squeezed with any desired degree of tightness round the piston rod, and can thus prevent the escape of steam. The opening by which the piston rod passes through the substance of the cover is lined with a gun-metal bush, *c*. In many engines, especially those of foreign manufacture, there is a stuffing box on the other cylinder cover, through which a prolongation of the piston rod works. The object of this arrangement is to prevent the piston bearing unequally on the lower side of the cylinder. It is also adopted in the

case of condensing engines, when the plunger of the air-pump is driven direct from the piston. It will be noticed that the interior faces of the covers are cast so as to fit into the corresponding faces of the piston. The reason of this arrangement is that the piston has in many cases to be formed in the shape shown in fig. 48, viz. with a broad rim or flange, and a thin disc; now if the cover faces were not shaped correspondingly, there would be at each end of the stroke a large space to be filled with steam before the piston began to move, which steam would do no work till expansion began.

Clearance.—The interior length of the cylinder bore, from cover to cover, is always a little longer than the stroke, plus the thickness of the piston, so that a small vacant space called the clearance is invariably left at the end of each stroke. If it were not for this precaution the covers might be knocked off whenever water accumulated in the cylinder. The clearance spaces and also the contents of the steam passages SS, between the valve face and the inner surface of the bore of the cylinder must at each stroke be filled with steam before the piston can be moved. This steam of course does no work till expansion begins, but a great portion of the loss due to this cause may be recovered by compressing the exhaust steam before the end of the stroke. This operation is called cushioning the piston, and is most essential for many reasons. It helps to bring the piston gradually to rest, and partially restores the temperature of the sides of the cylinder which become cooled during the exhaust. It further tends to produce uniformity of tangential effort on crank-pin in the case of quick running engines; see p. 193.

The joints of the covers are made steam tight by placing between the flanges a layer of red lead cement, or soft copper wire, or one of the numerous patent packings, and then tightening up the bolts.

Steam passages.—The design of the steam ports and passages is a matter of the greatest importance. It is desirable

to make the length of the passage as short as possible, so that its cubic contents may not add unduly to the contents of the clearance spaces, and on the other hand it is essential that the area of the passages should be ample, so that the fresh steam may not be throttled on its way into the cylinder, nor the exhaust steam on its way out, the result of which would be a considerable loss of power by reducing the pressure of the incoming steam and increasing the back pressure. It is not easy to reconcile these desiderata with the use of a single slide valve; for, if the passages were made as short as possible, the ports would necessarily be situated near the ends of the cylinder, and a valve that would cover both of them would be of unwieldy dimensions. Again, if the ports were made very wide so as to give a very free passage to the steam, the distance which the valve would have to move over—commonly called the travel of the valve—in order to fully uncover the ports would be so great that the work of moving the valve would absorb no inconsiderable portion of the power of the engine. Consequently in engines which are worked with a short slide valve,—and these constitute the great majority of engines in actual use,—a compromise has to be effected between conflicting evils The area of the steam ports is obviously connected with the piston speed of the engine; for, the greater the speed the greater the quantity of steam which has to be admitted in a given time; and consequently a port which would be found ample for a slow running engine might be totally inadequate if the speed were considerably increased. The following empirical rule has been found to give satisfactory practical results in the case of engines having long steam passages.

The area of steam port : area of piston :: speed of piston in feet per sec. : 100.

$$\text{or area of port} = \frac{\text{area of piston} \times \text{speed of piston}}{100.}$$

In some types of modern engines, which are designed

with the object of economising fuel, the steam ports are placed quite close to the cylinder ends, and separate passages are provided for the escape of the exhaust steam. In such case the simple slide valve is, of course, dispensed with, and each passage is worked by a separate valve (see page 264 and fig. 99). The object of providing separate exhaust passages is, that the fresh steam on entering the cylinder may not be cooled down by coming in contact with the sides of passages through which the cold exhaust steam has just escaped.

Valve boxes.—The valve box is generally flanged and provided with a cover, which is bolted to the face of the box, just as are the covers to the cylinder ends. The valve box is provided with a stuffing box, through which works the rod which actuates the valve, and also with an opening to which is attached the steam pipe from the boiler. The exhaust steam, after it has done its work, escapes through each steam passage alternately, and passes through the hollow portion of the valve V into the exhaust port E, whence it is led through an exhaust passage cast on the body of the cylinder, and is suffered to escape through a pipe into the open air, or else in the case of condensing engines, is conducted by a pipe to the condenser.

The face on which the slide valve works is, in large engines, often cast separately from the body of the cylinder, to which it is attached by bolts. In this manner it is possible to secure a sound face for the valves to work on, and to renew it when it becomes worn.

Jacketing.—In many engines of the better class, the cylinder is surrounded by an outer casing, sometimes cast in one piece with it, and sometimes attached to it, in such a manner as to leave an annular chamber between casing and cylinder. In this case the cylinder is said to be jacketed. The annular chamber is filled with fresh steam direct from the boiler. The object of this arrangement is to keep the body of the cylinder proper at, as nearly as possible, a uniform

temperature, and thus to avoid the injurious effects due to the cooling of the cylinder sides by the expanding and exhaust steam. Fig. 49 illustrates, in a longitudinal section, a jacketed cylinder. Where jacketing is carried out very thoroughly, the covers are jacketed as well as the body of the cylinder. For further examples of jacketed cylinders see pages 456 to 463.

In many modern marine engines, the inner surface or barrel of the cylinder is cast as a separate piece and fitted into the outer body. One end is attached by a flange to the outer body, while the other end is left free to expand and contract with the variations in temperature that take

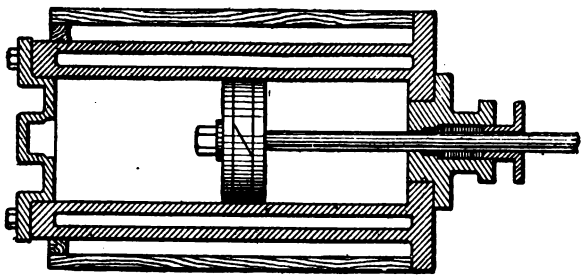

Fig. 49.

place. The free end must be packed steam tight. This inner portion is called the liner. There is usually a space between the liner and the body which forms the steam-jacket. Cylinders thus constructed possess many advantages. They are easier to make. When the inner surface wears down, it can be easily taken out, and a new one inserted. The cylinder is not unequally strained by variations of temperature in its different parts. Lastly, the liner can be made of cast steel, which is the strongest and most durable material for the purpose. For further on the subject of jackets, see page 450. The exterior surface of the cylinder, or of the jacket casing, should always be covered with a layer

of non-conducting material, such as wood, felt, or asbestos cloth, to prevent losses by radiation.

In order to discharge condensed water, or water which may have lodged in the cylinder on account of priming in the boiler, a small opening is made in the bottom of each end of the cylinder, and is provided with a cock called the 'pet-cock.' Sometimes, instead of the pet-cocks, small relief valves are provided, which open outwards, whenever the pressure within the cylinder exceeds the pressure which holds the valve down.

In order to lubricate the rubbing surfaces of the piston

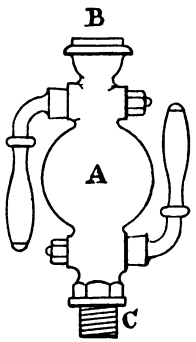

Fig. 50.

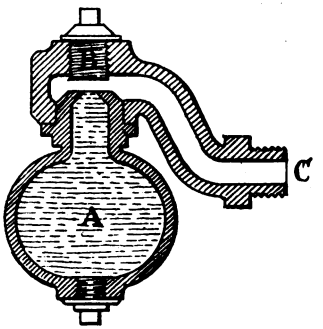

Fig. 51.

and slide valve, small openings have to be provided, into which are screwed lubricators containing oil or tallow. Fig. 50 represents a form of lubricator in common use. The bulb A contains the oil or tallow which is introduced through the funnel B. The lubricator is screwed into the cylinder or valve box by means of the screw C. The cocks, shown in the sketch, make or close communication between the bulb and cylinder, or bulb and funnel.

Fig. 51 represents Mr. Ramsbottom's continuous feed lubricators. The bulb A is filled with oil introduced by the plug B. The pipe C is screwed into the cylinder. The

steam, entering this pipe, condenses by degrees on the surface of the oil, and the water thus formed sinks to the bottom of the bulb, displacing a small quantity of oil. This process continues till the bulb becomes entirely filled with water, which can then be drawn off through the lower plug.

Great care should be devoted to the choice of a good material for lubrication. Tallow, even of the best quality, and animal oils generally, are unsuited for use with high-pressure steam, as they are decomposed by the high temperature into stearic and other acids, which readily attack the iron of the cylinders. If the waste steam from the cylinders is mixed with the feed water for the boilers, the effects of the decomposed tallow on the boiler are fatal and rapid. Sometimes the iron plates are eaten away, and sometimes a soapy deposit, of a very non-conducting nature, forms on the crowns of the furnaces, which permits the crown plates to become overheated, and to collapse. Whenever high-pressure steam is employed, some preparation of mineral oil should be used as the lubricating material.

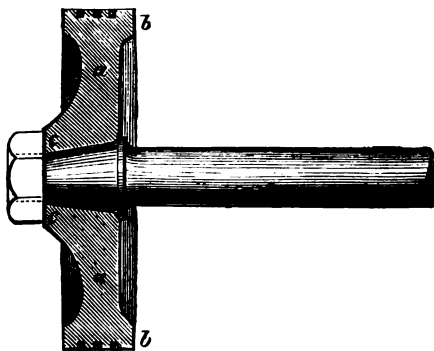

Fig. 52.

Pistons.—The piston is the metallic disc or moveable diaphragm which accurately fits the bore of the cylinder, and which receives and transmits the pressure of the steam to the other moving parts of the engine. Fig. 52 illustrates

in section a piston which is used for locomotives on the London & North-Western Railway. It consists of three principal parts, viz. the central disc *a*, *a*, which forms the body of the piston, the circumferential portion *b*, *b*, which contains the packing that enables the piston to move steam-tight along the cylinder, and which is so shaped as to form a large surface of contact with the sides of the latter, and the central boss *c*, *c*, which receives the coned end of the piston rod.

The forms of pistons are innumerable, and depend altogether upon the purpose for which the engine is intended, and the size of the cylinder, which in different classes of engines varies from a few inches to over 9 feet in diameter. The chief points to attend to in the design of a piston are the following:—it should be strong enough to withstand the pressure of the steam, and to hold the end of the piston rod immovably;—the packing round the circumference should be steam-tight, without causing undue friction, and not liable to get out of order;—the width of the circumferential portion should be such that the pressure per square inch—due, in the case of horizontal engines, to the weight of the piston—be not sufficient to cause undue wear of the inner surface of the cylinder. In many engines the weight of the piston is taken off the surface of the cylinder by prolonging the piston rod backwards through the back cover of the cylinder; in such cases the weight is partly transferred to the stuffing-boxes and slide-bars, which are easily got at for purposes of examination and adjustment.

The importance of keeping the surface of the cylinder true, and of keeping the piston in steam-tight contact with it, will be readily recognised when it is borne in mind that a leakage of steam past the piston means that during the whole time the engine is at work there is an open passage from the boiler to the condenser or outer air, through which steam is continuously escaping without doing any work.

Methods of packing pistons.—In the early days of steam engines, when the pressure used was low, a good packing was

obtained by winding plaits of hemp, soaked in tallow, round the flange of the piston. These plaits were secured in place and packed tightly together by a ring, called the junk ring, which was bolted down to one face of the piston. Such a system would be quite inapplicable to modern high-pressure engines. One of the simplest and best modes of packing pistons of moderate dimensions was devised by Mr. Ramsbottom ; it is illustrated in fig. 53. The flange of the piston *b* (fig. 52) is broad enough to allow of three rectangular channels being turned in it. Into each of these channels or grooves is placed a steel ring of rectangular section, as shown in the figure. These rings are not bent into an exact circle of the same diameter as the piston, but are so formed that they have a tendency to spring outwards, and consequently, when the piston is confined in the cylinder, they press against the working surface of the latter, and thus effect a steam-tight joint. Fig. 53 shows one of the rings, before it has been sprung into its place. The inner complete circle represents the diameter of the cylinder. The three rings are so arranged that the joints are not in a straight line. It has been found that if the spring on the rings is sufficient to cause them to press against the cylinder with a pressure of 3 lbs. to the square inch, the joint will be steam-tight.

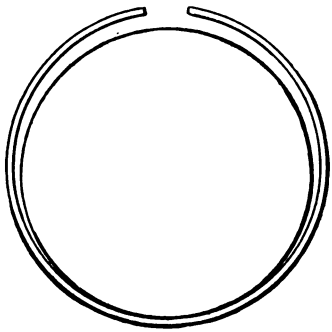

Fig. 53.

It is not necessary to make the rings of steel. In many cases they are made of cast iron, the ring when cast being slotted across. The outer face of the ring is turned, and the inner face left as it comes from the mould. This insures a good amount of spring.

There are many varieties of metallic packing rings in use. In some cases they are made without any initial outward spring of their own, but are pressed outwards by the action of the steam, which is admitted behind the ring by small holes in the body of the piston. In other cases the metallic packing, instead of being a simple ring, is made in the form of a spiral which makes rather more than two complete turns round the circumference of the piston, the groove in the latter being of course formed to correspond. In this manner the butt joint of the simple ring is avoided, and the leakage of steam through this joint is prevented.

In the case of marine engines, the packing ring is usually pressed against the barrel of the cylinder by means of a

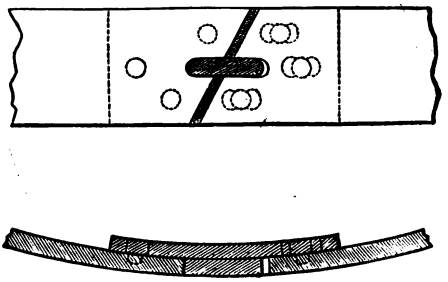

Fig. 54.

series of independent adjustable springs contained within the body of the piston. Figs. 54 and 55 [1] show the details of such an arrangement. The spring ring, which is of considerable depth, is held up against the sides of the cylinder by a series of steel springs, *aaa*, fig. 55. The joint in the ring is formed as in fig. 54 to prevent leakage. An oblique slot is taken out of the ring. A plate, fitted with a tongue piece, is fastened behind the slot, as shown in the section, fig. 54, and the tongue piece, which slides in a groove, allows the ring to expand and contract, and at the same

[1] Taken by permission from Mr. R. Sennett's work, *The Marine Steam Engine.*

time makes a steam-tight joint. The ring, with its springs, is covered by a flat circular piece of iron called the junk ring, which is shown in plan on one half of fig. 55. This enables the springs to be got at easily for examination and repair. The junk ring is attached to the body of the piston by bolts which work into brass nuts embedded in the metal of the piston. When the threads work loose, the nuts can be easily replaced. In the case of horizontal marine engines the part of the spring ring which is in contact with the lower

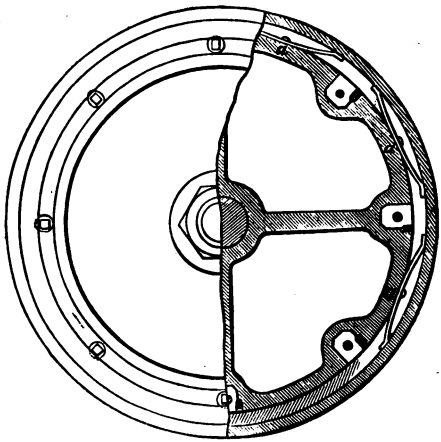

Fig. 55.

side of the cylinder is not backed by springs, but by solid blocks. These are necessary in order to support the weight of the piston, as otherwise the springs at the bottom would be abnormally compressed and those at the upper part of the circumference correspondingly slack. Within the last two or three years it has been possible, owing to the improvements introduced into the methods of casting steel, to manufacture pistons of that metal instead of cast iron, which is the material which has hitherto been almost invariably used. In marine engine pistons, it has been found possible

by the use of cast steel to effect a saving in weight of between thirty and forty per cent. as compared with cast iron.

Piston rods.—The piston rod is the member which transmits the motion imparted to the piston to the mechanism outside the cylinder. It consists of a truly cylindrical bar of wrought iron or steel, one end of which is fastened securely into the piston. The rod passes through the cylinder cover by means of a steam-tight stuffing-box, as shown in fig. 48, and the outer end terminates in the cross-head which will be described presently. There are various methods in vogue of fastening the rod into the body of the piston. Sometimes the end of the rod is turned cylindrical and a hole bored in the piston slightly less in diameter than the rod. The piston is then heated, which causes it to expand, when the rod can be inserted. After cooling, the piston contracts, and holds the rod firmly in its place. In the majority of cases the end of the rod is turned conical as in fig. 52, with a screw thread on the extreme end, by means of which, together with a nut, the rod is firmly embedded in a conical recess bored in the piston. Sometimes instead of a nut the rod is fastened by means of a cotter, and occasionally it is screwed and pinned into the boss of the piston.

The strength of piston rods has to be fixed with special reference to the fact that they are subject to alternating strains. When the piston is making the stroke towards the crank shaft the rod is in compression, and when making the return stroke the rod is in tension. The maximum stress per square inch of cross section at any part of the stroke is equal to the total pressure of the steam on the piston divided by the area of the rod. It is usual in designing pieces of machinery which have to bear alternating strains of tension and compression to make them much stronger than would be necessary were the strain always of one sort.

Cross-heads and slide-bars.—The outer end of the piston rod is attached to the cross-head, or motion block, which serves the double purpose of forming the means of connection

between the piston rod and the connecting rod, and of guiding the piston rod so as to keep it straight and in the line of the axis of the cylinder, in spite of the bending moment due to the angular position of the connecting rod.

The cross-head generally consists of three principal parts, viz. (1) the body which often contains a conical hole into which the coned end of the piston rod is fastened by means of a cotter ; (2) the part by which the joint with the connecting rod is effected ; and (3) the guides or motion blocks which travel between fixed bars parallel to the axis of the cylinder, called slide-bars, and which prevent the end of the piston rod from being deflected as the connecting rod assumes an angular position. Fig. 56 shows a piston rod, cross-head, slide-bars, and connecting rod in position.

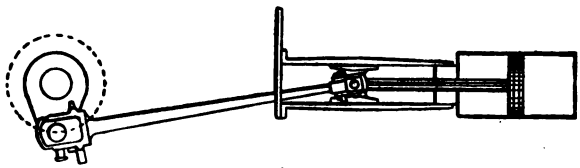

Fig. 56.

Pressure upon slide-bars.—Before giving examples of cross-heads, it is necessary to explain the nature of the strains which they transmit to the slide-bars. As the crank-pin revolves, the connecting rod assumes an angular position, the amount of the angularity increasing till the crank is at right angles to the axis of the cylinder and then again diminishing to nothing.

In the accompanying diagram (fig. 57) suppose the piston to be making a stroke towards the crank axle, the crank-pin revolving in the direction of the arrow. In this case the piston and connecting rods are both in a state of compression, and consequently there is a downward resulting force acting at the cross-head, which is opposed by the resistance of the slide-bar. Next take the return stroke, the connecting rod being in the position shown by the dotted line. In this case

both piston and connecting rods are in a state of tension, and consequently we have again a downward resultant force acting at the cross-head. Therefore so long as the engine runs in the direction shown by the circular arrow the pressure is always downwards. Similarly if the engine were reversed so as to run in the opposite direction, there would always be an upward resultant pressure acting at the cross-head and we should require a top slide-bar in order to prevent this pressure from deflecting the end of the piston rod. If the steam ceased to act on the piston during a portion of the stroke, and if at the same time the valves were so set that there should be considerable compression or back pressure on the other face of the piston, the piston rod might cease to be in a state of

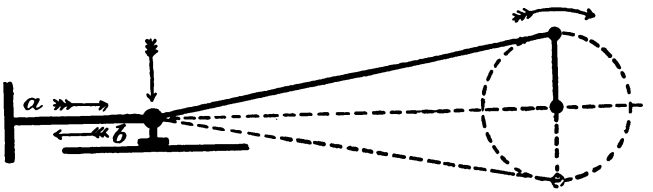

Fig. 57.

compression (or tension as the case might be) at a certain point in the stroke, and the above reasoning would not apply. In such a case we might have a downward pressure on the slide-bars during one portion of the stroke and an upward pressure during the remainder (or *vice versâ*), and thus two slide-bars are often necessary even in factory engines which are usually not reversed. Of course in the case of locomotives, marine and winding engines, which are constantly reversing, two slide-bars or some corresponding arrangement are an absolute necessity ; but in factory engines if the valves are so set that there is no fear of a reversal of the direction of stress on the piston, one slide-bar is often sufficient. Of course in such cases the engine is arranged to run so that the pressure may act downwards, as it is much easier to keep

the working face of the bottom bar lubricated than the corresponding top face.

The amount of the pressure on the slide-bar depends upon the angle (α) made by the axis of the connecting rod with the axis of the piston rod. The greater the angle (other things being equal) the greater the pressure. The size of the angle depends on two things: first, the position of the crank, the angle constantly increasing till the crank is at right angles to the axis of the piston rod, and then again diminishing; and second, the relative lengths of the crank arm and the connecting rod; the shorter the connecting rod, as compared with the crank arm, the greater will be the value of α for a given position of the crank. In the great majority of engines the connecting rod lies between 6 times and 3·5 times the length of the crank. The amount of the pressure on the slide-bar equals the vertical component of the pressure or tension in the connecting rod. When the crank is at right angles to the axis of the piston rod this vertical component equals the horizontal pressure or tension in the piston rod multiplied by the tangent of the angle between the piston and connecting rods. With the proportions of length of connecting rod to length of crank arm usually adopted, this angle is so small that there is no material difference between its sine and its tangent. Hence the usual rule is: Maximum pressure on slide-bar = pressure on piston × sine of angle between piston and connecting rods = pressure on piston × crank radius ÷ length of connecting rod.

We are only concerned to know the maximum pressure because the slide bar must be made wide enough to bear this pressure with safety. The safe amount of working pressure per square inch of slide-bar surface varies with the material of which the bar is constructed. In locomotives with steel or case-hardened wrought-iron bars the pressure is sometimes as high as 120 lbs. per square inch. This is, however, too high a pressure, and occasions very rapid wear

of both bars and motion blocks. In the best modern practice the width of bars and areas of motion blocks are so chosen that the pressure shall not exceed 40 lbs. per square inch when the bearing surfaces are of cast iron, and half as much again when they are of steel. The actual pressure acting on the piston rod when the crank arm is at right angles to the axis of the cylinder must be calculated from diagrams constructed as explained in Chapter V. page 194 ; it can only be taken from the indicator diagrams when the piston speed is very moderate.

The cross-head should be so designed that the point at which the vertical pressure is transmitted by the connecting rod shall be exactly over the centre of the guide or motion block, otherwise an undue proportion of pressure will be transmitted to the edge of the block nearest to the point

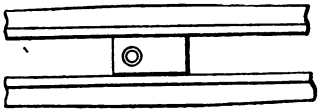

Fig. 58.

where the pressure is applied. Fig. 58 is an illustration of a cross-head in which this consideration has not been attended to.

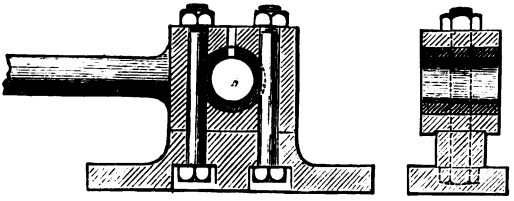

Fig. 59.

Fig. 59 shows a longitudinal and transverse section of a type of cross-head commonly in use in factory engines which require only one slide-bar.

Fig. 60 is an example of a cross-head provided with but one slide-bar. The motion block is so arranged as to envelope the bar, and the pressure comes on the upper or lower surface of this latter, according to the direction in which the engine is running.

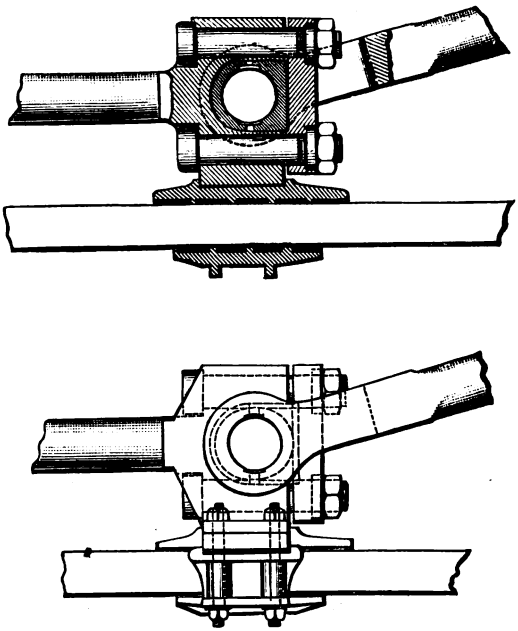

Fig. 60.

Fig. 61 is an illustration of the cross-head of the largest type of marine engine showing also the ends of the piston and connecting rods, and sections of the large slides and massive slide bars, the latter being attached to the main frames of the engines. Specimens of cross-head and slide bars are also given in pages 458 to 461.

In the older types of engines slide-bars were not used. The end of the piston rod was kept straight by means of a

parallel motion, which is an arrangement of link-work so contrived that a pressure is always brought to bear on the end of the rod equal in amount, but opposite in direction to the pressure due to the angular position of the connecting rod. Parallel motions are now seldom used for this purpose, and need not therefore be described in this volume. A

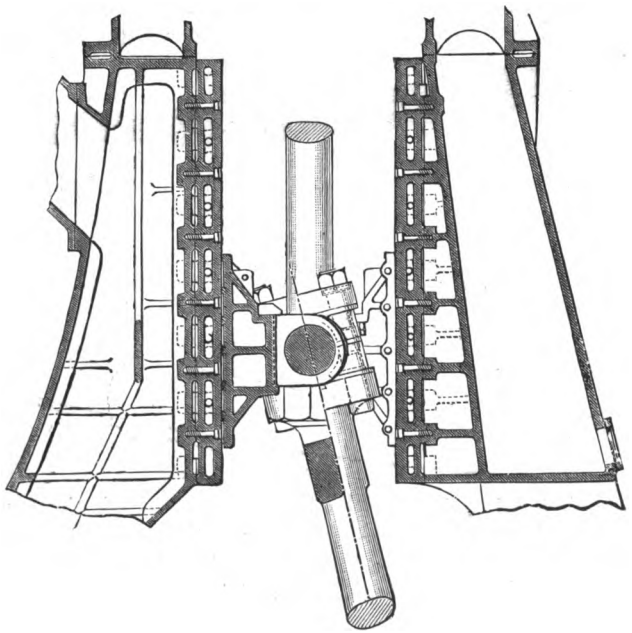

Fig. 61.

simple form of parallel motion, as used in the construction of indicators, is illustrated on page 319.

Connecting rods and cranks.—The connecting rod is the link which enables the reciprocating rectilinear motion of the piston to be converted into the circular motion of the crank pin. It is a link or rod of metal so formed at the two ends that it can be jointed to both the cross-head and

the crank-pin. Suppose the piston to be at the commencement or end of the stroke as shown in the upper diagram fig. 62. Corresponding to these positions, the crank lies in the direction of the axis of the piston rod prolonged. If pressure be applied to the piston when at rest in either of these positions, that pressure will be transmitted direct to the bearings of the crank axle, there will be no turning force applied to the crank, and consequently the latter will have no tendency to revolve. Hence these two positions of the crank are called its dead centres. Let, however, the crank occupy the position *a'c*, fig. 62. The connecting rod will now be inclined to the directions of both piston rod and crank. If when in this position pressure be applied to the

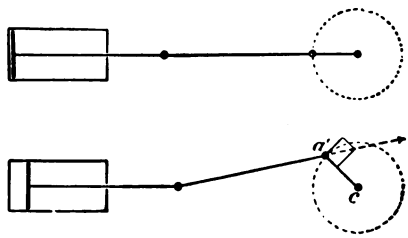

Fig. 62.

piston, the pressure will be transmitted through the connecting rod to the crank-pin, and can there be resolved in two directions, viz. along the crank and at right angles to it, i.e. tangential to the crank circle. The former component merely exerts pressure on the bearings of the crank axle, but the latter tends to cause the crank to revolve. The two components are constantly varying in size and in relation to each other, as is fully explained in Chapter V. When the crank is on either dead centre the tangential component vanishes, while the radial one is a maximum; and *vice versâ*, when the crank occupies such a position that the axis of the connecting rod forms a tangent with the crank circle, the radial component vanishes, while the tangential is a maximum.

This position, which occurs twice in each revolution, depends upon the length of the connecting rod relatively to that of the crank. If the connecting rod were of infinite length it would form a tangent with the crank circle when the crank was at right angles to the direction of the axis of the piston rod. The shorter the connecting rod relatively to the crank, the nearer to the dead centre will be the position of the crank when the tangent is formed. This is

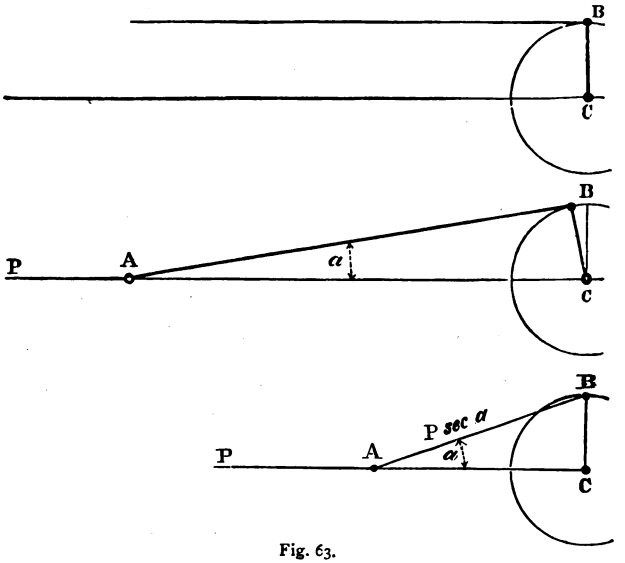

Fig. 63.

illustrated by the three diagrams, fig. 63, which represent the tangential positions for connecting rods having respectively lengths equal to infinity, and to six times and three times the length of the crank.

The force transmitted by the connecting rod is estimated as follows. Let P be the force acting on the piston rod, and *a* the angle which this latter forms with the connecting rod, then P sec *a* is the force in the connecting rod. If P were

constant the value of this expression would increase with a. The maximum value attained by a is when the crank is at right angles with direction of the line PC. Let the connecting rod be r times the length of the crank arm. In this position ABC is a right-angled triangle and sec a $= AB \div AC = \frac{r}{\sqrt{r^2-1}}$. Suppose that $r = 6$, then the force in the connecting rod $= P \frac{6}{\sqrt{35}} = P \times 1{\cdot}0142$. If $r = 3$, then the force $= P \frac{3}{\sqrt{8}} = P \times 1{\cdot}06$.

It will thus be seen that the connecting rod transmits a constantly varying force to the crank-pin, and the component of the force which causes the crank to rotate is also constantly varying from zero, which value it attains at the dead centres, to P sec $a = P \frac{\sqrt{r^2+1}}{r}$, which it attains at the two positions when the connecting rod is tangential to the crank circle. The proper method of exhibiting graphically the tangential effort transmitted by the connecting rod to the crank-pin is explained in detail in Chapter V., which also shows how to take account of the varying steam pressure on the piston, the effect of the inertia of the reciprocating parts, and the length of the connecting rod relatively to the crank arm.

It will be noticed that while the piston is moving once backwards and forwards in the cylinder, the crank-pin is describing a circle, having a diameter equal to the length of stroke. Consequently the relative lengths of the paths travelled by piston and crank-pin are as the diameter to the semicircumference of a circle, or as $2r : \pi r = 2 : 3{\cdot}14159$. Now, by the principle of work, the pressure on the piston multiplied by the length of the path it travels equals the tangential pressure on the crank pin multiplied by the length of its path; hence the average pressure on the piston

is to the average tangential pressure on the crank pin as 3·14159 : 2. The fact that the average tangential pressure on the crank-pin was less than that on the piston led old writers on the steam engine into the fallacy of asserting that the oblique action of the connecting rod caused a loss of power in the steam engine. They forgot, however, that though the full pressure on the piston does not in general appear tangentially to the crank, the path travelled by the latter is correspondingly greater than that of the piston.

In addition to the stress due to the pressure of steam on the piston, which is alternately one of compression and tension, the connecting rod is also subject to stresses, due to the inertia of the piston and piston rods and also due to its own inertia. These stresses, which are dealt with in Chapter V., may become very considerable in high-speed engines. The rapid vibration of the rod from side to side, somewhat after the manner of a pendulum, also tends to produce a peculiar motion of the rod called whipping. This arises from a bending stress due to the resistance of the rod to acceleration in a direction at right angles to the motion of the piston. The tendency to whip is greatest in rods of great relative length. It may be guarded against by sparing unnecessary weight in the rod, by making it strongest where the tendency to whip is greatest, and by so distributing the material in the cross-section of the rod that its resistance to bending stresses may be a maximum, For instance the rod is usually tapered from the piston rod end to the crank-pin end ; and instead of being made round, it is usually given a rectangular section, the side of the rectangle which lies in the plane of the motion being the greatest. In this manner a given weight of material is distributed so as best to resist a bending stress. Sometimes the rod is given a section resembling that of a flanged girder, the greater part of the material being distributed in the two flanges.[1]

[1] The rules for estimating the stresses on and fixing the proportions of the various moving parts of engines are given with great clearness in Professor Unwin's Elements of Machine Design, published in this series.

Fig. 64 represents a connecting rod, and its various details as used in a stationary engine. The parts surrounding

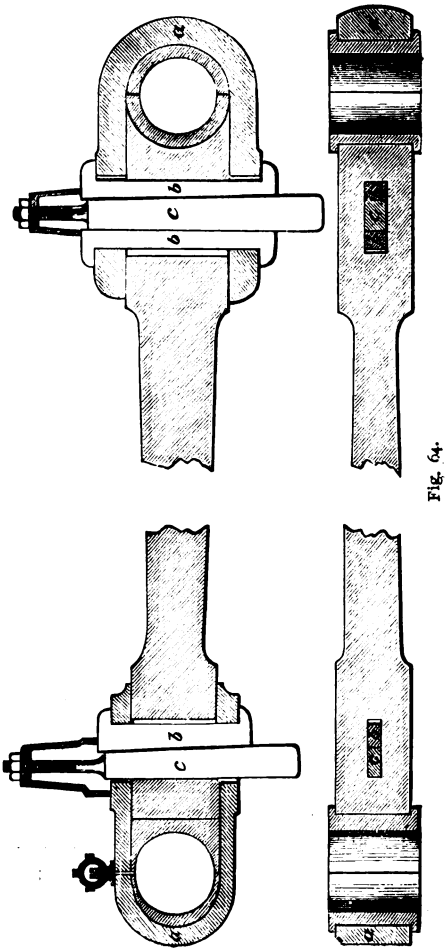

Fig. 64.

the crank and cross-head pins are made of gun metal, brass, or white metal so as to diminish friction. They are made in separate pieces, called steps, and are held in place by the straps *aa*, which are fastened to the rod by means of the gibs *bb*, and the cotters *cc*. When the brasses wear they can be tightened up by driving in or screwing up the cotter, which draws up the strap, and thus tends to shorten the rod.

Fig. 65 shows a solid ended rod. In this case the brass steps are placed in the rectangular opening formed in the end of the rod, their flanges being partly cut away so as to allow of their being inserted. They are held in position by a cotter, which in its turn is prevented from moving by one or more set screws. When the brasses wear they can be tightened up by driving in the cotter, which has the effect of lengthening the rod.

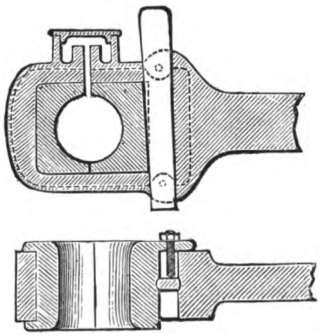

Fig. 65.

Fig. 66 shows a fork-ended connecting rod which is necessary with certain sorts of cross-heads. Fig. 67 is the ordinary type of marine connecting rod. In this rod the brass steps are not held by gibs and cotters, but by bolts, which are clearly shown in the sketch.

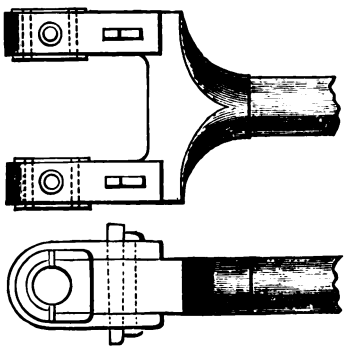

Fig. 66.

The lubrication of the surfaces of the brass steps where they rub on the crank and cross-head pins must be very carefully attended to.

The ends are for this purpose provided with lubricators, which are sometimes of brass, and can be removed, but are more commonly forged solid on to the straps or ends. If

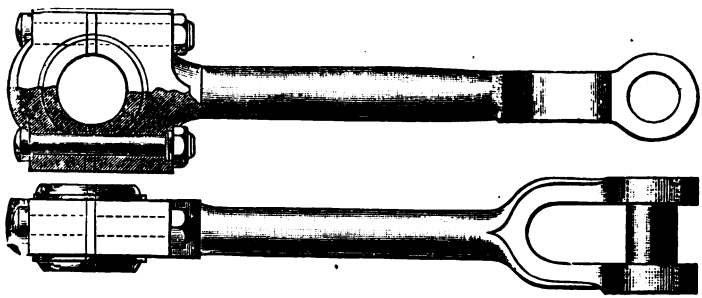

Fig. 67.

the lubrication is well attended to, and if the diameters and lengths of the pins are made sufficiently large so as to keep the pressure per square inch within moderate limits, the wear of the brass steps should give very little trouble.

In old-fashioned engines the connecting rod was often made of cast iron, but nowadays it is invariably of case-hardened wrought iron, or of mild steel.

Cranks and eccentrics.—The crank is simply a lever of the first order, either attached to, or forged in one piece with the main shaft of the engine. By means of it, the reciprocating motion of the piston is finally converted into circular motion, as has been explained in the previous section. The manner in which the force, transmitted by the connecting rod to the crank, is resolved into a radial component which has no effect but to exert pressure on the main bearing, and a tangential component which alone tends to revolve the shaft, is also explained in that section, and more fully in Chapter V.

Fig. 68 shows two views of one of the simplest forms of crank ; *a* is the crank shaft, *c* the crank-pin. The distance from the centre of *a* to the centre of *c* is the length of the

crank arm, which is, of course, equal to half the stroke of the piston. *b* is the web of the crank, *d*, *d'* the bosses. Cranks of this form are generally of cast iron, and are attached to the

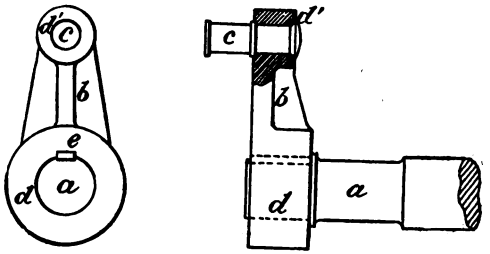

Fig. 68.

main shaft by means of a key *e*. The fastening of a moveable crank on a shaft requires the greatest care, because all the stresses thrown on the crank are liable to reversion during each stroke, especially in the case of a slow-running engine working with a considerable cushion of steam. Very severe reversals of pressure also occur if water is allowed to accumulate in the cylinder. In such cases the piston is brought up dead before the end of the stroke is reached, while the crank endeavours to pull it on, thus throwing a heavy strain on all the connections, and amongst others on the key. Cranks of this type in addition to being keyed are generally shrunk on to the shaft, or else are forced on by hydraulic pressure.

Fig. 69 shows an end elevation and cross-section of another form of cast-iron crank, called a disc crank. It is, as its name implies, formed of a disc of cast iron, attached to the shaft by the methods just described, and provided with a wrought-iron or steel crank-pin. The portion of the disc opposite to the pin is usually much thicker and heavier than the remainder of the disc, this extra weight being used as a balance to the weights of the reciprocating parts.

Very often it is impossible to arrange that the crank shall overhang the end of the shaft, and in such cases it has

to be forged in one piece with the latter. A great number of engines also work with two or more cylinders, and a corresponding number of cranks. In the case of two-cylindered engines, the cranks are, as a rule, set at right angles to each other, so that when one of them is on its

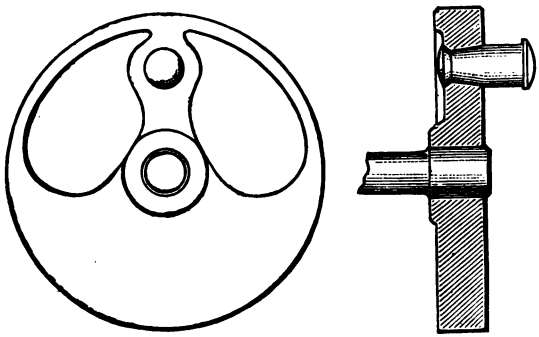

Fig. 69.

dead centre, the other is in the most favourable position, and thus, the resultant force tending to turn the shaft is made more uniform. Fig. 70 shows two views of the crank

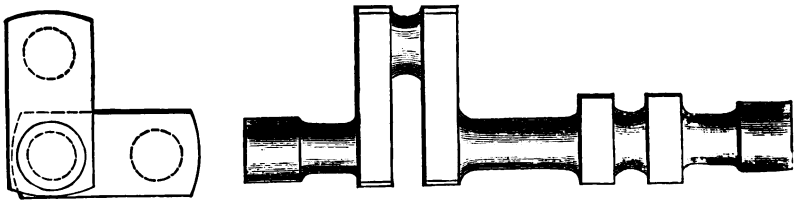

Fig. 70.

shaft of a locomotive engine, with two cranks forged in one piece with the shaft, the whole being made of steel, which metal, on account of its great strength, is especially useful in cases where, from want of space, the thickness of the webs or side pieces of the crank is limited. It will be noted

that in the case illustrated in fig. 70 the want of thickness in the webs is made up for by increasing their width, which is nearly double the diameter of the shaft.

The crank-pin is the portion of the engine which receives the greatest stress, and special care must therefore be given to its design and lubrication. The pressure which comes upon it varies in practice, according to the type and speed of the engine, from 500 lbs. to 2,000 lbs. per square inch. This latter figure is far too high for safety. The diameter and length of the pin should be so chosen that the pressure may not exceed 1,000 lbs. per square inch. It is desirable to make the pin as long as is consistent with strength and other considerations, because bearings retain their lubrication better when long than short. In high speed-engines, there is a great tendency to expel the lubricating oil from the crank pin by centrifugal force, and this tendency must be carefully guarded against, otherwise excessive wear of the connecting rod brasses will be the result.

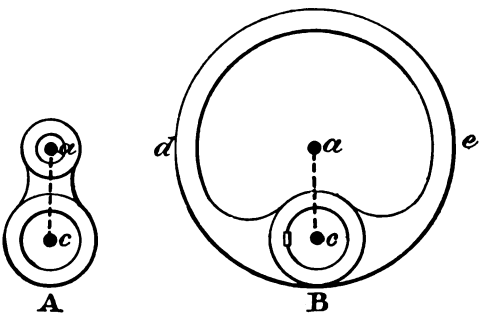

Fig. 71.

In the crank proper, the pin *a*, fig. 71 (A), is of such a diameter that it does not come in contact with the shaft. There is, however, a species of crank called the *eccentric*, in which the pin is so large that it completely envelopes the shaft. Such a crank is shown at fig. 71 (B). The distance

ac is the same as at A, but the pin has assumed the diameter of the outer circle *de*. In such a case, of course, the web is impossible and unnecessary. A crank is used for converting the rectilinear motion of the piston into circular motion; the eccentric, on the other hand, is usually employed for converting the circular motion of the main shaft back into rectilinear motion. The ordinary crank is, of course, equally capable of effecting this conversion, but it would in many cases be impossible to apply it to such a purpose. A crank always weakens a shaft except when overhung; hence, the fewer the number of cranks made use of, the better. Moreover, when circular motion is converted into rectilinear, as a rule, the rectilinear path travelled is short, relatively to the diameter of the crank shaft; hence a crank would be an impossibility, because the crank-pin would fall either wholly or partly within the section of the shaft. With an eccentric, however, the case is different, the distance *ac*, fig. 71 (B), corresponding to the length of the crank arm, may be as small as we please. The most frequent uses to which eccentrics are put are to drive slide valves and pumps, the travels of which are very much less than that of the piston.

The distance *ac*, from the centre of the shaft to the centre of the eccentric, is called the eccentric radius, the eccentricity, or the half-throw of the eccentric, and is equal in length to the half-travel of the part to be driven, such as the pump plunger, or slide-valve.

Fig. 72 represents side elevation and a longitudinal section of an eccentric and rod as used for driving an ordinary slide-valve. The circular portion *a*, which corresponds to the crank pin, is called the sheave of the eccentric. It is usually made of cast iron in two halves, which are bolted together round the shaft, and keyed on in the proper position. The piece *bb* is called the strap, and corresponds with the big end of a connecting rod. The strap is made of cast iron, wrought iron, or steel, according to circumstances,

and is lined with a brass or white metal ring, where it comes in contact with the sheave. This ring is grooved, as shown in the section at *cc*, so as to prevent it from getting off the

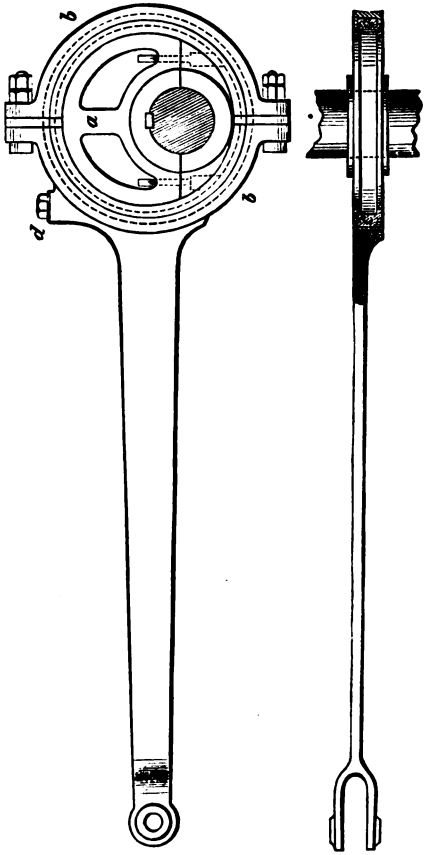

Fig. 72.

sheave. The strap is made in two halves bolted together so that it can be readily put on, or taken off the sheaf. It should be made sufficiently rigid not to spring when the

engine is running, as the effect of this would be to cause a great deal of local friction between the sheave and strap. An oil cup *d* is usually forged solid on one half of the strap, and particular care must be given to the lubrication, the friction of eccentrics being much greater than that of cranks. The eccentric rod, which corresponds with the connecting rod of a crank, is sometimes forged solid with one half of the strap ; it is, however, often made separate, and attached by bolts to the strap. The length of the eccentric rod, relatively to the half throw of the eccentric, is always much greater than that of the connecting rod, relatively to the crank arm. Hence, the disturbing influence on the point driven, due to the finite length of the rod, is usually very slight.

The manner in which the eccentric, or combinations of eccentrics, are used to actuate the valves, will be explained in the next chapter.

Crank Shafts.—The shaft of the engine is the part which receives circular motion from the crank and the reciprocating pieces. By means of the shaft, the power generated in the cylinder is transmitted to the machinery intended to be driven. Thus, in the case of factory engines, a pulley is usually keyed on to the shaft, and by means of a leather belt passing over this pulley, the various lines of shafting throughout the building are driven. In locomotive engines, the driving wheels are keyed direct on to the shaft, and rotate with it, and in the case of marine engines, the paddles or screw are also attached direct on to the shaft or its prolongation.

Strains in Crank Shafts.— Shafts are subjected to a variety of strains. In the first place, they undergo bending stresses from any weights which may be attached to them, the most considerable of which is that of the fly-wheel, acting vertically downwards. Also the pull of the driving belt causes a bending stress, which acts in the line joining the driving and the driven shaft. The most important stresses,

however, are due to the direct thrust and pull of the connecting rod, or rods, which, when at their maximum, act in the line of the axes of the cylinders. These various bending stresses may act in such directions as either to partly neutralise or to reinforce each other. In any case a resultant stress can always be found which represents their combined action. A shaft under the action of these stresses behaves in exactly the same manner as a loaded girder.

The foregoing strains act in the planes of the axis of the shaft. Shafts are, however, subjected to a totally different class of strains, which act in planes at right angles to the axle, viz. those which result from the twisting or torsional effects of the power and resistance acting at the ends of levers represented by the length of the crank arm, and the radius of the pulley, over which the main belt passes in the case of factory engines ; or the radius of the driving wheels, or of the centres of effort of propellers, in the cases of locomotive and marine engines respectively. The tendency of these torsional strains is to shear the metal, composing the shaft, in a plane at right angles to the axis. They are consequently opposed, by the resistance of the metal to

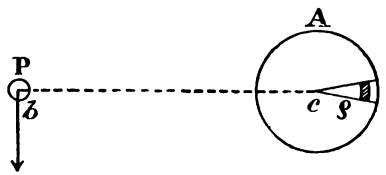

Fig. 73.

shearing. In fig. 73 let A represent a section of a shaft. Let $bc=l$ be the length of the crank arm, and P be the maximum force in lbs. transmitted to the crank pin by the connecting rod, estimated in the direction of the arrow. Then, $P \times l$ is the twisting moment applied to the shaft. If l is expressed in feet, then the moment is in foot-pounds, and if in inches, then the moment is in inch-pounds. The forces

at work are prevented from shearing the shaft by the resistance to shearing of all the metal in the section A. If the shaft were just upon the point of shearing, the moment got by multiplying the resistance of the metal by the distance of its centre of resistance from c, would be exactly equal to the twisting moment Pl. To calculate the value of this moment of resistance, we must proceed in the following manner: Conceive the section of the shaft to be divided up into a number of triangles (one of which is shown), and the triangle to be divided up by lines parallel to its base into a series of bands (one of which is also shown shaded); then, the power of any one of these bands to oppose the shearing effect of the twisting moment Pl, relatively to the corresponding power of the exterior band, depends upon the area of the given band, and its distance from the centre, relatively to the exterior radius of the shaft. Call the variable distance of the bands from the centre ρ, and the angle at the apex c of the triangle, $d\vartheta$, then the breadth of the band, measured along the radius, is $d\rho$, and its length $\rho.d\vartheta$; consequently, its area is $\rho.d\rho.d\vartheta$, and if the band were carried right round the circle, its area would be $2\pi.\rho.d\rho$. The distance of this annular band from the centre, relatively to that of the exterior circumference of the shaft $=\frac{\rho}{r}$; hence, the power of the strip to resist shearing, relatively to that of the outside, $=2\pi.\rho.d\rho.\frac{\rho}{r}$ $=\frac{2\pi\rho^2}{r}d\rho$, and its moment about the centre $c=\frac{2\pi\rho^3}{r}d\rho$. Similarly with every other circular band into which the section of the shaft can be divided. Hence, the moment of resistance of the whole shaft to resist shearing about the centre c, is proportional to the above expression integrated between the limits $\rho=0$ and $\rho=r$, or

$$\frac{2\pi}{r}\int_0^r \rho^3 d\rho = \frac{2\pi}{r}\frac{r^4}{4} = \frac{\pi r^3}{2}.$$

If, instead of the radius r, we take the half diameter $\frac{d}{2}$ the above expression becomes $\frac{\pi d^3}{16} = 0{\cdot}196d^3$.

In order to make this expression practically useful we require to know first, the ultimate resistance to shearing of the metal of which the shaft is made; and second, the factor of safety to be employed, that is to say, the number of times which the moment of resistance of the shaft should exceed the twisting moment, in order that it may be safe in practice. The following are usually taken as the ultimate shearing strengths of the metals usually employed in making shafts:

Cast iron	28,000	lbs. per square inch.
Wrought iron	54,000	
Steel	80,000	

As a rule the factor of safety employed for shafts is 6, therefore in designing a shaft for a given purpose we must only take one sixth of the above figures. Thus to find the proper diameter for a wrought-iron shaft subject to a given twisting moment Pl in inch lbs., we have

$$0{\cdot}196d^3 \times \frac{54000}{6} = Pl,$$

$$\therefore d \text{ (in inches)} = 0{\cdot}08275 \times \sqrt[3]{Pl}.$$

Sometimes the horse-power (HP) transmitted by the shaft and the number of revolutions (N) are given. One horse-power = 33,000 foot-pounds per minute. The total horse-power equals the average pressure (P) applied to the end of the crank arm (l) × by the path of the crank pin in feet $\left(\frac{2\pi l}{12}\right)$ × by the number of revolutions (N) per minute,

$$\therefore 33{,}000 \times \text{HP} = \frac{P \, 2\pi l \, N}{12},$$

$$\therefore Pl = 63{,}024 \times \frac{\text{HP}}{N}$$

and, equating this latter expression to $\cdot 196d^3 \times 9000$,

we obtain $$d \text{ (in inches)} = 3\cdot 295 \sqrt[3]{\frac{HP}{N}}.$$

Journals.—The part of the shaft which is supported by the bearing is called the journal. The usual form of the journal of an engine crank is shown in fig. 74. The part

Fig. 74.

which runs in the bearings is turned so as to be truly cylindrical. The end play of the shaft is limited by the two raised collars. The length of the journal, or the distance between the inner faces of the collars, relatively to the diameter depends principally upon the number of revolutions which the shaft has to make per minute. For slow-running engines the length is sometimes equal to the diameter, whereas in cases of high speed it may be as much as from two to three times the diameter of the journal. The following figures give the proportions usually adopted [1] for wrought-iron journals.

No. of revolutions per minute 50, 100, 150, 200, 250, 500.

Ratio of length to diameter 1·2, 1·4, 1·6, 1·8, 2·0, 3·0.

Great care must be taken in designing journals not to pass abruptly from one section of the metal to another. All such differences should be gradually rounded off as shown in fig. 74.

The strains to which the journals of crank shafts are subjected are due to the combined action of the twisting forces and the transverse loads.

Shaft Bearings and Pedestals.—The bearing usually consists of brass steps supported by a cast-iron pedestal or

[1] *Elements of Machine Design.* By Professor W. Cawthorne Unwin.

plummer block. Fig. 75 shows three views in half elevation and half section of a common form of pedestal which is used with a masonry foundation. It consists of a wall plate

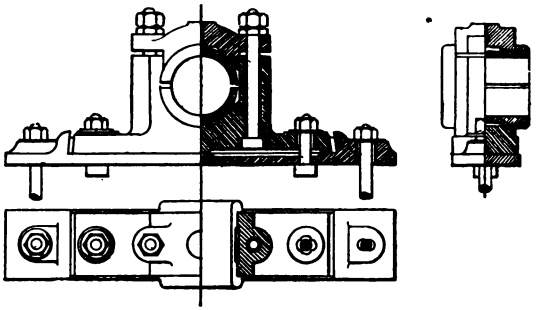

Fig. 75.

which is bolted to the foundation and on which is fixed the pedestal proper. The nature of the arrangement and the means by which the steps are adjusted and secured are sufficiently explained by the drawing.

In most stationary engines one or both of the pedestals are attached to the cast-iron framework as shown in fig. 76, which represents the principal pedestal of a horizontal engine. In this case the steps are not divided horizontally, but in an oblique plane, so that the direction of the resultant force of the pull or thrust in the connecting rod and of the other forces which act on the shaft, may pass through the solid metal of the step and not through the junction between the steps.

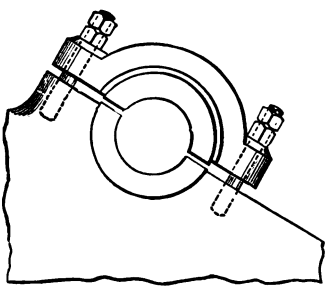

Fig. 76.

In the case of locomotives the bearings are not fixed,

but are free to slide up and down in a vertical plane, within the limits allowed by the springs. These bearings are called axle boxes. The whole weight of the engine is transmitted through them to the journals by means of the springs. Fig. 77 explains the structure of an axle box, which consists of an outer case, arranged so as to be capable of sliding up and down, between guides called horn plates which are bolted to the frame of the engine. The casing

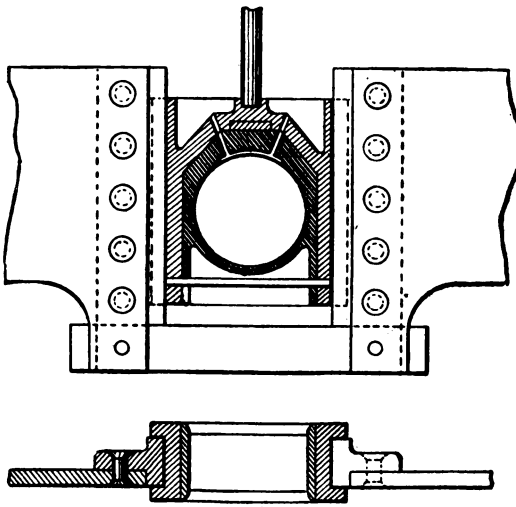

Fig. 77.

contains one brass step, the whole of the pressure being of course on the upper half of the journal. The lower part of the casing contains a receptacle for the oil which escapes after lubricating the bearings. The upper portion contains the oil box, and has also a socket formed in it which receives the foot of a spindle by means of which the pressure from the springs is transmitted.

Governors.—If, during the working of a steam engine, the load were wholly or partially removed while the supply

of steam to the cylinder remained undiminished, the engine would commence to race. If, on the contrary, the load were increased, the speed of the engine would be reduced below the proper rate. To prevent such variations in the speed, a contrivance called a governor is made use of which acts upon the steam supply in one of two ways; viz. either by partially closing or opening the throttle valve which regulates the flow of steam from the boiler; or else, by acting directly on the valve gear in such a way as to vary the point in the stroke where the steam is cut off, and thus alter the rate of expansion.

The most common form of governor was invented by Watt. It consists (see fig. 78) of two heavy metal balls A, D, attached to two inclined arms, which latter are jointed at the point E, to the central vertical spindle. The latter is connected by gearing with the main shaft of the engine so as to revolve at a rate strictly proportional to that of the shaft. The effect of rotation is that the balls tend to fly away from the vertical spindle and, being controlled by the arms, they can only rise and fall in arcs of circles about the centre E.

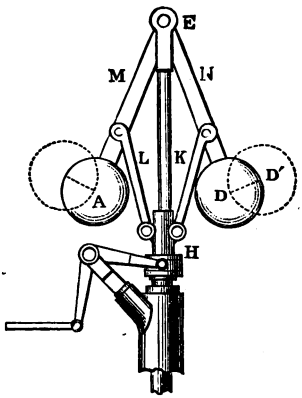

Fig. 78.

Supposing that the velocity of rotation were increased beyond the normal rate, the balls would fly out and occupy some new position D', at the same time lifting the collar H which slides on the central spindle and which is attached by the links L and K and to the ball arms M and N. Into the collar H gears the forked end of a bell crank lever which is connected by a link with the throttle valve. When H is lifted the link acts upon the throttle valve, partly closing

it, and reducing the supply of steam. Conversely when the balls fall, H falls also and the throttle valve is opened.

The theory of the conical pendulum governor is as follows. Let us suppose that the weight of the arms may be neglected. When the balls occupy any position as in fig. 79, each of them is maintained in position by the three following forces. The weight of the ball W acting downwards; the tension T in the inclined arm; the centrifugal force $\frac{Wv^2}{gr}$ (where v is the velocity of rotation of the ball), acting radially and horizontally outwards. Since the ball is in equilibrium, the three forces may be represented in magnitude and direction by the three sides of the triangle ABC. Let the radius BC be denoted by r, and the height of the cone of revolution AC by h feet,

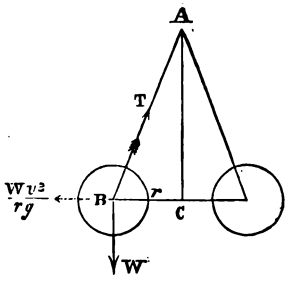

Fig. 79.

then $$\frac{h}{r}=\frac{W}{\frac{Wv^2}{gr}}=\frac{gr}{v^2}, \quad \therefore \frac{r^2}{v^2}=\frac{h}{g}, \quad \therefore \frac{r}{v}=\sqrt{\frac{h}{g}}.$$

Also since the ball is supposed to move in a circle in a horizontal plane with a uniform velocity, let t = the time in seconds occupied in making one revolution,

then $$\frac{2\pi r}{v}=t, \quad \therefore t=2\pi\sqrt{\frac{h}{g}}.$$

Consequently the time of a revolution is proportional to the square root of the height of the cone of revolution.

If we are given the number of revolutions N per minute,

then $$Nt=60, \quad \therefore \frac{60}{N}=2\pi\sqrt{\frac{h}{g}},$$

$$\therefore N=\frac{30}{\pi}\sqrt{\frac{g}{h}}=\frac{54{\cdot}29}{\sqrt{h}}.$$

If h be given in inches instead of feet, the above formula becomes

$$N = \frac{188{\cdot}2}{\sqrt{h}}.$$

As the speed of rotation of the governor and consequently of the engine is inversely proportional to the square root of the height of the cone of revolution, it is clear that the possible variations in the height of the cone have a very direct influence upon the sensitiveness of the governor.

For instance, if the governor were so contrived that the height of the cone were a constant quantity, the speed of the engine would remain constant. The object aimed at in the practical design of governors is to keep the variations in the height of the cone of revolution within convenient limits. It will be noted that in fig. 78 the ball arms are jointed on the axis of the vertical spindle, while in other cases the joints are at some distance from the axis ; in other instances the arms are crossed as in fig. 81, so that the joints are on the sides of the spindle opposite to the corresponding balls.

Each of these arrangements affects the height of the cone of revolution for a given position of the balls.

It is evident, from a mere inspection of the figures, for a given deviation of the balls from the axis of revolution, that the variation in the height is greatest in fig. 78, and least in fig. 81.

It is quite possible to make a governor so inconveniently sensitive, that it is never still for a moment, but is affected by even the small periodic changes of velocity, which occur in each revolution of the engine. As the governor can, by its nature, never act until after the change of velocity actually occurs, which it is designed to control, and as, moreover, the periodic changes of velocity above referred to are only momentary in their duration, it may easily happen that the effect of a hypersensitive governor is only felt by the engine when it has resumed its normal speed of rotation, or

even attained a rate of speed which has fluctuated in the opposite sense to that which had affected the governor. In such cases, only harm can be done by the sensitiveness of the governor: for, supposing that it is affected at a period of the revolution when the crank-pin velocity is slightly above the average of the revolution, the supply of steam will be diminished when the crank-pin has re-attained its average speed, or even sunk below it, in which case the effect of the governor will be to aggravate the evil it was designed to cure.

Many advantages are found to attend the use of high-speed governors. They are more sensitive to alterations in speed, the parts may be made lighter and move with less friction. In order, however, to prevent the balls from flying out too far, in consequence of the increased speed of rotation,

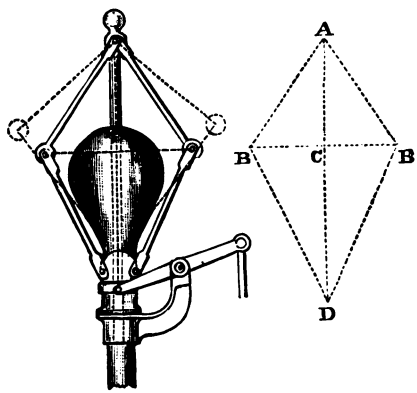

Fig. 80.

a weight, or else a spring, is so arranged as to act on the ball arms in such a manner as to develope a radial force in the contrary direction to the line of action of the centrifugal force. Fig. 80 shows a loaded high-speed governor. Each ball is attached to two sets of links. The weight is arranged

to slide on the central spindle, and presses directly upon the lower pair of ball links. To find the height of the cone h, corresponding to a given speed of rotation, we reason as follows : Each ball is at rest under the action of its weight acting downwards,—the centrifugal force acting radially outwards, and the tensions in the two ball arms due to the weights which they support.

Calling the weight of each ball, W, that of the load, W′, the radius BC$=r$, and the height of the cone AC$=h$, the tension in BA$=$T, in BD$=$T′, the angle BAC$=\alpha$, and BDC $=\beta$, we have :—

The centrifugal force in BC is balanced by the components of the tensions in the two arms, estimated in the direction BC

$$\therefore \frac{Wv^2}{gr}=T\sin\alpha+T'\sin\beta.$$

Also, the vertical component of the tension in BA balances the weight of the ball W, and the vertical component of the tension in BD

$$\therefore T\cos\alpha=W+T'\cos\beta.$$

The tension in BD is due to half the weight W′

$$\therefore \tfrac{1}{2}W'=T'\cos\beta, \quad \therefore W'=2T'\cos\beta,$$

and, finally, $r=BA\sin\alpha=BD\sin\beta$, from which equations it is possible to eliminate the values of the tensions in the ball arms, and also the angles α and β, so that h, for any velocity v, may be expressed in terms of the weights of the balls and load, and of the radius r. The simplest case is when $\alpha=\beta$, whence

$$\frac{Wv^2}{gr}=(T+T')\sin\alpha,$$

$$W=(T-T')\cos\alpha,$$

$$W'=2T'\cos\alpha.$$

Also, $h=r\cot\alpha$.

Eliminating T, T', and cotα, we obtain

$$\frac{Wv^2}{gr} = \frac{r(W+W')}{h},$$

$$\therefore\ h = \frac{r^2 g}{v^2}\left(1 + \frac{W'}{W}\right).$$

It has been already shown how the governor can be arranged to act on the throttle valve. In many modern

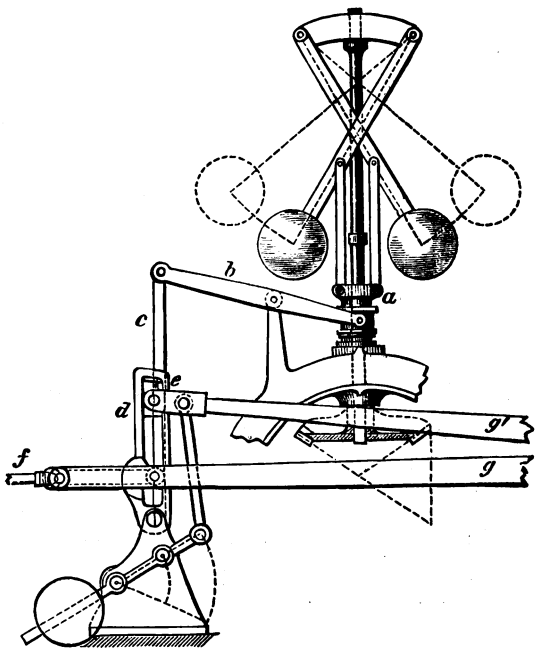

Fig. 81.

engines the throttle valve is, however, not interfered with during the working, and the governor is arranged to act directly on the expansion gear of the slide valves. Fig. 81 shows a simple method of effecting this object. The collar

a, on the vertical spindle of the governor, works a lever *b*, which is connected by a link *c* with the end of the sliding block *d* which works in a rocking link as shown. The block *d* is attached to the end of the eccentric rod *g'*. On the position of *d* depends the amount of swing given to the rocking link. An expansion valve, working on the back of the main valve (see p. 263) is driven from another point of the rocking link, and on the travel of this expansion valve depends the point at which the steam is cut off. The main valve is driven by the eccentric rod *g* in the ordinary manner.[1]

The forms of governors are so numerous, that it has been impossible here to do more than explain the principles upon which they act.

Locomotive engines are never fitted with governors, but in marine engines they are very necessary, as racing may ensue whenever the propeller is partially out of water, or whenever the propeller or crank shaft may give way. On account of the motion on board ship, the forms of governors used on land engines could not be employed for marine purposes. Marine governors are of two principal sorts, viz. those that are actuated by variations in the water pressure at the stern of the ship, and those which depend for their motion on variations in the velocity of the engine. The former class only provide for cases due to the incomplete immersion of the propeller, but the latter will guard against every contingency. In consequence of the great size of the throttle valves and expansion gear of marine engines, an ordinary governor cannot conveniently be employed to act directly on the controlling parts; hence, in this class of engines, what are called steam governors are now generally employed. The governor proper is arranged to move the slide valve of a small steam cylinder, which, in its turn, actuates the throttle valve.

[1] This will be better understood after reading the succeeding chapter on valve gearing.

Fly-wheels.—The functions of fly wheels have been explained in Chapter V., pp. 157 and 196. It is only necessary now to consider the principles involved in their construction.

The greater portion of the mass of a fly wheel is concentrated in its rim, and when revolving, every particle of the rim is under the action of centrifugal force, and tends to fly away radially from the centre ; hence the rim, when in a state of revolution, resembles the condition of a ring put in a state of tension by a force from within acting outwards. The tension developed in the rim is opposed by the tensile strength of the metal of which it is formed, and should the former exceed the latter the rim will inevitably burst asunder, just as a boiler would burst if the steam pressure were too great for the strength of the shell plates.

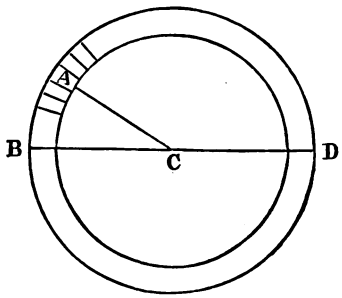

Fig. 82.

Suppose the rim in fig. 82 to be divided up into a number of segments. The centrifugal force on any one of them, such as A, acts radially along the line AC, and may be resolved into two components, one along the diameter BD, and the other at right angles to it, and similarly for all the other segments. The sum of all the components at right angles to BD is the force which tends to tear the ring asunder at the sections B and D.

It is well known that if a force press uniformly outwards all along the semicircumference of a ring, the components at right angles to a given diameter equal the total radial force multiplied by the ratio of the diameter to the semicircumference. In the case of a fly wheel, the radial pressure acting along any given semicircumference is half the

centrifugal force of the entire wheel, and the sum of the components at right angles to the diameter BD

$$= \tfrac{1}{2} \text{ centrifugal force of wheel} \times \frac{2r}{\pi r}.$$

Hence, the tension at either B, or at D, is half the above quantity = half centrifugal force of wheel $\times \frac{1}{\pi}$.

If W = the weight of the wheel in lbs., r its mean radius in feet, and N the number of revolutions per minute, we have (see p. 161)

$$\text{Tension at B, or at D} = \frac{W \times N^2 \times r \times \cdot00034}{2 \times \pi}.$$

EXAMPLE.

A fly wheel, the mean radius of which is 8 feet, weighs 15,000 lbs., the whole of which weight is supposed to act at the mean radius; what is the tension on the metal of the rim at either end of any diameter, the wheel making 60 revolutions per minute?

Answer. $\text{Tension} = \frac{15000 \times 60 \times 60 \times 8 \times \cdot00034}{2 \times 3\cdot14159} = 23376$ lbs.

Taking the tensile strength of cast iron at 15,680 lbs. per square inch and allowing a factor of safety of 5, it is evident that the section of the rim of the wheel must not be less than about 8 inches. In order to attain the given weight, the section of the rim would have to be far greater than this quantity, hence the rim would possess ample strength.

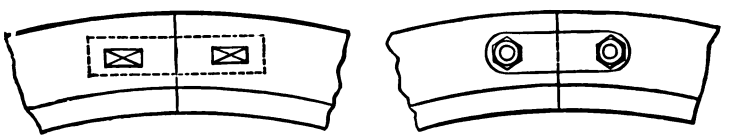

Fig. 83.

Small fly wheels are usually cast in one piece, but when so large that the weight would be unwieldy, the rim is cast in pieces which are afterwards put together by bars and cottars, or by bars and bolts, as shown in fig. 83.

The rim in very large wheels is generally fastened to the arms, as shown in fig. 84. The arms are fastened to the

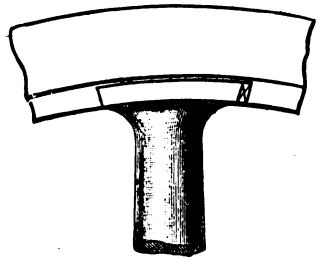

Fig. 84.

boss in a similar manner, and the latter, in the case of large wheels, is often cast in two halves, which are either bolted together or attached by wrought-iron rings shrunk on.

CHAPTER VII.

VALVES AND VALVE GEARS.

Action of the simplest form of D slide valve driven by single eccentric—Definitions of 'lap' and 'lead'—Position of eccentric as affected by the lap and lead of the valve—Effect on the steam distribution of the lap and lead of the valve—Effect of ratio of length of connecting rod to length of crank in modifying steam distribution—Means of varying the rate of expansion and of reversing—Stephenson's link motion—Effect of diminishing the throw of the eccentric—Reversing lever—Ramsbottom's reversing screw—Variations in the details of Stephenson's link motion—Other systems of link motion—Other means of varying the rate of expansion—Meyer's separate expansion valve—Corliss's valve gear—Varieties of valves—Valve gears in which eccentrics are dispensed with—Joy's gear—Geometrical representations of the action of slide valves—Zeuner's valve diagrams—Case of valve without lap or lead—Case of valve with lap and lead—Problems on simple valve setting—Zeuner's diagrams applied to valves driven by link motions—Analytical method of fixing centres of valve circles—Graphical method—Problems in link motion—The method of suspending link motions—Zeuner's diagrams applied to Meyer's valve gear—Reversing by Meyer's gear—Problems on valve setting with Meyer's gear.

THE successful and economical working of a steam engine depends in a very large degree upon the design and adjustment of the valve or valves which regulate the distribution of the steam in the cylinders. The subject is, perhaps, more complicated in its nature than any other question affecting the design of the engine. In order to treat it simply and, at the same time, systematically, it is intended in this chapter, first to explain the simplest examples, and then to proceed to the description of the cases which more frequently occur in practice.

Action of the simplest form of slide valve driven by a single eccentric.—As the motion of a slide valve is modified by the length of the connecting and eccentric rods relatively to the

length of the crank arm and to the throw of the eccentric respectively, we will suppose, in the first case, that these rods are infinite in length. Fig. 85 represents a portion of a cylinder with steam chest, slide valve, and passages, in which the above condition as to length of rods is supposed to obtain. The slide valve D is exactly the length contained between the outer edges of the steam ports. This point is important to notice, as will appear presently. Also the faces *d d'* of the valve are just sufficient to cover the width of the steam ports and no more.

The general arrangement of the slide valve, steam chests, steam and exhaust passages, in relation to the piston and

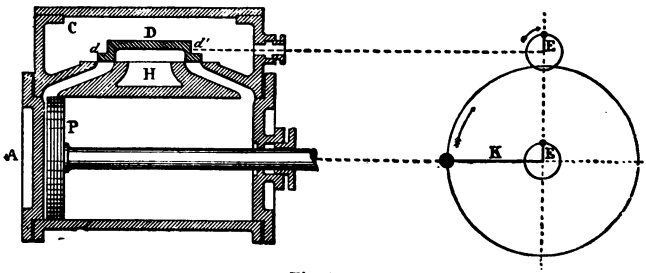

Fig. 85.

cylinder having been already explained (see page 202), the reader will have no difficulty in understanding what follows. In practice the eccentric is always mounted on the crank shaft, and revolves in a plane parallel with that containing the crank arm, while the slide valve works on the side of the cylinder. In figs. 85 to 93 the valve is shown on the top of the cylinder, and the eccentric E revolves in the same plane as the crank K, and above the latter, as in this way alone can the relative motions of valve, piston, crank, and eccentric be shown simultaneously. The arrangement of the eccentric and crank circles is not one that could be carried out in an actual steam engine.

The piston P is represented at one end of the cylinder,

and the valve is shown in its middle position exactly covering the two steam passages, so that no steam can pass from the steam chest C to either side of the piston ; while, on the other hand, none can escape from the cylinder through either of the passages into the exhaust. As the piston is at the end of its stroke the crank is on the dead centre, and as the valve is at mid stroke the arm of the eccentric must also be in the position midway between the two dead centres ; that is to say, it is at right angles to the crank. Let the piston, however, be moved in the slightest degree to the right-hand side, turning the crank K and the eccentric radius E in the direction of the arrows, and two things will in-

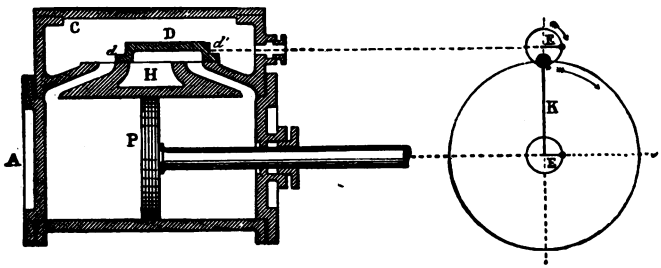

Fig. 86.

stantly happen. The eccentric, whose action has been already explained (see page 229), will pull the valve D slightly to the right, so that the outside edge of the face *d* will open the left-hand steam port, and thus admit steam to the left side of the piston, while the inner edge of the face *d'* will uncover the right-hand steam port, and thus permit whatever steam may be in the cylinder on the right side of the piston to escape along the passage to the under or hollow side of the valve, whence it finds its way to the exhaust passage H. The result will be that the entering steam will propel the piston forward, and the crank and eccentric will continue to rotate in the direction of the arrows.

When the crank reaches the position shown in fig. 86,

which is at right angles to its first position, the piston will be at half-stroke, and the eccentric radius will also be at right angles to its first position. From an inspection of the diagram it is evident that the eccentric has now pulled the slide valve as far to the right as it will go, and that, as the crank continues to revolve, the eccentric will commence to travel back from right to left. Also it is evident that the total space through which the eccentric has moved the valve from its original or central position is exactly equal to the half-diameter of the circle described by the eccentric radius. As will be seen from the figure, the left-hand steam port is now fully open to the entering steam, while the other is fully open to the exhaust.

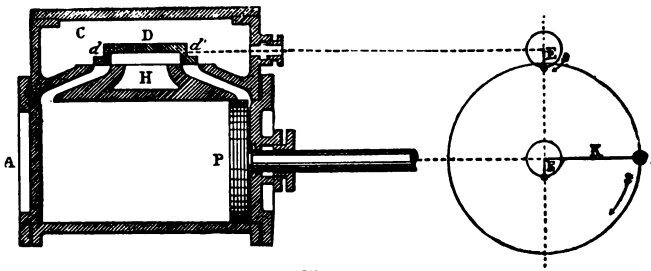

Fig. 87.

At the end of another quarter-revolution the piston will have reached the end of its stroke. The crank and eccentric will occupy the positions shown in fig. 87, while the valve has now regained its central position, closing both ports. The slightest motion to the left will now admit the steam to the right side of the piston, and cause it to commence to move backwards from right to left, and will at the same time open the left-hand port to the exhaust. At the end of the next quarter-revolution the crank and eccentric will occupy the positions shown in fig. 88 ; the right-hand port is now fully open to the entering steam, while the other is fully open to the exhaust, and the valve has reached the farthest limit of its travel to the left. From the above we see that

the total distance moved by the valve, called *the travel of the valve, is exactly equal to the diameter of the circle described by the radius of the eccentric.*

The next quarter-revolution will bring everything to the positions occupied at starting, and the whole series of operations may be repeated over and over again so long as the steam supply lasts. We thus see that by means of a slide valve and a single eccentric, the operations of opening and closing the steam and exhaust inlets can be satisfactorily accomplished.

It will be noted, however, that the valve just described, which only just covers the steam ports when in its central

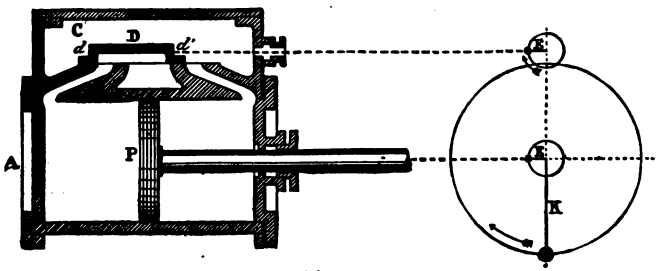

Fig. 88.

position, when driven by an eccentric set at right angles to the crank, keeps the admission steam port open during the whole length of the stroke, and thus does not permit of the expansive working of the steam. Similarly it keeps the exhaust open during the whole length of the stroke, thus rendering compression or cushioning of the exhaust steam impossible. Moreover, it only opens the admission and exhaust ports just *after* the stroke has commenced. In practice these features are inadmissible. In order to effect economy in working the supply of steam must be cut off comparatively early, and expanded during the remainder of the stroke. Similarly the exhaust should be closed before the end of the stroke, so that the steam left in may be

compressed before the advancing piston, and aid in bringing the reciprocating parts to rest. Moreover, the steam must be admitted just before, instead of just after, the commencement of the stroke.

These objects may all be accomplished, within certain limits, by adding to the length of the slide valve so that it overlaps the outer edges of the steam ports, by diminishing the width of the hollow portion D so that the faces of the valve overlap the inner edges of the ports, and lastly by altering the position of the eccentric on the shaft relatively to the crank.

Before investigating this question the following definitions must be stated.

Outside lap.—Any portion added to the length of a valve more than is absolutely necessary in order to cover

Fig. 89.

the outside edges of the steam ports, is called the outside lap of the valve. In fig. 89 the portions *c c* are the outside lap.

Inside lap.—Any portion added to the hollow portion D of the valve more than is necessary in order to cover the inner edges of the steam ports, is called the inside lap of the valve. In fig. 89 the portions *i i* are the inside lap.

Lead.—The amount by which the admission steam port is open when the piston is at the commencement of the stroke is called the lead of the valve. Thus in fig. 89 the space *b* is the lead.

Action of a slide valve provided with lap and set with lead.—Referring back to fig. 85, it is obvious that if the valve were provided with outside lap, and had a certain lead, it would have to be moved out of its central position by an amount

equal to the lap and the lead together when the piston was at the commencement of its stroke Consequently the position of the radius of the eccentric can no longer be central, that is to say, at right angles to that of the crank, but must be inclined forward at such an angle, DCE, fig. 90, that the space CL intercepted between the perpendicular EL and the centre C shall be equal to the lap and lead added together. It is also obvious that the travel of the valve will have to be increased, for it has to move sufficiently to uncover fully the steam port, and in order to do so it must travel over a space equal to the width of the port *plus* the lap. The result of altering the position of the eccentric is that all the operations effected by the valve will be completed earlier than in the previous examples. Consequently, not only will the admission steam port be partly open at the commencement of the stroke, but it will also be entirely closed to the steam before the end of the stroke, and the steam will consequently expand during the interval. Similarly the exhaust will be closed before the end of the stroke, and the exhaust steam remaining in the cylinder will undergo compression. This is shown in the following four diagrams, which show the points of the stroke at which the steam and exhaust ports are opened and closed.

The relative positions of valve and piston during the stroke will of course be very materially modified if we take into account the ratios of the lengths of the connecting and eccentric rods to the length of the crank and the throw of the eccentric respectively. As a rule the length of the connecting rod is from three to six times the length of the crank, according to the class of engine employed. The travel of the valve, and consequently the throw of the eccentric, is, however, always kept as small as possible, so as to diminish to the utmost the waste work expended in overcoming the friction of the valve on its seating; hence the ratio of the length of the rod to the throw of the eccentric is usually from $\frac{1.5}{1}$ to $\frac{3.0}{1}$; and it will be easily seen

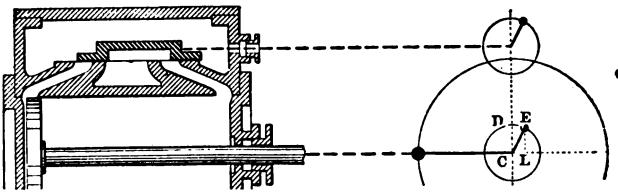

Fig. 90.—Left-hand port just opened to steam by the amount of lead.

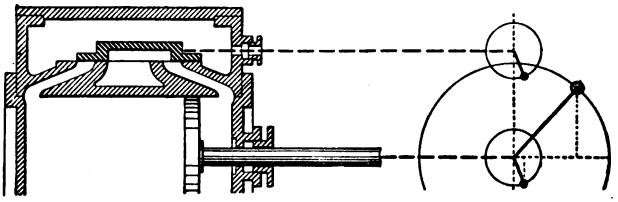

Fig. 91.—Left-hand port just closed to steam; expansion commencing.

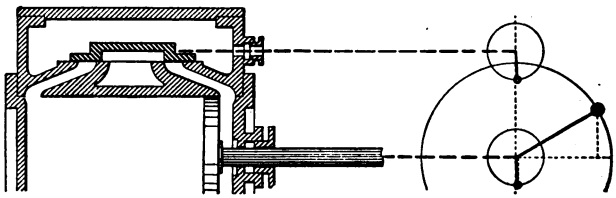

Fig. 92.—Right-hand port just closed to exhaust; compression commencing.

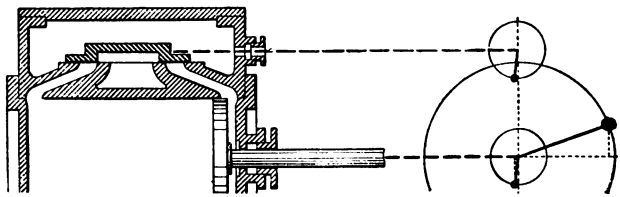

Fig. 93.—Left-hand port just about to open to exhaust: piston not yet at end of stroke.

that the position of the valve is much less affected by the obliquity of the eccentric rod than is the piston by that of the connecting rod; also the effect on the position of the valve is most apparent at those points in its travel which cause the least possible disturbance to the steam distribution. The effect of the obliquity of the connecting rod upon the position of the piston has already been explained (see page 183). The general effect of a connecting rod of finite length is that it causes the piston in its advance towards the crank to be always in advance of the position which it would occupy were the rod of infinite length; and *vice versâ*, in the return stroke the piston lags behind. Hence, as the movements of the valve are practically the same, but those of the piston quite different relatively to the positions of the crank, so also will the steam distribution be different in the two strokes.

Means of reversing the engine and of varying the rate of expansion.—The arrangement above described of a single eccentric driving a properly proportioned slide valve will answer very well for engines which have always to work in one direction at a uniform rate of expansion. In many engines, however, the rate of expansion has to be constantly varied, and in some types, such as locomotives, marine, winding, and rolling-mill engines, the direction of working has to be constantly changed. In order to provide for these requirements other arrangements have to be adopted. Referring to fig. 86 it will be readily understood, that if there were a second eccentric keyed on the shaft exactly opposite to the original one, and if the valve were by any means connected with this second, and disengaged from the first eccentric, the valve would be moved to the opposite end of its travel, and the steam port, which in the figure is shown as open to the exhaust, would be opened to the fresh steam, and *vice versâ*; the result of which would be that the engine would commence running in the reverse direction.

A convenient arrangement for effecting this reversal and

for regulating the distribution of the steam is the link motion invented by Stephenson, which is illustrated in fig. 94.

The centre of the crank shaft is at C, and the centres of the two eccentrics at *o o'*. It will be noticed that the two latter are not exactly opposite to each other, as they would be if the valve had no lead, but are brought nearer to each other by twice the amount of the angular advance. The ends of the two eccentrics are connected to a slotted link, L, at the two points P P'. The link is curved, the radius of curvature being the length of the eccentric rod, and is suspended from the point P by means of a system of levers

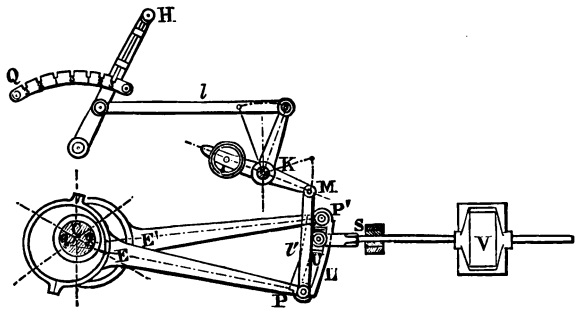

Fig. 94.

and link work clearly shown in fig. 94. In the slotted portion of the link is a block, U, which fits the slot exactly; and when the link is raised or lowered in position by means of the hand lever H, acting through the bell-crank lever K, and the two rods *l l'*, the block U slides in the slot, and is capable of taking up an intermediate position between P' and P, as is shown in the two diagrams, fig. 95, which represent the link raised to such positions that the block occupies first the centre of the link, and then a point opposite the end of the lower eccentric.

The block U, fig. 94, is connected directly to the spindle S, which drives the valve V. Now when the block occupies

the position nearest to P′ it is almost wholly under the influence of the eccentric E′, and the engine will run in one direction. Let, however, the link be raised so that U comes into the position nearest P, the block is then under the influence of the eccentric E, and the position of the valve will be so shifted that the engine will run in the reverse direction.

When U occupies a position intermediate between P′ and P it is under the control of both eccentrics, but most under

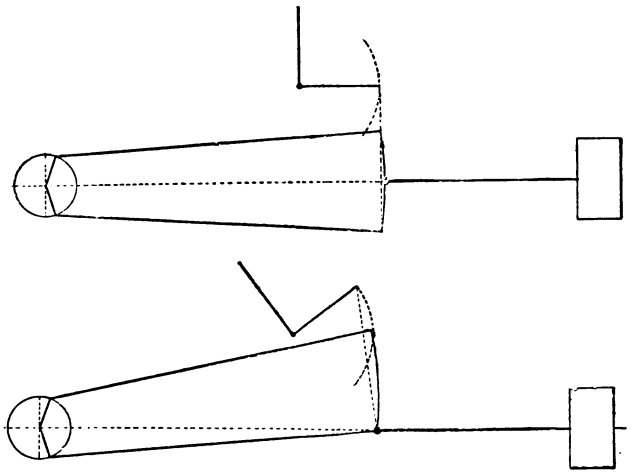

Fig. 95.

the control of the one to which it is nearest. When it occupies the exact centre of the link it is equally under the influence of both, and the consequence will be that the engine will not run in either direction. When, however, it occupies intermediate positions the effect is very curious and most important. The block, being actuated by two eccentrics working to a great extent against each other, will not travel the full horizontal distance due to the throw of either eccentric, but a distance which gets gradually less as

the block approaches the central point of the link. The effect on the valve is the same as if it were driven by a new eccentric of lesser throw than the original one. Now if we refer back to figs. 90 to 93, and keep everything, with the exception of the throw of the eccentric, the same as in these figures, it can easily be proved, by drawing the positions of the valve corresponding to the positions of the crank, that the distribution of the steam is entirely altered, the cut-off being effected at a much earlier period of the stroke, and the rate of expansion consequently increased. It will thus be seen in a general way, that the result of raising the link so as to bring the block nearer to its central position is to effect the cut-off of the steam at an earlier period of the stroke. It will hereafter be shown that not only is the point of cut-off affected, but also the angle of advance of the ideal eccentric which represents the combined action of the two actual eccentrics, as well as the lead and the points of compression and release of the exhaust steam. The combination of a pair of eccentrics with a link and sliding block is, therefore, capable not only of effecting the reversal of the direction of running of an engine, but also of entirely altering the distribution of the steam, and for this reason we find it very generally employed in locomotives, marine, winding, and rolling-mill engines.

In order to fix the position of the block U in the slide, the hand lever H, fig. 94, is provided with a catch and a notched quadrant Q. Each notch corresponds with a separate rate of expansion in forward or back running. The central notch is the position of no motion of the engine. By dropping the catch on the hand lever into any given notch the link is kept in its new position relatively to the block. Thus a quadrant with three notches on either side of the centre is capable of running an engine at as many different rates of expansion. In order to provide for any possible rate of expansion the arrangement shown in fig. 96 was invented by Mr. Ramsbottom. In this plan a

screwed spindle works the lever, which also makes it easier to reverse the engine when running.

The above is a general description of the action of a particular sort of link motion. The actual motion of the link

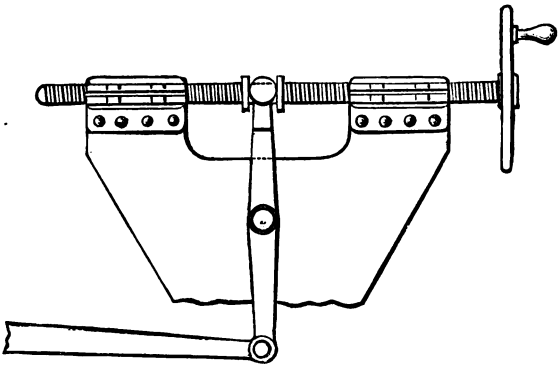

Fig. 96.

under the influence of two eccentrics is extremely difficult to follow accurately, and is further complicated by the method of suspension. So complicated, indeed, is the motion, that the travel of the slide valve can only be approximately expressed by mathematical calculation or by geometrical construction. The approximate geometrical method of determining the motion of a slide valve driven by link motion will be given hereafter.

There are many different types of link motion in use, and even considerable variations in the details of Stephenson's motion. For instance, in some cases the shape of the link is that shown by figs. 94 and 97A, in which the points of $p'p$ are respectively above and below the extreme positions of the block U, the result of which arrangement is that the valve is never exclusively under the control of either eccentric, and never receives its full motion. Consequently with this arrangement the throw of the eccentric requires to be greater than the full travel of the valve. Fig. 97 represents a link

in which the block in its extreme positions is exactly opposite the ends of the eccentric rods. Sometimes the link is suspended from the middle, and sometimes from one or other of the ends. Sometimes the eccentric rods are worked in the open position as shown in fig. 94, and sometimes the rods are crossed. All of these details effect variations in the distribution of the steam. The other link motions best known in this country are Gooch's, in which the link is not shifted up or down by the reversing gear, but the block U slides within a link suspended to a fixed point; and Allen's, in which both link and block are made to shift in opposite directions.

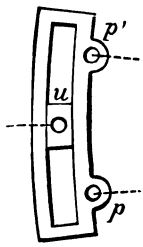

Fig. 97.

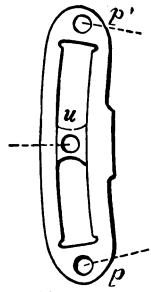

Fig. 97A.

It will be readily perceived from the description that the link motion is a very efficient method of reversing an engine, and a convenient means of regulating the rate of expansion. Its performance of the latter function is, however, attended with certain drawbacks. In addition to altering the point at which the valve cuts off the steam, each new position of the link also alters the lead, and the periods of release and compression of the exhaust steam. As, moreover, the travel of the valve is altered while the lap remains constant, the extent to which the steam port is opened is very seriously curtailed as the sliding block approaches its central position. These peculiarities, coupled with the fact that all simple slide valves open and close the ports with a comparatively slow motion, thus causing the corners of the indicator diagrams to assume a rounded instead of a sharp appearance, have led to the adoption of modifications of the above, or other methods of effecting the dis-

tribution of the steam when economy of fuel is much sought after.

Meyer's valve gear.—The method in most common use to obviate the disadvantages belonging to the common D slide valve and link motion is the adoption of a second valve to control the cut-off, while the ordinary valve, or a modification of it, regulates all the other points connected with the steam distribution. There are several varieties of these double valve motions in use, but the one selected for description, on account of its simplicity and efficiency, is that known as Meyer's valve gear, illustrated in fig. 98.

In fig. 98, A, A′, E are the two steam and the exhaust ports, which are worked by a valve B B, differing from an ordinary slide valve only in the fact that it is much longer, and that two steam passages *a a′*, for enabling the fresh steam to get to the ports, are made through the substance of the prolonged portions. This valve is driven in the usual manner either by one eccentric or, when the engine is required to reverse, by two eccentrics set opposite to each other in the ordinary way and connected by a link, which latter, however, is not used for altering the rate of expansion. The valve B B is called the distribution valve, and effects the admission, the release, and the closing of the exhaust precisely in the same way as an ordinary slide valve. The cut-off is effected by closing the passages *a a′* at the proper point in the stroke by means of the expansion valve C C′, which slides on the back of B B. The expansion valve is formed in two halves, which, by means of the threaded valve spindle D can be caused to approach or recede from each other, thus varying the point at which they close the passages *a a′* and cut off the steam. The expansion valve is usually driven by a fixed eccentric,

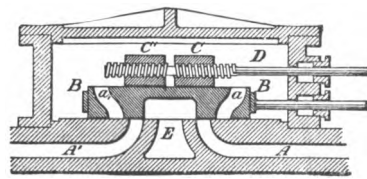

Fig. 98.

and means are provided exterior to the valve box for turning the spindle D by hand, so as to put the point of cut-off under the control of the driver, or, in some cases, of the governor.

It is evident that, with this gear, the lead and the points of admission, release, and compression are quite independent of the rate of expansion, and, when once fixed, are invariable. Moreover, as the expansion valve, when closing the passages *a a'*, is always moving in the opposite direction to the distribution valve, the cut-off can be effected much more rapidly than with the ordinary D slide. The proper method of proportioning the two valves, and of setting the eccentrics so as to provide for any desired range of expansion, will be explained in the theoretical portion of the chapter (see page 302).

Corliss valve motion.—Meyer's valve gear, though it possesses many advantages, does not remedy the evil of admitting the fresh steam to the cylinder through the passages which have been cooled down by the low temperature of the exhaust steam. Moreover, the extra power required to drive it, in consequence of the friction of two sliding surfaces is so considerable, that many eminent authorities doubt the advantage of using a comparatively complicated gear, the chief object of which is to prevent the compression curve increasing too rapidly with the rate of expansion. In order to provide against the evil of admitting the fresh steam through comparatively cold passages, and at the same time to diminish friction and retain the good points of the Meyer class of gearing, the Corliss valve motion was brought out, and is now very often employed, in some of its modifications, in non-compound expansive engines when fuel economy is a point of primary importance.

The leading peculiarities of the Corliss gear are that there are four steam ports and four valves, the two of the latter which control the steam admission and cut-off being driven by a special mechanism which ensures a very sharp closing of the steam ports when the cut-off takes place. The

two upper ports are intended solely for the admission of the steam, the remaining two serving for the exhaust. Thanks to this arrangement, the fresh steam on entering the cylinder does not come in contact with surfaces which have just been cooled down by the passage of the comparatively cold exhaust steam. The valves do not belong to the class of D slides, but are formed of portions of cylinders, A A, B B, fig. 99,

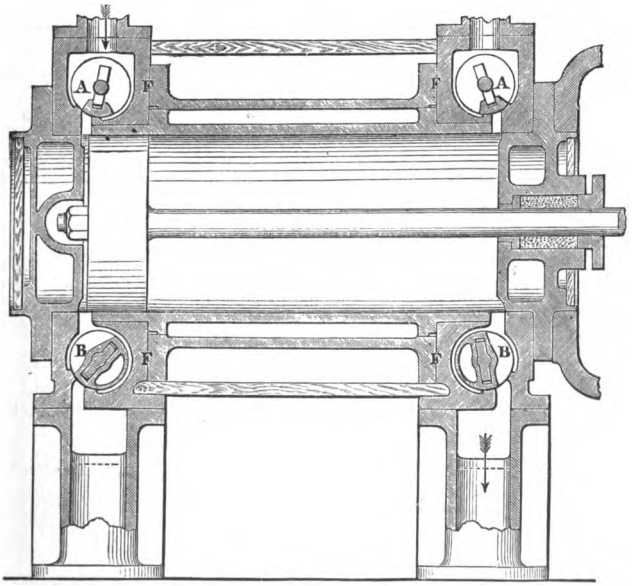

Fig. 99.

which oscillate on a cylindrical face. The valves are so arranged that the pressure of the steam forces them against their seats only when the port is closed. In all other positions there is no friction due to steam pressure, consequently the operation of these valves absorbs very little of the power of the engines. Fig. 99 is a section of a cylinder provided with Corliss gear. It shows very clearly the four steam

passages, and the cylindrical valves with the peculiarities of seatings just explained.

Fig. 100 gives a general view of the mechanism by which the valves are worked. The disc L is caused to oscillate in an arc of a circle round its own centre by means of the eccentric and the eccentric rod F. To the disc are jointed four valve rods, G G and H H, which are also attached to the four valves. The two belonging to the lower valves B B are perfectly simple; all they have to do is to open the

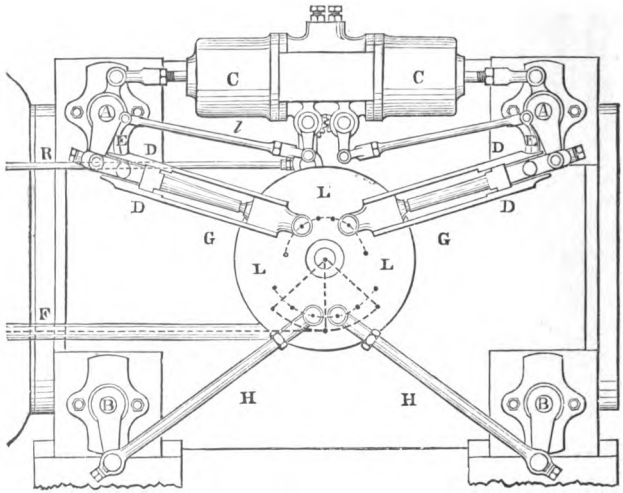

Fig. 100.

exhaust ports at the proper moment, and keep them open during the required fixed period of the stroke. The two upper rods are more complicated. Their function is, not only to open the steam ports at the proper moment, but also to keep them open during whatever period of the stroke is required by the rate of expansion, and then to liberate the valve so as to permit of its being closed sharply by an independent piece of mechanism which will be described hereafter. The upper rods are made in two independent pieces, one of which slides within the other. One of these pieces

is attached to the disc L, and the other to the rocking arm of the steam valves A A. By means of two side-clip springs D D, fig. 101, which are permanently fastened to the disc end of the rod, and which engage with the projecting shoulders *e e*, on the valve end of the rod, the two halves can at will be united into one rigid rod, or be disunited, so that the motion of the disc no longer controls that of the valve. At a certain

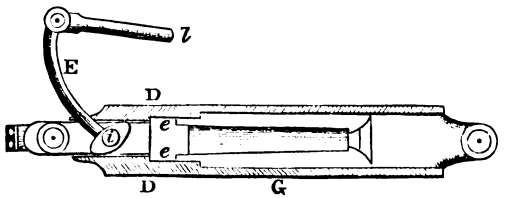

Fig. 101.

period in each stroke, dependent on the action of the governor, the clip springs D D are prised apart; the valve rod ceases to act as a whole, the valve is consequently liberated from the control of the disc L, and is instantly closed by the action of the mechanism contained in C. The mechanism by which the clip springs D D are prised apart is shown on an enlarged scale in fig. 101; it consists of a toe lever centred on *i*, and having an arm E, the inclination of which is capable of being altered by the rod *l*, which in its turn is under the immediate control of the governor rod R, fig. 100. The toe lever rocks during each stroke of the valve rod, and the period that it reaches its extreme position, and prises open the clip springs, is determined by the inclination of the arm E, and consequently by the governor. Hence we see that in this type of valve gear the rate of expansion is controlled automatically. When the clip springs are opened the valve is closed in the following manner. The valve spindle A, fig. 100, is provided with a second arm which is attached to a piston or plunger contained in C. The back of this piston is always in communication with the condenser, while the front receives the atmospheric pressure; and, consequently, whenever the valve is released from the action of the valve rod, the piston

is sharply forced inwards by the atmospheric pressure, and actuates the arm which closes the valve. There is also an air buffer, or dash pot, contained in C, which prevents the concussion, due to the very sharp action of the piston, which would otherwise ensue when the valve is closed.

Varieties of valves.—There are many varieties of valves besides the short D slides which have been described above. These latter, though in common use in the smaller types of land engines and in locomotives, would be quite inapplicable to the large cylinders and high pressures of modern marine engines. It will be readily understood that the friction to be overcome in driving the ordinary type of D slide valve is very considerable when high pressures are used, as is the case in locomotives. In fact, it frequently happens that each of the valves of a large locomotive is pressed against the surface on which it slides with a total net pressure of nine or ten tons, so that the mere driving of the valves probably absorbs some five or six per cent. of the total power exerted by the engine. This defect would be very much exaggerated if the D type of slide valve were applied without modification to marine engines.

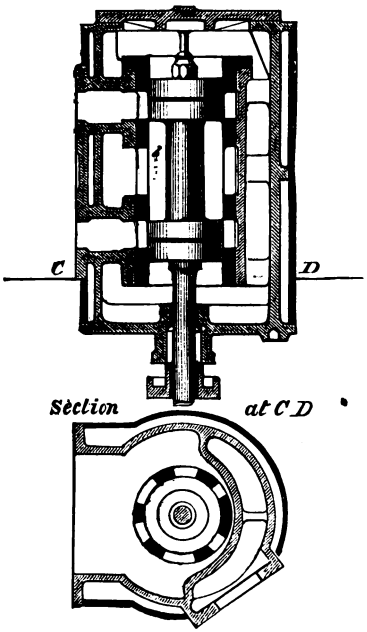

Fig. 102.

The types of valve generally made use of nowadays in marine engines are either the double ported slide valve or else what is known as the piston valve, an example of which is shown in fig. 102. Here it will be seen the valve consist of two pistons, one to each port, connected together by a common spindle. The pistons are provided with the ordinary spring ring packings to keep them steam-tight; they move up and down in the two cylindrical spaces shown, in which are cast the openings to the steam ports. The latter are not made continuously open as is the case with ordinary slide valves, but are cast with bars of metal as shown in the section, to prevent the packing rings of the pistons from springing out into the ports, and also in order to afford a continuous guide to the pistons. It will be seen that this type of valve is perfectly balanced, as far as the steam pressure is concerned. The only friction to be overcome in its motion is that due to the pressure of the spring packing rings. In the piston valve shown in fig. 102 the steam is admitted between the two pistons, the inner edges of which cut off the steam.

The double ported slide valve is an ingenious modification of the ordinary D slide already described. Examples are shown in section in the valves of the low-pressure cylinder, pages 456 and 458, and also in fig. 103. It will be noticed that the steam passages of these cylinders have each two ports or openings on the valve face. The steam is admitted to the outside ports in the usual way, over the outer edges of the valves, but the two inner ports get their steam from the passages cast in the body of the valve, and which are shown in section in the figures above referred to. The advantage of this arrangement is, that for a given movement of the valve twice the area of steam port is uncovered as with the ordinary single ported valve, and consequently for a given area of opening the travel of the valve may be greatly reduced.

In order to relieve the faces of the slide valves of marine

engines of a portion of the pressure brought to bear on them by the action of the steam on their backs, it is usual to fit relief rings on to these latter, in order to cut off a portion of their area from the pressure of the steam. It will be readily understood that if the back of the slide valve were planed perfectly flat, and if it could be so arranged as to work in perfect contact with the inner face of the valve box cover, so that no steam could get between the two faces, then the valve would only be held against its seating by the unbalanced portion of the pressure acting on the lips of the valve

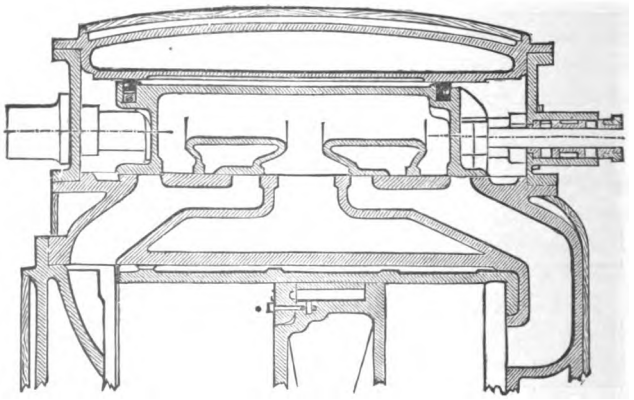

Fig. 103.

c c, fig. 89. Such an arrangement could not however be made to work satisfactorily in practice, because, in consequence of the different expansions under heat of the metals of the valve and the box, the former would either jamb or work loose. The relief ring is intended to get over this difficulty. It consists of a flat ring on the back of the slide valve, which is pressed outwards against the face of the valve box cover by means of a spring. The ring fits steam-tight into the back of the valve and works steam-tight against the face of the valve box, and thus excludes a large portion of the back of the valve from the direct pressure of the

steam. In case any steam should leak into the hollow space within the ring, the latter is generally placed in communication with the condenser. Fig. 103 is a section of a double ported valve fitted with a relief ring, which is shown in section in black.

Joy's valve gear.—There are other methods in common use for driving valves besides the eccentric system which has been described. It has constantly been an object with inventors to get rid of the complications of the two eccentrics, link, &c., required for an expansion gear. Several successful gears have been brought out, both in this country and the Continent, in which the valve is driven from some moving part of the engine. One of the best known of these

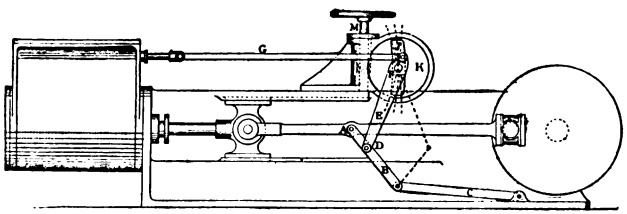

Fig. 104.

is the Joy valve gear, which has been largely used both for locomotives and marine engines. Fig. 104 illustrates a simple form of this gear applied to a horizontal stationary engine. A vibrating rod or link, B, is attached at one end to a point A, near the middle of the connecting rod; while the lower end is jointed to the radius rod C, which compels B to move in a vertical plane. To a point D in the link B is jointed the end of the long arm of a lever E F, of which the end of the small arm works the valve rod G, and the fulcrum F is attached to a block which slides in the curved slot J. This slot is formed in a disc, the centre of which is the position of the fulcrum F when the piston is at either end of its stroke. The radius of the slot is equal to the length of the valve rod G. The disc can be made to

rotate through an arc by means of the worm and wheel shown. Thus the slot can be inclined to either side of the vertical. The slot allows the fulcrum of the lever to move up and down with the motion of the point A of the connecting rod. The forward or backward motion of the engine, and the rate of expansion, are controlled by inclining the slot to one or other side of the vertical, the central position corresponding with mid-gear. If the end D of the lever were attached direct to the connecting rod, the motion of the fulcrum F about the centre of the slot would not be symmetrical, and the result would be that the cut-off would

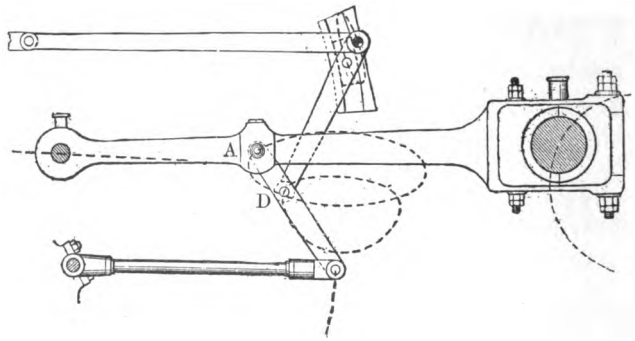

Fig. 105.

be unequal in the two strokes. This error is corrected by attaching the end of the lever to the point D of the vibrating link. For while the point A on the connecting rod describes a nearly true ellipse, as shown in fig. 105, the point D describes a bulged figure, and the amount of the bulge is so regulated as to correct the unequal motion of the fulcrum above and below its central position. It is obvious that by shifting the point D the amount of the bulge may be altered, and thus the error may be corrected too little, or too much, and by taking advantage of this circumstance a later cut-off may be given to either end of the cylinder if found desirable.

Several advantages are claimed for this type of gear over the ordinary link motion driven by eccentrics. Foremost among these is the fact that it gives an almost mathematically correct motion to the valve, which the older gear does not. It is also considerably cheaper; and from the peculiarity that the valve boxes are on the top of horizontal cylinders, and in front of vertical marine engine cylinders, instead of being at the sides, as is the case when the valves are driven by ordinary eccentrics, a considerable saving of space is effected.

Geometrical Representation of Action of Slide Valve.

There are few points connected with the successful working of steam engines of greater importance than the design of the valve gearing. Mathematical calculations intended to show the connection between the dimensions of the valve, the position and throw of the eccentric, and the various points connected with the distribution of the steam—such as the lead, the period of admission, the cut-off, release, duration of exhaust and compression—are too complicated and cumbersome to be of general use. Many geometrical diagrams have been designed to meet the drawbacks of mathematical calculation, and of these the most simple and comprehensive are those designed by Dr. G. Zeuner, which we will now proceed to describe and illustrate.

Zeuner's valve diagrams.—It will be assumed for the sake of simplicity that the obliquity of the eccentric rod has no appreciable effect upon the position of the valve. The diagram will be made to show the particular angles of the crank at which the various critical points of the steam distribution take place, from which the corresponding positions of the piston can be deduced for the forward and backward strokes when we know the ratio of the length of the connecting rod to the arm of the crank.

Referring to fig. 85, which represents a valve without lap or lead and an eccentric set at right angles to the crank, let AO, fig. 106, represent the position of the crank at one of its dead centres, and OE at right angles to AO, the corresponding position of the eccentric, the valve being then in its central position; also let the amount of eccentricity or half throw of the eccentric equal OE. If the crank now occupies successively the positions OB, OC, &c., the eccentric will take up the corresponding positions OF, OG, &c.; and if from the points F, G, &c. we let fall perpendiculars F*f*, G*f'*, &c., upon the diameter AOH, the lengths O*f*, O*f'*, &c., intercepted between the centre of the circle and the feet of the perpendiculars, will represent the distances by which the valve has been moved from its central position when the eccentric takes up successively the positions OF, OG, &c.

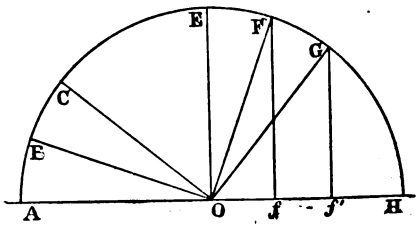

Fig. 106.

Now, however, suppose the eccentric to be fixed in position independently of the crank shaft and suppose the latter to revolve with the engine cylinder and all the moving parts attached round the centre O. It is evident that the fixed eccentric will in this case impart exactly the same motion to the valve as was done in the former case, only

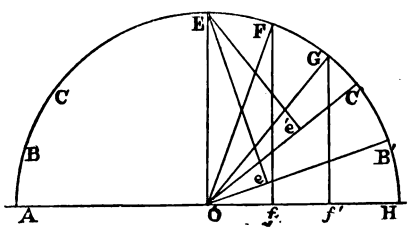

Fig. 107.

that in this instance the revolution will have to start from the dead centre H, fig. 107, and proceed in the reverse direction, so that when the crank occupies successively the positions OB, OC, &c., it will be represented in the diagram as occupying the positions OB′, OC′, &c. From the centre of the eccentric E let fall perpendiculars E*e*, E*e*′, &c. upon the lines OB′, OC′, &c.; then the lines O*e*, O*e*′ represent the distances travelled by the valve from its central position when the crank occupies the respective positions OB′, OC′, &c. For in the diagram, fig. 106, we saw that the distances moved by the valve were represented by the lines O*f*, O*f*′; and it can easily be proved that O*f*, O*f*′ are respectively equal to O*e*, O*e*′. In the triangles OE*e* and OF*f* we have OF = OE, both being radii of the same circle; also the right angle O*e*E = the right angle O*f*F. Also by construction the angle EOF = the angle B′O*f*. Therefore, adding to each of these the common angle FO*e*, we have the whole angle EO*e* = the whole angle FO*f*, and consequently the two triangles are equal, and the side O*e* = the side O*f*, and similarly O*e*′ = O*f*′, and so on for every other position of the crank; therefore O*e*, O*e*′, &c. represent the distances travelled by the valve from its central position when the crank occupies positions opposite OB′, OC′, &c. Hence we see that the distances moved by the valve for any positions of the crank OB, OC may be found graphically by dropping perpendiculars from the centre of the eccentric E upon the opposite positions OB′, OC′, and measuring the lines intercepted between the feet of these perpendiculars and the centre O.

Now it will be noticed that in this way a series of right-angled triangles, O*e*E, O*e*′E, &c. are constructed upon a common base OE; and it is a well-known fact (depending upon Prop. 31, Euclid, Bk. iii.) that when such a series of right-angled triangles is constructed, their apices, *e*, *e*′, *e*″, &c., all lie upon the circumference of a circle of which the base-line is the diameter.

The above suggests a very simple method of ascertaining the position of the valve for every angle occupied by the crank, and this method is the basis of all Zeuner's valve diagrams. We have only to draw the line OE, connecting the centre of the eccentric with the centre of the crank-shaft, when the crank is at either of the dead centres, and upon this line as a diameter to describe a circle; then the chords of this circle, O*e*, O*e'*, O*e''*, &c., will represent the spaces traversed by the valve from its central position when the crank occupies successively the positions opposite to O*e*B', O*e'*C', O*e''*D', &c. During the return stroke the motion of the valve will be indicated by the corresponding chords of the circle described on the line OK.

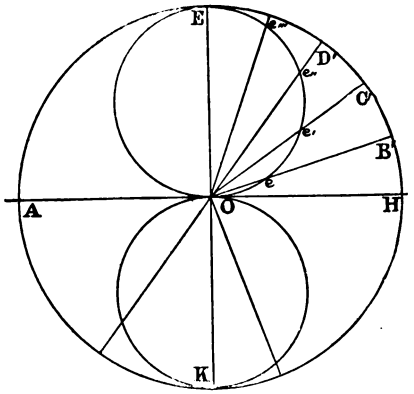

Fig. 108.

The application of this diagram to the valve shown in fig. 85 is very easy, it being remembered that the valve has no lap, and that it occupies its central position when the piston is at the commencement of the stroke. Commencing with the edge of the valve *d* which works the left-hand steam port, we see that when the crank occupies the position opposite OH, fig. 108, in consequence of the position of the circle on OE, there is no chord intercepted by any part of the line OH, and consequently when the crank is on the dead centre the port is not open at all; but directly the crank moves through any arc, no matter how small, there will be a chord intercepted by the periphery of the circle, and the valve

will be opened by an amount equal to the length of the chord. The port will continue to open till the crank-pin reaches the position E, at which point the valve will have travelled to one end of its beat, for no arc of a circle can be drawn longer than the diameter. From this point the valve commences to close the port, but does not completely close it till the end of the stroke. Similarly the motion of the valve during the return stroke can be ascertained by means of the circle on OK.

The diagram is equally useful in tracing the exhaust side of the valve. While the crank is travelling in the direction of the arrow, fig. 85 (represented of course in the diagram by the opposite direction), the outer edge *d* of the valve is keeping the steam port open, but as soon as the piston reaches the end of its forward stroke the valve has returned to its central position, and during the first half of the return stroke continues its motion towards the left. Directly the crank is over the dead centre the inner edge of the valve opens the left-hand port to the exhaust, and the arcs intercepted between O and the periphery of the circle OK, fig. 108, measure the extent to which the port is opened. Following the motion as before, we see that the exhaust is fully opened when the crank is at OK, and that it remains open till the end of the stroke (compare fig. 88).

How to indicate lap and lead on the valve diagram.— When a valve is provided with outside lap, and when the port has to be opened by the amount of the lead at the commencement of the stroke, the valve can no longer be in its central position when the crank is on the dead centre. This has been shown in fig. 90, which also illustrates the manner of setting the eccentric. Describe a circle, fig. 109, with radius OA equal to the half-throw of the eccentric. From O measure off OB equal to the outside lap, and BC equal to the lead. When the crank-pin occupies the dead centre A, the valve has already moved to the right of its central position by the space OB + BC. From C erect the

perpendicular CE, and join OE. Then will OE be the position occupied by the line joining the centre of the eccentric with the centre of the crank-shaft at the commencement of the stroke. On the line OE as diameter describe the circle OCE ; then, as before, the chords O*e*, OE, O*e'*, will represent the spaces travelled by the valve from its central position when the crank-pin occupies respectively the positions opposite

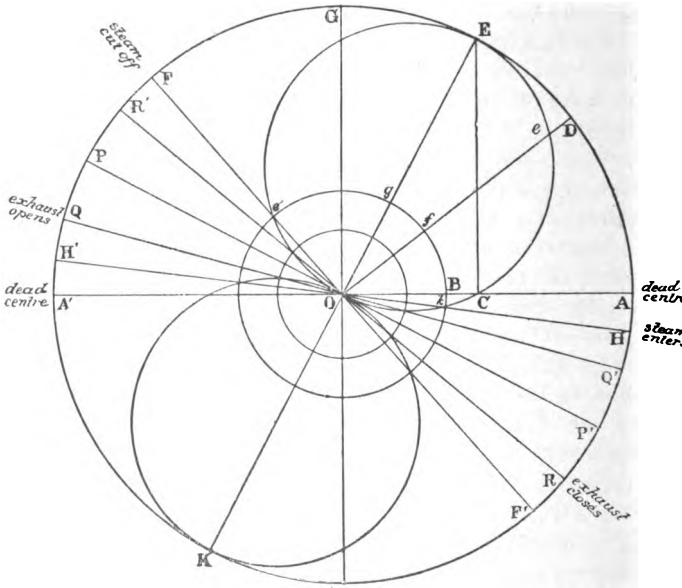

Fig. 109.

to D, E, and F. But these chords will no longer represent the extent to which the outer edge of the valve has opened the steam port, because before the port is opened at all the valve must have moved from its central position by an amount equal to the lap OB. Hence to obtain the space by which the port is opened we must subtract from each of the arcs O*e*, OE, &c., a length equal to OB. This is re-

presented graphically by describing from centre O a circle with radius equal to the lap OB; then the spaces *fe*, *g*E, &c., intercepted between the circumferences of the lap circle B*fe'* and the valve circle OCE, will give the extent to which the steam port is opened. Tracing the motion of the valve as before, and remembering that, when we speak of the crank occupying the position, say, OD, it really occupies the position symmetrically opposite on the other side of the diameter OG, we shall see at once how different is the distribution of the steam to that illustrated in the last case.

To begin with, take the point *k*, at which the chord O*k* is common to both valve and lap circles. At this point it is evident that the valve has moved to the right by the amount of the lap, and is consequently just on the point of opening the steam port. Hence the steam is admitted before the commencement of the stroke, when the crank occupies the position OH, and while the portion H A of the revolution still remains to be accomplished. When the crank-pin reaches the position A, that is to say at the commencement of the stroke, the port is already opened by the space OC — OB = BC, called the lead. From this point forward till the crank occupies the position OE the port continues to open, but when the crank is at OE the valve has reached the furthest limit of its travel to the right, and then commences to return, till when in the position OF the edge of the valve just covers the steam port, as is shown by the chord O*e'*, being again common to both lap and valve circles. Hence when the crank occupies the position OF the cut-off takes place and the steam commences to expand, and continues to do so till the exhaust opens. For the return stroke the steam port opens again at H' and closes at F'.

So far we have traced the action of the valve in admitting and cutting off the steam. There remains only the exhaust to be considered. When the line joining the centres of the eccentric and crank-shaft occupies the position opposite to OG at right angles to the line of dead centres, the crank is

in the line OP, at right angles to OE; and as OP does not intersect either valve circle the valve occupies its central position, and consequently closes the port S, fig. 89, by the amount of the inside lap *i*. The crank must, therefore, move through such an angular distance that its line of direction OQ must intercept a chord on the valve circle OK, equal in length to the inside lap, before the port can be opened to the exhaust. This point is ascertained precisely in the same manner as for the outside lap, namely, by drawing a circle from centre O, with a radius equal to the inside lap; this is the small inner circle in fig. 109. Where this circle intersects the two valve circles we get four points which show the positions of the crank when the exhaust opens and closes during each revolution. Thus at Q the valve opens the exhaust on the side of the piston which we have been considering, while at R the exhaust closes and compression commences, and continues till the fresh steam is readmitted at H.

Thus we see the diagram enables us to ascertain the exact position of the crank when each critical operation of the valve takes place. Making a *résumé* of these operations for one side of the piston, we have—

Steam admitted before the commencement of the stroke at H.

At the dead centre A the valve is already opened by the amount BC.

At E the port is fully opened, and valve has reached one end of its travel.

At F steam cut off, consequently admission lasted from H to F.

At P valve occupies central position, and ports are closed both to steam and exhaust.

At Q exhaust opened, consequently expansion lasted from F to Q.

At K exhaust opened to maximum extent, and valve reached the end of its travel to the left.

At R exhaust closed and compression begins, and continues till the fresh steam is admitted at H.

Solution of problems relating to simple valve gearing by Zeuner's diagrams.—All problems bearing on valve gearing involve relations between the following variables :—

The inside and outside laps of the valve.

The angle of advance and the throw of the eccentric.

The angles of the crank or points of the stroke at which take place the admission and cut-off of the steam, and the opening and closing of the exhaust.

Occasionally, and more especially when the valves of an old engine have to be altered, we have also to take account of the width of the steam ports, and the extent to which they have to be opened.

PROBLEM I.—The simplest problem which occurs is the following. Given the length of throw, the angle of advance of the eccentric, and the laps of the valve, find the angles of the crank at which the steam is admitted and cut off and the exhaust opened and closed.

This problem is solved in the manner shown in fig. 110. Draw the line OE, representing the half-throw of the eccentric at the given angle of advance with the perpendicular OG. Produce OE to K. On OE and OK as diameters describe the two valve circles. With centre and radii equal to the given laps, describe the outside and inside lap circles. Then the intersection of these circles with the two valve circles give points through which the lines OH, OF, OQ, and O R can be drawn. These lines give the required positions of the crank.

PROBLEM II.—Given the points at which the steam is to be admitted and cut off, and the exhaust to be opened, also the throw of the eccentric, find the proper angle of advance and the laps of the valve, also the point at which the exhaust closes.

Describe a circle AGA′, fig. 110, with radius equal to the half-throw of the eccentric, AA′ being the dead centres. Let

OF be the crank angle when steam is cut off, OQ when the exhaust opens, OH when the steam is admitted. Now the valve is in exactly the same position when the steam is admitted and cut off; consequently it reaches the end of its travel midway between these positions. Bisect the angle HOF by the line OE, then OE is the direction of the crank when the valve is at the end of its stroke; that is to say, the chord of the valve circle made by the line of direction of the crank will be a maximum at E, or in other words, OE is the diameter of the valve circle. Produce EO to K; on OE and OK describe the valve circles. Now the circle on OE intersects the line OF at the point *e*. Therefore O*e*=the outside lap. Similarly OQ intersects the circle on OK at the point *i*, therefore O*i* is the inside lap. Describe the two lap circles with radii O*e* and O*i*. The intersection of the smaller lap circle with the valve circle OK gives the direction of the crank OR when the exhaust closes. The line *cd* gives the lead, that is, the extent to which the steam port is admitted when the stroke commences.

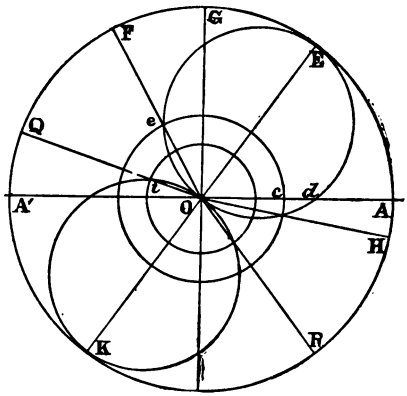

Fig. 110.

If the amount of the lead *cd* had been given instead of the angle of lead we should have had to proceed in a different manner.

First assume that there is no lead, but that the port opens when the crank is on the dead centre. Bisect the arc AOF at the point E', fig 111. From E' let fall the perpendicular

E′*c*′ on OA. From *c*′ mark off *c*′*d* equal to half the required lead. From *d* erect a perpendicular cutting the circumference in E. Mark off the arc EH equal to EF. Then OH is the required angle of lead. The remainder of the construction will be as before.

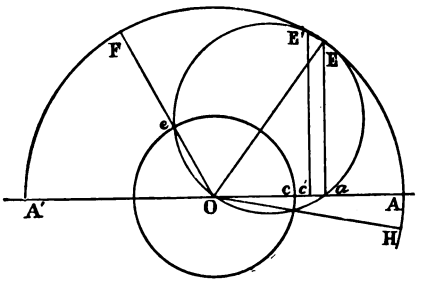

Fig. 111.

Another, and a very simple method of finding the position of the crank when steam is admitted, the amount of the lead being given, is to describe a small circle with centre A and radius equal to the amount of the lead. Call this the lead circle. From F draw a straight line tangential to the lead circle, and prolong it to meet the circumference of the circle AFA′ in the point H. Join OH, then OH is the required position of the crank.

Problem III.—Given the throw of the eccentric, the external lap, and the lead, find the point where the steam is cut off, and the angle of advance. Let OA=the half-throw. With this radius describe a circle. Let O*c* = the external lap. With this radius describe another circle. Let *cd* = the lead. From *d* erect the perpendicular *d*E, cutting the circumference of the outer circle in E. Join OE, and on OE as diameter

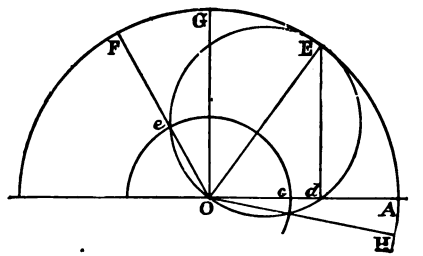

Fig. 112.

describe the primary valve circle. The angle GOE is the angle of advance, and the steam is cut off at the point F, obtained by joining O with the point of intersection of the primary valve and the lap circles.

PROBLEM IV.—Given the outside lap, the lead, and the point where the steam is cut off, find the throw of the eccentric and the angle of advance.

Let O*c*, fig. 112 = the external lap, *cd* = the lead, and OF the position of the crank when the steam is cut off. The problem then resolves itself into describing a circle *e*EO which shall pass through the three points *e*, O, *d*. The diameter of this circle OE gives the length and position of the half-throw of the eccentric, and the line OH gives the position of the crank when the steam is admitted so as to secure a lead = *cd*.

PROBLEM V.[1]—Given the position of the crank when the steam is cut off, the lead, and the amount the valve is to be open for any particular position of the crank, find the throw of the eccentric, the angle of advance, and the lap.

Let *c*E, fig. 113, represent the required lead, and *ca* the extent to which the port is to be opened when the crank occupies a position parallel to EG′. Also let EF′ be parallel to the position of the crank when steam is cut off.

Draw *cc′* and *aa′* at right angles to *ca* ; E*g* at right angles to EF′ ; E*b* at right angles to EG′, intersecting *aa′* in the point *k*. Bisect the angles *gic′*, E*ka′*. The point O where the bisecting lines meet will be the centre of the lap circle.

Draw OA parallel to *ca*. OA will represent the direction of the cranks when on their dead centres. With centre O describe a circle to touch the two lines *ic′*, *ig*. The radius of this circle represents the lap. Join OE, and on OE as diameter describe the valve circle E*n*O. Then OE represents the half-throw and position of the eccentric. From O draw through the point of contact *e* of the lap circle with the line

[1] The very beautiful geometrical solutions of this and the two following problems are due to Mr. Cowling Welch.

E*g* the line O*e*F. Then O*e*F is evidently parallel to the line EF′, because it cuts *g*E at right angles. Also the point *e* is on the circumference of the primary valve circle because the angle O*e*E is a right angle, and as *e* is also on the circumference of the lap circle, therefore the steam is cut off when the crank is at OF parallel to the given direction EF.′

Also the lead is evidently equal to the given lead *c*E ; for, joining E*d*, we have the angle O*d*E in a semicircle equal to a right angle, therefore E*d* is parallel to *cc*′, and therefore *c*E equals the lead as shown by the part of OA intercepted

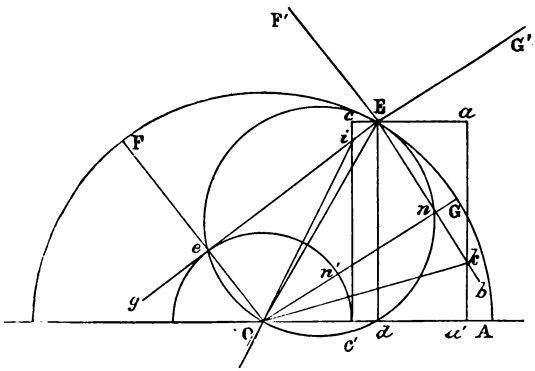

Fig 113.

between the circumferences of the lap and the primary valve circles.

Also through O draw OG parallel to EG′. Then *n*′*n* equals *c*′*a*′, because the two triangles *k*O*n*, *k*O*a*′ are equal, and as *c*′*a*′ equals *ca* by construction : therefore *n*′*n* equals *ca* as required, and all the conditions are fulfilled.

Problem VI.—Given the lead, the angle at which the steam is cut off, and also the angles at which the release takes place and compression begins, find the angle of advance and thow of the eccentric, and the outside and inside laps of the valve.

This problem is useful in solving questions connected with Meyer's valve gear, in which the angles of release and compression, as well as the lead, are regulated by the main or distribution valve.

Take any point E in a horizontal line. Mark off E*c* equal to the lead, and draw EF′, EQ′, and ER′, parallel respectively to the positions of the crank when the steam is cut off, the release takes place, and the compression commences.

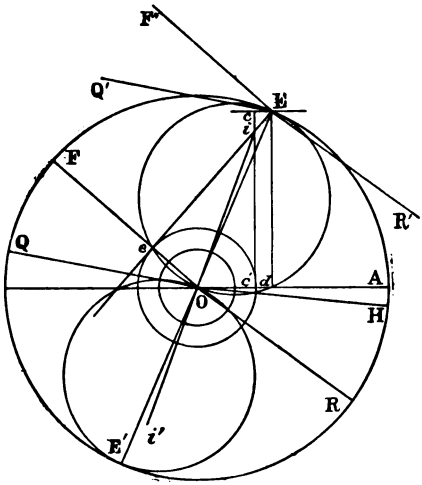

Fig. 114.

At *c* erect a perpendicular *cc*′. At E erect a perpendicular to EF′. Then the direction of the radius of the eccentric must lie halfway between the points of lead and cut-off, or, what is the same thing, between the points of release and of compression. Bisect the angle Q′ER′ by the line EE′. Bisect also the angle *c*′*ie* by the line *ii*′. Through the point O where these two lines intersect, draw the horizontal line OA. From O draw the lines OF, OQ, and OR parallel to EF′, EQ′, and ER′ respectively. Upon OE as diameter describe the primary valve circle. This will of necessity pass through *d*, because the angle O*d*E is a right angle. With centre O and radius O*c*′ describe the outer lap circle. This will of necessity pass through the point of intersection *e* of the valve circle with the line OF. Hence with the

dimensions arrived at the lead will be $c'd = cE$, and the steam will be cut off at F. In order to provide for the release taking place at Q and for the compression commencing at R we have only to draw the other valve circle on OE′, and with centre O to draw the inner lap circle through the intersections of OQ and OR with the circumference of the circle OE′,

The following problem is of great practical utility in the design of steam engines.

PROBLEM VII.—Given the position of the crank when the steam is cut off, the lead, and the maximum opening of the steam port, find the throw of the eccentric, the angle of advance, and the outside lap.

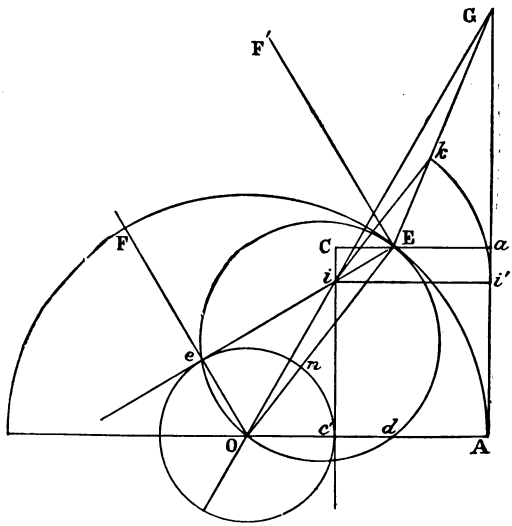

Fig. 115.

Let CE represent the given lead, and C*a* the maximum opening of the port. Let EF′ be drawn parallel to the direction of the crank when steam is cut off. Through C

and *a* draw C*c'* and GA at right angles to C*a*, and from E draw E*ie* at right angles to EF'. Bisect the angle *eic'* by the line O*i*, which produce till it intersects A*a*G in G. Join GE. With centre *i* and radius = C*a* describe an arc intersecting GE in *k*. Join *ki*, and from E draw EO parallel to *ki*, and intersecting *i*O in the point O. Then O is the centre of the lap circle. Through O draw OA parallel to C*a*. With centre O and radius OA describe a circle. Join OE, and on OE as diameter describe the primary valve circle, and with O as centre describe the lap circle touching the line *ie* in *e*, and C*c'* in *c'*. Join O*e* and produce it to F. Then it can be proved, as in the last problem, that the steam is cut off when the crank occupies the position OF, which is parallel to EF'. Also that *c'd* = CE, the given lap.

It remains only to prove that *n*E, which is evidently the greatest opening of the port = *ca*. Through *i* drawn *ii'* parallel to *ca*. Then, because *ik* is parallel to the side OE of the triangle GOE, we have—

$$GO : Gi :: OE : ik.$$

Also, because *ii'* is parallel to the side OA of the triangle GOA, we have—

$$GO : Gi :: OA : ii'.$$

But *ii'* = *ik*, both being radii of the same circle; therefore also OA = OE; and subtracting from each the equal radii O*c'* and O*n*, we have the remainder *c'*A = *n*E. But *c'*A is by construction equal to C*a*, the maximum port opening; therefore also *n*E = C*a*.

Before passing on to the consideration of the more complicated diagrams used to explain the action of expansion gears it may be useful to show how, from any diagram such as fig. 110, to set out the dimensions of the valve and ports. The valve travels to and fro on a plane surface, which is in general made of such a length that when the valve reaches the end of its beat, its edges do not overhang the end of the plane. Take any straight line AB, and suppose the valve to occupy

its central position. It will travel from this position in either direction by an amount equal to the radius of the eccentric. From A mark off AC equal to the radius OE, fig. 110. From C set off CD, equal to O*c*, the radius of the lap circle. Now the greatest amount of opening which can be given to the steam port equals the diameter OE, *minus* the radius of the lap circle. From D, therefore, set off DE equal to this difference ; this will be the width of the steam port. Next, to determine the width EF of the bridge between the steam and exhaust ports. It is evident that this width must be great enough to render it impossible for the outer edge of the valve, when at the right-hand end of its travel, to open the exhaust port to the fresh steam. Therefore the length CF

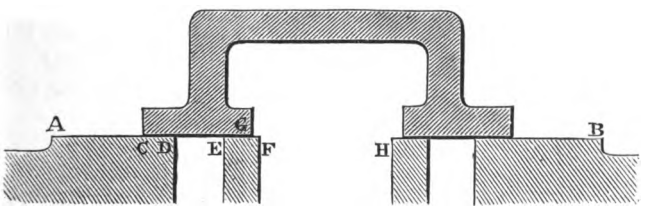

Fig. 116.

must in any case be greater than AC. The only other condition to observe in proportioning EF is that the bridge must be strong enough to withstand the pressure of steam upon it. If the width of the steam port has been properly set off as described, the above-mentioned contingency can never arrive, for the lap of the valve *plus* the width of the port, should together equal the travel. In the present instance the width is three quarters of that of the steam port. From E set off EG equal to the radius O*i* of the inner lap circle, fig. 110. In proportioning the width of the exhaust port the principal point to remember is that it must never be throttled, when the valve is at the end of its travel, to such an extent as to affect the back pressure. It is consequently usual to make it of such a width, that when the

valve is at the end of its beat the port is still open to the same extent as the steam port. From G, therefore, set off the length GH equal to the half-travel of the valve *plus* the width of the steam port, then will FH represent the width of the exhaust port. The remaining dimensions are of course the same as those of the corresponding parts which have just been found.

Zeuner's Diagrams applied to Link Motion.—When a slide valve is driven by two eccentrics connected to a link, as

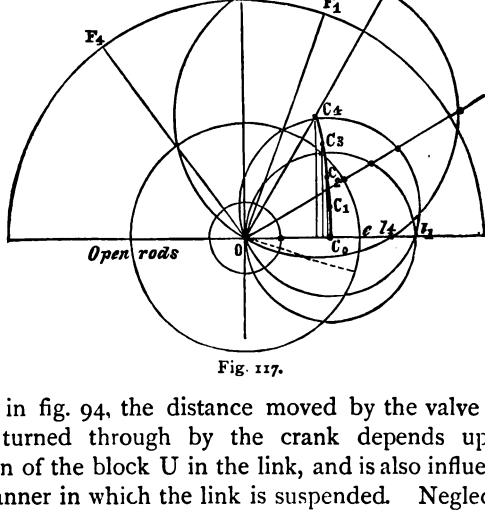

Fig. 117.

shown in fig. 94, the distance moved by the valve for any angle turned through by the crank depends upon the position of the block U in the link, and is also influenced by the manner in which the link is suspended. Neglecting for the moment the latter consideration, it is capable of analytical proof[1] that the distances moved by the valve from its central position may be represented by the chords of a pair of circles touching each other, precisely as in the case of a

[1] The proof is too long and too complicated for insertion in an elementary work. Those who wish to study it should refer to the English translation of Zeuner's work.

simple slide-gear. Moreover, to each new position of the block in the link corresponds a separate pair of valve circles, which differ both in their diameter and angle of lead from the circles corresponding to any other position of the block. It has also been found that the centres of these valve circles, figs. 117, 118, all lie upon a curve, which in the case of Stephenson's link motion with open arms is a parabola, $C_4 \ldots .C_0$, concave to the centre of the crank circle, fig. 117; while for the same motion with crossed arms the parabola is convex to the centre (see fig. 118).

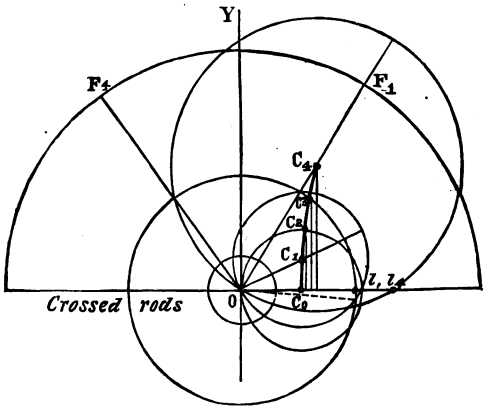

Fig. 118.

Calling the angle of advance α, the throw of the eccentric t, the length of the eccentric rod l, and the half-length of link $= k$, the parameter of this parabola $= \frac{lt \cos \alpha}{2k}$; while the distance between the vertex of the parabola and the centre O

$$= \frac{t}{2}\left(\sin \alpha \pm \frac{k}{l} \cos \alpha\right),$$

the *plus* or *minus* sign being used respectively according as the arms are crossed or open.

Analytical method of finding the centres of the valve circles in link motions.—One of the main problems in connection with the diagrams for link motion is to fix the positions of the centres of the primary valve circles, fig. 117, corresponding to the given positions of the sliding block in the link. These circles and the inside and outside lap circles being drawn in place, it is perfectly easy to trace the variations in the lead, release, compression, &c., corresponding to the varying points of cut-off. These points may be fixed either by analytical means, or with very approximate accuracy by a graphical method.

Let x and y be the co-ordinates of the centres, and u the distance which the slide has been moved in the link for the given degree of expansion; and using all the other letters in the same sense as above, we have the following formulæ, which are obtained from the investigation above referred to; viz.—

$$x = \frac{t}{2}\left(\sin a + \frac{k^2 - u^2}{kl}\cos a\right);$$

$$y = \frac{tu \cos a}{2k}$$

by means of which we can describe the primary valve circles when the angle of advance, the throw of the eccentric, the lengths of link and eccentric rods, and the position of the block in the link are given.

Let, for instance, the link be of the sort illustrated in fig. 97, so that the block when fully raised or lowered in the link comes exactly opposite the ends of the eccentric rods.

Let the half-length k of the link be divided into say four divisions, called grades of expansion. Then u may be expressed as a fraction of k. Thus, at the third grade, $u = \frac{3k}{4}$

At the fourth grade $u = \frac{4k}{4} = k$; and so on.

Substituting these values of u in the formulæ given

above, we have, when the fourth grade of expansion is used,—

$$x = \frac{t}{2} \sin \alpha ;$$

$$y = \frac{t}{2} \cos \alpha .$$

At the third grade, when $u = \frac{3k}{4}$, we have—

$$x = \frac{t}{2} \left(\sin \alpha + \frac{\frac{7}{16}k}{l} \cos \alpha\right) ;$$

$$y = \frac{3t}{8} \cos \alpha .$$

For the middle or dead point $u = 0$, and—

$$x = \frac{t}{2} \left(\sin \alpha + \frac{k}{l} \cos \alpha\right) ;$$

$$y = 0 ;$$

which shows that for this position of the slide block the centre of the valve circle lies in the straight line OX.

Let, for example, the angle of advance be 30°, the throw of the eccentric $1\frac{1}{4}$ inches, the half-length of link 3 inches, and the length of the eccentric rod 30 inches. Substituting these numerical values in the above formulæ, we obtain the centres of the primary valve circles as shown in fig. 117. Thus, for the largest circle,

$$x = \frac{1{\cdot}25}{2} \times \frac{1}{2} = {\cdot}312 \; ; y = \frac{1{\cdot}25}{2} \times {\cdot}866.$$

The ordinates of the centres c^4, c^3, &c., in fig. 117, are obtained in this way. In order to avoid unnecessary complications of the diagram, the valve circles for the second and third grades of expansion are omitted.

We can see at a glance from fig. 117 how completely all the critical points connected with the distribution of the steam are altered by the position of the sliding block. For instance, with the laps as given in the fig., and the link

in full forward gear, the steam is cut off at F^4, and the lead equals el_4 ; while at the first grade of expansion the cut-off is earlier, viz. at F_1, while the lead is increased to el_1, and similarly the alteration in the points of compression and release may be ascertained by tracing the intersections of the primary valve circles with the inner lap circle.

The alteration of the lead with the rate of expansion is one of the peculiarities of the Stephenson link motion. In the example first given, in which the eccentric rods are open the lead increases with the rate of expansion ; but if the rods are crossed, the contrary takes place, the lead decreasing. It will also be observed that the travel of the valve, which is represented by twice the diameter of the primary valve circles, varies with each rate of expansion, continually diminishing till it reaches a minimum, when the block occupies the middle of the link. Consequently, if in full forward gear, the valve completely uncovers the steam port and no more, for each succeeding rate of expansion the maximum opening of the port is reduced, till, at the central position, the maximum opening only equals the lead. As a consequence of this peculiarity it is necessary to make the ports of engines provided with link motion unusually broad, as otherwise the steam would be dangerously throttled at the higher rates of expansion.

The distribution of the steam when the block is in mid-gear is very peculiar. By reference to fig. 117 it will be clear that the steam is cut off at about a quarter-stroke, while it is released shortly after half-stroke, and admitted to the other side of the piston at about three-quarter stroke ; also compression commences on one side of the piston very soon after expansion begins on the other. The consequence is that with the block at mid-gear it is impossible for the piston to make a stroke.

Geometrical method of finding the centres of valve circles in link motions.—It is found practically more convenient to fix the positions of the centres of the primary valve circles

by a graphic method of construction rather than by calculation. The method in common use, which will now be explained, though not theoretically exact, is quite accurate enough for all practical purposes.

As before, the throw and position of the eccentric and the lengths of the link and eccentric rods are supposed to be given. One of the results obtained by the analytical investigation of the subject is that the radius of the curvature of the link should always be equal to the length of the eccentric rod. This fact is made use of in the following construction. Let LL′ (fig. 119) represent the length of the link. Bisect LL′ in O, and through O draw OA at right angles to LL′. With centre L, or L′, and radius equal to the length of the eccentric rod, describe an arc intersecting OA in C. With centre C, and radius equal to the length of the eccentric rod, describe the arc LL′, which represents the centre line of the link. With centre C

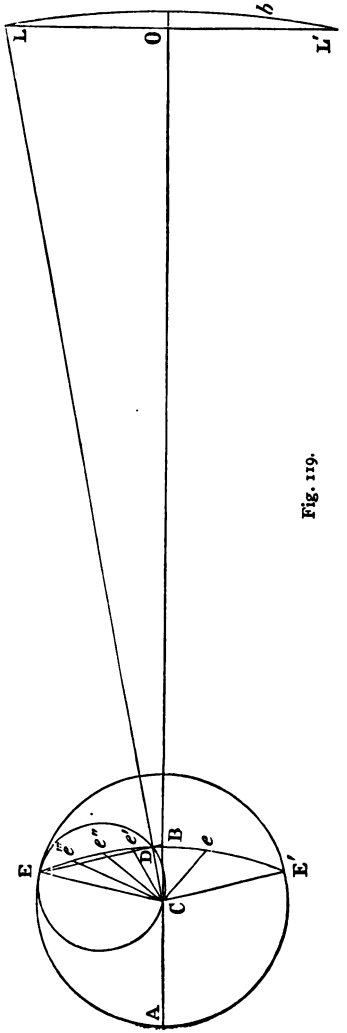

Fig. 119.

and radius CE equal to the half throw of the eccentrics describe a circle EE′A, and draw CE, CE′, having the given angle of advance. Upon CE as diameter describe a primary valve circle. Through the point E draw the line EDB, passing through the primary valve circle at the point D where the circumference is intersected by the line OL. Describe an arc of a circle through the points EBE′. Divide the arc EB into four equal parts at the points e', e'', e'''. Join these points with the centre C; then the lines Ce', Ce'', Ce''', CE, will represent the lengths and positions of the diameters of the primary valve circles, which will show the distribution of the steam when the slide block occupies the corresponding positions in the link. Generally speaking, if we wish to know the effect on the steam distribution due to the slide block occupying any position b in the link, we have only to divide the arc EE′ into two portions,

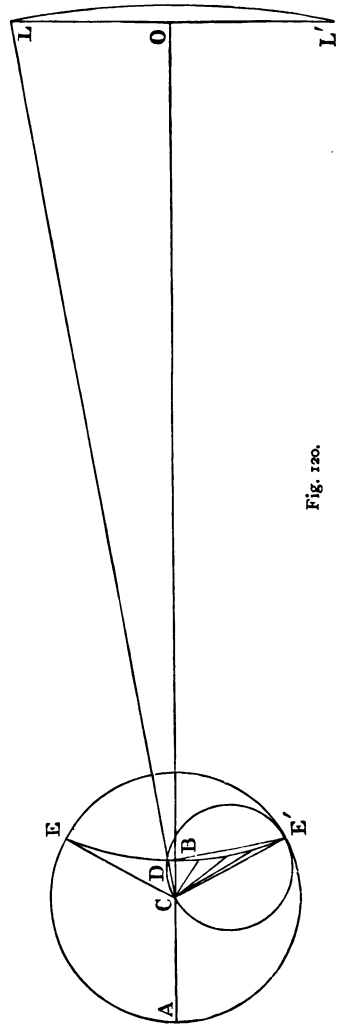

Fig. 120.

at *e* bearing to each other the same proportion which L*b* bears to *b*L′. By connecting the point *e* with the centre C we obtain the length and position of the primary valve circle, which will illustrate the distribution of the steam.

If the eccentric rods had been crossed instead of open we should have proceeded as above, except that to find the position of the point B we should have joined C with L; then the line joining E′ and the intersection of CL with the circumference of the primary valve circle will, at its intersection with the line CO give a point B, such that the curve drawn through EBE′ is convex to C, instead of being concave as in the former case. This is illustrated in fig. 120, which is merely a portion of fig. 119, the rods being supposed to be crossed.

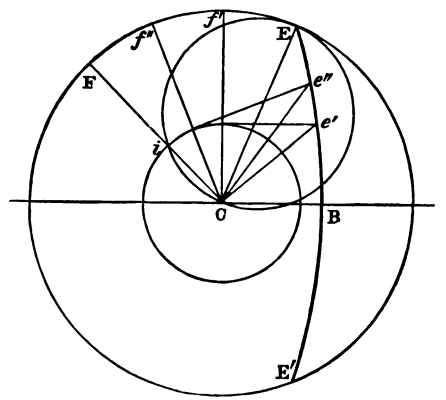

Fig. 121.

Supposing that, instead of having to find the distribution of the steam when the position of the block in the link is given, the problem be reversed, and we are given the positions of the crank at which the steam is to be cut off, and are required to find the proper method of dividing the quadrant of the reversing lever, so as to provide for the given rates of expansion, the other data being as above, we should proceed as follows. The link is supposed to be of the type illustrated in fig. 97, which admits of the block being put into full gear.

Let CE, fig. 121, be the radius and position of the eccen-

tric. Let the arc EE′ be drawn in the manner already described from the known dimensions of link and eccentric rods. Let CF be the position of the crank when the steam is to be cut off, when the block is in full forward gear, and let f' f'' be the points where the cut-off is required to be effected when the engine is worked at higher grades of expansion. The length C*i* gives the radius of the lap circle. The points where the lap circle intersects the two radii cf' cf' give points on the circumferences of the two primary valve circles corresponding to the points of cut-off f'', f'. One end of the diameters of these valve circles is in O, the centre of the circle EAE′, and the other ends are situated in the arc EE′. From the points of intersection of Cf'' and Cf', with the lap circle draw tangents to this circle intersecting the arc EE′ in e'' and e'. Join Ce'' and Ce'. These lines are the diameters of the valve circles required. It now only remains to divide the half-link LO, in the same ratio as the arc EB is divided at e'' and e'. These points of division will give the positions of the slide block necessary in order to effect the required expansion ; and the half of the expansion lever quadrant, fig. 94, requires to be divided in the same ratio, in order to obtain the positions of the notches necessary to bring the slide block into the required positions.

If we had not originally been given both the length and position of CE, but had been furnished instead with some of the data given in Problems II. to VI., we could have proceeded to find the throw and angle of advance in the manner explained in those problems by means of a separate diagram, afterwards proceeding as above.

We have hitherto considered cases in which the data relate either to the position and throw of the eccentric, or else to the lead, cut-off, and opening of the valve when the link is in *full gear*. We might, however, have to solve a question, the data referring to, say, the lead, point of cut-off, and maximum opening of the valve when the slide block

occupied some position in the link *intermediate between full and central gear*, the lengths of link and eccentric rods being given as before. Find the centre C, fig. 122, and describe the curve of the link as explained. Next, with the aid of a separate diagram, find the position and throw of an eccentric which, if connected direct to the valve spindle, will give the required lead, cut-off, and maximum opening of the valve (see Problem VI.). Transfer the result to fig. 122, C*e* being the position and length of the throw as found. On C*e* as diameter describe a primary valve circle. Let *b* be the given intermediate point in the link which has to drive the valve in the manner required. Join *b*C, intersecting the circumference of the primary valve circle in D. Join *e*D, and prolong it to intersect the line CO in B. Describe an arc of a circle which shall pass through *e* and B, and be

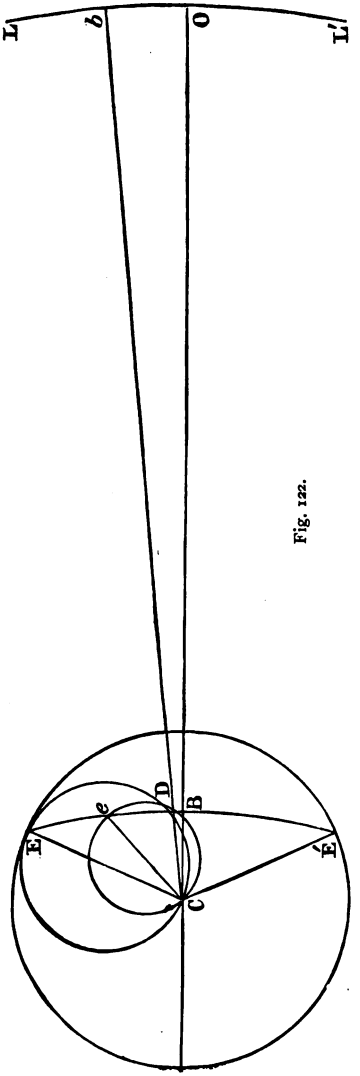

Fig. 122.

symmetrical about the line CO. Prolong this arc, and find in it a point E such that E*e* is to *e*B as *b*L is to *b*O. Join EC, then EC and the corresponding line E′C will give the positions and throws of the eccentrics, which with the given length of link and eccentric rods will produce the required effects when the slide block is at *b*. This construction is useful in solving problems connected with the working of links which are joined to the eccentric rods in such a way that the slide block can never be brought opposite to the connecting-rod ends. Such a link is shown in fig. 97A. Once the position and length of the line CE are fixed, other problems can be solved in the same manner as in the preceding example.

In all the diagrams which have been given to illustrate the action of link gearing it has been assumed that the motion of the link was in no way affected by the manner in which it was suspended. This assumption is, however, far from being justified by practice, for, the link being held up by a rod *l′*, of finite length (see fig. 94), which oscillates about a point M, the point of suspension P consequently moves in an arc of which M is the centre, and must therefore move up and down a little during each stroke. The point of the link which drives the valve spindle must therefore also slide a little up and down in the link, instead of keeping to the position intended. It is very easy to aggravate this irregularity by adopting a wrong method of suspension. The end of the rod *l′* moves in an arc of a circle of which K is the centre. The object aimed at will be best attained when the point of suspension is made to oscillate —for every position of the block in the link—in arcs, the chords of which are parallel to the line of the valve spindle. In practice the point of suspension is either the lower end or else the central point of the link. Theoretical investigation proves that in the former case the point M, fig. 94, should move in a parabolic curve, the highest position of M (when the block is at the bottom of the link) being the

vertex of the parabola, and the parameter being equal to twice the length of the eccentric rods; the co-ordinates of the vertex referred to C, fig. 123, as origin and to CX and CY as axes, being Cx = the length of the eccentric rod, and Cy = the length of the lifting link l.

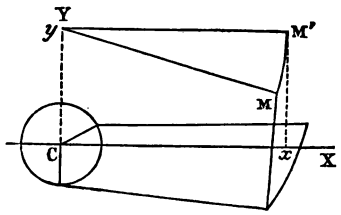

Fig. 123.

In practice, instead of the parabolic curve we may make use of a circular arc, the radius of which is equal to the length of the eccentric rod, and the centre of which is vertically above the point C at a distance = l.

When the link is suspended from its central point it is found that the point M, fig. 124, should also move in a parabola, the middle position of M corresponding with the vertex, and the parameter being twice the length of the eccentric rod.

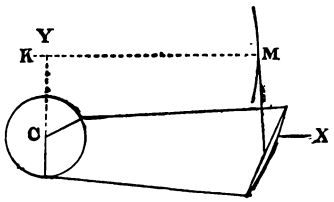

Fig. 124.

The axis of the parabola is parallel to the line CX, and the co-ordinates of the vertex are Cx = the length of the eccentric rod $-\frac{k^2}{2l}$ (where k = the half-length of the link, and l = the length of the eccentric rod), and Cy = the length of the lifting link l. In practice we may substitute for the parabola a circular arc of which the centre lies in a line KM parallel to CX, and at a distance above it equal to l, the central point K lying to the left of Y at a distance $=\frac{k^2}{2l}$.

In actual practice it is never possible to give the arm

KM a length equal to the eccentric rod, but it is always desirable to make it as long as possible.

Zeuner's Diagrams applied to Meyer's Valve Motion.—These diagrams are peculiarly applicable to the investigation of those gears which work with an expansion valve on the back of the ordinary distribution valve. It is true that in consequence of the large number of circles employed the diagrams look somewhat complex, but the principles on which they are constructed are very easy to understand and to apply.

It is perfectly obvious, from what has gone before, that a separate primary valve circle may be employed to show the distance which each valve circle has travelled from its central position, for every position of the crank. Consequently the difference between the lengths of the chords of the two circles got by drawing the direction of the crank in any position, gives the distance apart of the centres of the two valves for that position of the crank. Thus in fig. 125 let CE denote the length and position of the radius of the eccentric which drives the main or distribution valve, and CK the corresponding throw and position of the eccentric for the expansion valve. On each of these lines as diameter describe a circle, and describe the circle AEA′, to represent the path of the crank-pin. Then the circle described on CE shows in the usual way, in conjunction with the lap circle, the lead, cut-off, &c., as provided for by the distribution valve. Also for any position of the crank such as CD, the chord Ce shows the distance moved by the main valve from its central position, while the chord Ce' shows the corresponding distance travelled by the expansion valve; therefore Ce − Ce' = $e'e$ shows the distance apart of the central lines of the two valves for the position CD of the crank.

We will next prove that it is possible to draw a third circle, viz. CG, the chords of which, such as Cϵ, will represent the differences Ce − Ce', and which chords will consequently

show at a glance the distances apart of the centres of the two valves for any position of the crank.

Join EK, and from C draw CG parallel to EK, and from E draw EG parallel to CK; then CG will represent the length and position of the diameter of the third or resultant circle. The problem is to prove that $C\epsilon = e'e$. Join $G\epsilon$ and Ke'. From E let fall a perpendicular EB on Ke produced. Then in the two triangles $GC\epsilon$ and KEB we have the side GC equal to the side KE, being the opposite sides of a

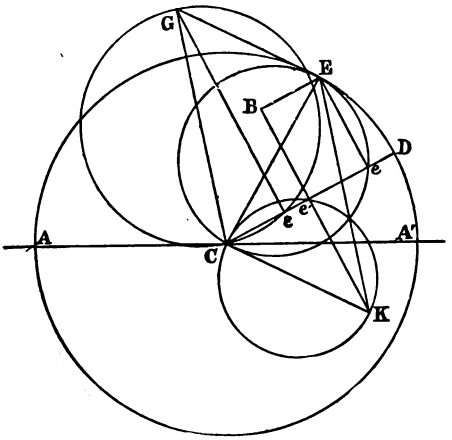

Fig. 125.

parallelogram; also the angle $GC\epsilon$ equals the angle BEK since each of them is the sum of two alternate and consequently equal angles. Also the angle $G\epsilon C$ is a right angle, being the angle in a semicircle; and EBK is a right angle by construction; therefore the two triangles are equal, and $C\epsilon$ equals BE. But BE can easily be proved to be equal to $e'e$. For join Ee; then in the quadrilateral Be we have the angle at B a right angle by construction; also the angle $Be'e$ is equal to the vertically opposite angle $Ce'K$, which being the angle in a semicircle is also a right angle. For

the same reason the angle C*e*E is a right angle, therefore the remaining angle B*e*E is a right angle, and the quadrilateral is a parallelogram, and consequently BE equals *e'e* ; and therefore Cϵ = *e'e* ; that is to say, the chord intercepted between C and the circumference of the circle CG in the direction CD is equal to the differences of the chords intercepted in the same direction between the point C and the circumferences of the circles CE and CK.

This proof is perfectly general for all cases where the

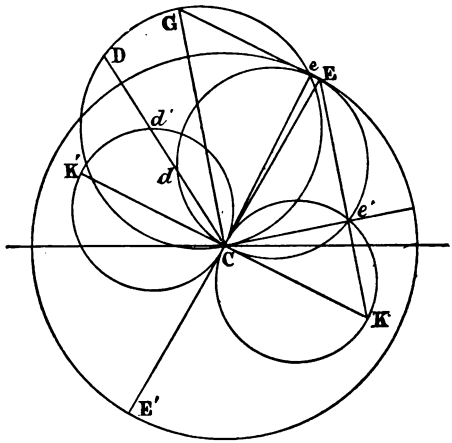

Fig. 126.

direction of the crank intercepts the circumferences of all three circles. Take, however, the case when it passes through the point of intersection *e*, fig. 126, of two of the circles. In this case the difference between the chords of the two circles on CE and CK equals C*e*, which is a chord to both circles CE and CG; therefore the line C*e* should not form any chord with the circumference of the circle on CK. Join *e*G and *e*E. Then the angle C*e*G, being in a semicircle, is a right angle ; and for the same reason C*e*E is a right angle ;

therefore GE is a straight line, and as CG is drawn equal and parallel to EK, therefore GE is also parallel to CK, and therefore the alternate angle G*e*C is equal to the alternate angle *e*CK ; therefore *e*CK is a right angle, and consequently the line *e*C is a tangent to the circle CK at the point C, and therefore the direction of the crank C*e* does not intercept any chord from the circle CK. A similar proof would hold good for the point *e'*.

It must be borne in mind that the chords of the circles CK and CE represent the movements of their respective valves to the right of their central positions, while the corresponding circles on CE produced to E' and CK produced to K' indicate movements to the left. If, therefore, any line of direction of the crank CD should intersect simultaneously say the expansion valve circle CK' and the distribution valve circle CE, then the distances apart of the centres of these two valves will not be represented by the difference of the chords C*d* and C*d'*, but by their sum ; and it may readily be proved that the chord CD of the circle CG equals this sum. Hence we see that for every possible position of the crank the chords of the resultant circles CG and CG' represent the distances apart of the central lines of the valves.

We must next show the connection between the distances apart of the centres of the valves, and the opening which the expansion valve EE', fig. 127, permits in the port of the distribution valve DD'.

First let the centres of the two valves coincide as in the upper diagram, fig. 127, and next let both valves be moved to the right, but of course to different extents, so that their centres no longer coincide as shown in the lower diagram. Let *k* be the distance from the centre of the distribution valve to the outer edge of the port. Let *l* be the length of the half of the expansion valve. Let *u* be the distance which the inner edge of the expansion valve is moved by the screw spindle (see fig. 98) from the centre line. The distance *u* is variable, so as to allow of the rate of expansion

being altered. Let r be the distance from the edge E of the expansion valve to the outer edge of the port in the distribution valve. The value of r is of course variable, and depends upon u, so that

$$r = k - l - u.$$

Now let both valves be moved to the right by their respective eccentrics; let the distance moved by the distribu-

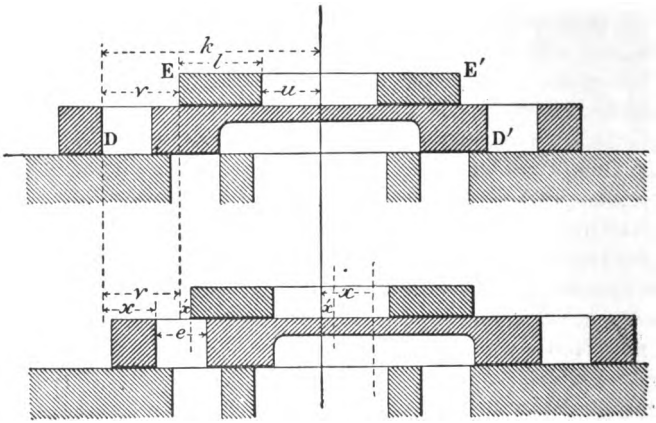

Fig. 127.

tion valve be x, and that by the expansion valve x'; also let the opening in the port in the distribution valve be e; then it is evident from the figure that—

$$r + x_1 = x + e.$$

$$\therefore e = r - (x - x_1) = k - l - u - (x - x_1),$$

which expression gives the opening of the port in the valve in terms of the two fixed dimensions k and l, the distance u which is under control, and the distances x and x', traversed by the valves.

It is very easy to show this opening of the port in the valve graphically. As has already been proved, the chords

of the two resultant circles CG, CG′, fig. 128, give the values of $x - x_1$ for any position of the crank. If with centre C and radius $CH = k - l - u$ we describe a circle, then the difference between any radius of this circle, such as C*e*, and the corresponding chord of the resultant valve circle, viz. C*e*′, gives the opening of the port in the distribution valve.

We have now the means of tracing the distribution of the steam throughout the stroke, and also of ascertaining the effect on the expansion due to the alteration of u, the half-distance apart of the two portions of the expansion valve.

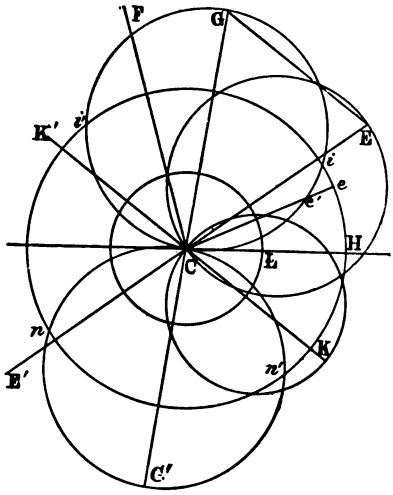

Fig. 128.

We see that when the crank lies in the direction C*i* the port in the distribution valve is just closed, for at the point *i* where the resultant valve circle intersects the circle described with $k - l - u$ as radius we have $k - l - u = x - x_1$, and consequently the value of e in the above equation is zero. Between the points *i* and *i* the port in the distribution valve remains closed, but at *i*′ it is reopened, and the steam would be readmitted to the cylinder were it not for the fact that by this time the cylinder port is closed by the distribution valve, as is proved by drawing the line CF through the intersection of the primary valve circle CE and the lap circle CL.

The tracing out of the steam distribution by means of

this diagram will be greatly facilitated if we bear in mind the following points.

1st. The opening and closing of the exhaust are determined solely by the distribution valve, are not affected in the slightest degree by the existence of the expansion valve, and may consequently be traced by noting the intersections of the valve circle CE with the outside and inside lap circles in the manner already explained.

2nd. No matter what the action of the expansion valve, the distribution valve *always* cuts off the steam on its own account at the fixed position of the crank determined by the intersection of the outside lap circle with the valve circle CE. In fig. 128 this position is denoted by the line CF.

3rd. Provided the plates of the expansion valve are not screwed so far apart as to cover the ports in the distribution valve at the commencement of the stroke (which would of course be absurd, as it would prevent all admission of steam), the lead is determined solely by the valve circle CE and the outside lap circle.

4th. The amount by which the port in the distribution valve is opened, and consequently the point at which the steam is cut off by the expansion valve, is determined by the resultant valve circle CG and the circle described from centre C and radius $CH = k - l - u$. That is to say, it is so determined for all positions of the crank between the dead point and the position CF ; for once the position CF is passed the steam is cut off by the action of the distribution valve, and the expansion valve can have no further effect. The amount by which the ports in the distribution valve are opened are measured for the port farthest from the crank shaft by the differences between the radii of the circle described with radius CH and the corresponding chords of the resultant valve circle CG, but for the other port the chords must be taken of the circle CG'. Thus we see that the far port in the distribution valve is, in the case of fig. 128,

opened during the whole of the stroke except while the arc ii' is being described, and similarly the near port is closed only during the arc nn'.

5th. The details of the valves and eccentrics having been fixed, the rate of expansion is altered solely by varying the distance apart of the two halves of the expansion valve, that is to say, by varying the radius CH.

We must now examine more closely the results obtained by varying this radius. Fig. 129 is similar to a portion of fig. 128 to a larger scale, but with the eccentrics set at different angles, and with all unnecessary complications left out.

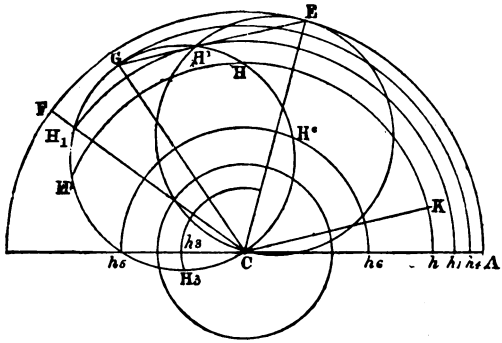

Fig. 129.

The length CA corresponds with the value $k-l$. The length Ch corresponds to the value $k-l-u$ shown in fig. 127. In this case we see that the steam is cut off at the position CH of the crank, the port in the distribution valve is closed while the arc HH$_1$ is described, and at the point H$_1$ the port in the valve is reopened, and steam would be readmitted into the cylinder were it not for the fact that the port in the cylinder itself is closed by the distribution valve when the crank is at CF. If now we diminish the value of u, so that $k-l-u = Ch_1$, the expansion valve is so set that it cuts off the steam at the position CH^1 of the crank and

reopens the valve port at the position CH_1 ; that is to say, at the same moment that the distribution valve covers the cylinder port. The position CH^1 is therefore the *latest* cut-off which it is possible to effect by means of the valve gear represented by the above diagram ; for it is obvious that if u is still further diminished, then the valve port will be re-opened before the cylinder port is closed by the distribution valve, and consequently steam will be readmitted to the cylinder during a portion of the stroke. If the value of u be still further diminished, say to Ah^4, the circle described with Ch^4 as radius will not intersect the circle CG at all, but will merely touch it at the point G, showing that the expansion valve then only closes the valve port for an instant, and consequently does not affect the expansion. While if u be still further diminished, the only effect of the expansion valve will be to reduce the available width of the valve port during a short portion of each stroke. Thus we see that with the dimensions actually chosen this valve gear ceases to have any useful effect when we wish to cut off steam *later* than at the angle CH^1.

On the other hand, its action between the positions CH^1 and CA is perfect. For instance, if the value of u be increased to Ah^6, then the steam is cut off at the position CH^6. If u be still further increased, so as to be greater than $k-l$, say to Ah^3, then the cut-off will be effected at a position found by prolonging the line H^3C backwards ; while if u be still further increased to Ah^5 the steam can be cut off at the commencement of the stroke, while in each of the latter cases the reopening of the valve port does not take place till after the position CF has been passed.

Hence we see that the valve gear represented above is perfectly efficient for any cut-off between the dead point and the position CH^1. If we wish to be able to cut off at later positions than CH^1, we must alter the positions and throws of the eccentrics. An inspection of fig. 129 will prove that the nearer the diameter CG of the resultant circle approaches

the position CF, where the distribution valve cuts off the steam, the later will be the position when the expansion valve can cut off the steam. Fig. 130 will show that it is

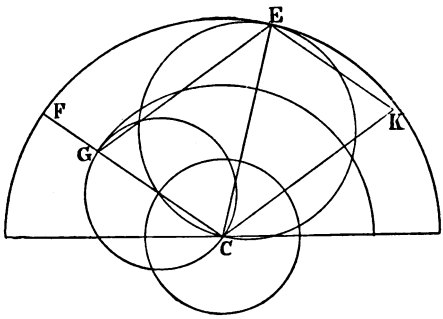

Fig. 130.

perfectly possible so to arrange the eccentrics that the steam may be cut off at any point that may be desired between the dead point and the position CF where the distribution valve closes the cylinder port. In the case of fig. 130 this result is attained by keeping the throw and angle of advance of the eccentric represented by CE as before, and by altering the angle and throw of the other eccentric to such an extent that the diameter CG of the resultant circle coincides with the direction of the line CF. It may here be noticed that in designing a Meyer valve gear it is unnecessary to give the length $AC=k-l$, fig. 129, a greater value than CG, the diameter of the resultant valve circle, for by so doing we merely admit of the possibility of the expansion valve being absolutely useless under certain conditions. This would be the case in fig. 129, for instance, whenever u had a less value than Ah^4.

Reversing by Meyer's valve gear.—If the engine to which Meyer's valve gear is applied is expected to be able to run backwards as well as forwards, the choice of the angle of advance of the expansion valve eccentric will require a good

deal of consideration, for it is evident that unless this latter eccentric is situated exactly midway between the two distribution valve eccentrics, the diameter of the resultant circle will be quite different according as the back or forward eccentric of the distribution valve is in gear. This is illustrated in fig. 131, where CE and CE′ represent respectively the positions and length of the radii of the forward and back gear eccentrics working the distribution valve, and CK that

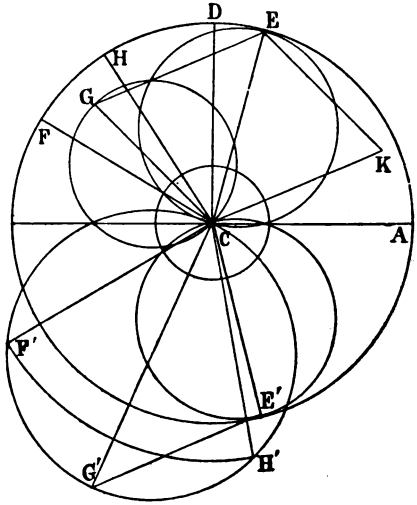

Fig. 131.

of the expansion valve. We see that owing to the angle of advance of the expansion valve eccentric DCK not being 90°, the length and position of the diameter of the resultant circle CG is quite different from that of the resultant circle CG′, and consequently the limits within which we can vary the expansion are quite different according as the engine is running backwards or forwards. For instance, when the eccentric CE is in gear, the expansion can be controlled by

the expansion valve as far as the position CH of the crank ; but when the eccentric CE′ is in gear, then the cut-off can only be effected by the expansion valve between the limits CA and CH′, which is a much smaller per-centage of the stroke.

It is easy to see that the more perfect the action of this gear when the eccentric CE is in gear, the less perfect will it be for the eccentric CE′. Hence, when an engine has to do the greater part of its work running in one direction, it is desirable so to arrange the eccentrics that the action of the valve gear will be most perfect for this direction.

As in the case of simple valve gears and link motions, Zeuner's diagrams may be applied not only to analyse the steam distribution when the dimensions, &c., of valves and eccentrics are given, but also to solve problems—that is to say, to find out some of the dimensions when other dimensions or conditions are given.

PROBLEM.—Given the lead, the angles of the crank when steam is cut off, the release takes place, and the compression commences, find the position and throw of the eccentric of the expansion valve which will permit of all degrees of expansion between the dead point and the point where the distribution valve closes the cylinder port. Also find the length of the plates of the expansion valve, and the distances apart of these plates for any given degree of expansion.

The data all refer to the distribution valve. From them we proceed to deduce the position and throw of the eccentric belonging to this valve in the manner explained in Problem VI., page 285. Having thus found the angle of advance and the length of the throw CE, fig. 132, the external and internal lap circles are found in the usual manner.

We have next to find the angle of advance and throw of the expansion valve eccentric. If the main valve is to close the cylinder port at the same instant that the expansion valve covers the port in the distribution valve, the diameter

CG of the resultant circle should coincide in direction with the line CF, which indicates the direction of the crank when steam is cut off by the distribution valve. The length of the diameter CG may be anything we like so as to suit the remainder of the design. The longer we make CG, the greater will be the angle ECK between the eccentrics of the two valves; and for engines which have to reverse, it is desirable that the angle of advance of CK should be as nearly as possible a right angle. Having fixed the length

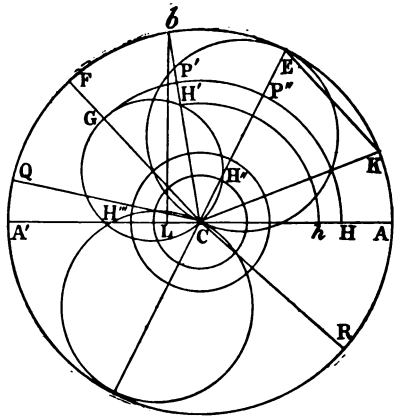

Fig. 132.

CG, join GE, and draw CK parallel to GE, and EK parallel to CG; then CK is the position and half length of throw of the expansion valve eccentric.

The minimum length of $k-l$ equals CG (see page 311). With centre C and radius CG describe an arc of a circle intersecting CA in H; then CH corresponds with the line CA, fig. 129, and equals $k-l$. We have now to fix the distance apart of the plates of the expansion valve for any degree of expansion. Let AL to AA′ be the required ratio of expansion; that is to say, supposing AA′ to represent the stroke, the steam is supposed to be cut off at L. Erect a perpen-

dicular from L intersecting the circumference of the circle described on AA′ in *b*. Join *b*C; then, neglecting the influence of the length of the connecting rod, C*b* is the angle of the crank when steam is cut off. C*b* intersects the arc GH in P′ and the resultant valve circle in H′. With centre C and radius CH′ describe an arc of a circle intersecting the line CA in *h*. Then it will be evident, from the description accompanying fig. 129, that H*h* is the half-distance apart of the plates of the expansion valve for the given ratio of expansion. But *h*H = P′H′, hence we can measure the distance required directly, as the part of the line C*b* intercepted between the arc GH and the circumference of the resultant valve circle. Similarly, if steam were cut off when the crank stood at the angle CE, the half-distance apart of the plates would be represented by P″H″. If steam were cut off at the commencement of the stroke the half-distance would be HH‴.

To find the length of the plate *l* of the expansion valve we must first fix the highest grade of expansion. Having done so, we can find the half-distance apart of the two valves in the manner above described. This gives the value *u* in the formula $r = k - l - u$. And as we know that $k - l = \text{CH}$, we can deduce *r*, that is the distance between the outer edge E of the expansion valve, fig. 127, and the outer edge D of the port in the distribution valve when both valves simultaneously occupy their central position. If the grade of expansion is very high, say one-eighth of the stroke, we shall find that *u* is greater than $k - l$, which means that E would stand to the left of D when the valves were simultaneously central. Having thus fixed the position of E when the two valves are supposed to be in their central position, we must find its position when the valves are at their greatest distance apart. Now the greatest distance apart is when the crank lies in the direction of the diameter CG of the resultant valve circle, and the distance apart equals CG. Hence it will readily be understood by refer-

ence to fig. 127 that when the crank occupies the position CG, fig. 128, the edge E of the expansion valve must lie still further to the left of the point D by the distance CG, consequently the edge E will be distant from D by the total length $r+CG$. But when the crank is in this position the plate of the expansion valve must *fully* cover the port in the distribution valve, otherwise steam would be momentarily readmitted into the cylinder. Hence the length of the plate must be equal to $r+CG+$ the width of the port in the distribution valve + a small overlap. Now the width of the port is supposed to be known, and the small overlap may be arbitrarily chosen, hence we know the length of the plate l; and as $r+l+u=k$ (see page 306), we can deduce k, the half-length of the distribution valve.

It would be possible to multiply examples of the application of Zeuner's valve gear to the solution of problems connected with Meyer's valve gear, but the explanations given are quite sufficient to enable the student to solve by himself the great majority of the questions which may arise.

CHAPTER VIII.

INDICATORS AND INDICATOR DIAGRAMS.

Uses of indicator diagrams—Richards' indicator—General character of indicator diagrams—The lines of admission, expansion, release, exhaust, and compression—How to measure the power exerted during a stroke of the piston—How to ascertain from diagram the horse-power exerted by the engine—How peculiarities and defects are revealed by the diagram—Loss of pressure during admission—Slow cut-off of steam—Effects of clearance on expansion curve—Effects of condensation and re-evaporation in cylinder on expansion curve—Usual form of the expansion curve—Late and early release—Effect of wet steam on the exhaust line—Causes affecting the back pressure—Cushioning or compression of exhaust steam—Principal causes affecting forms of diagrams—Examples of diagrams from defective engines—How to draw the hyperbolic curve of expansion—Initial condensation and re-evaporation shown by diagram—Leaky pistons and slide valves—Gross and net indicated power—Cause which limits the economical rate of expansion—How to deduce from indicator diagrams the effective pressure on piston—How to ascertain the expenditure of steam accounted for by the diagram.

IT has been already explained (p. 79) how the work done during the expansion of a gas can be represented graphically by an area, while the exact way in which the work is done is shown by the nature of the lines bounding the area. It is of the greatest importance to be able to take diagrams of the work done in the cylinder of a steam-engine, for by computing the area of the diagram we can ascertain exactly the power exerted by the engine, and by examining closely the bounding lines we are enabled to see how the work is being done at each successive instant, and can tell, for instance, when the steam is cut off, whether expansion proceeds in the proper manner, when the cylinder is open to the exhaust, what is the amount of the back pressure, when the valves are closed to the exhaust, when the readmission of steam

takes place, as well as numerous other details connected with the working of the engine—a knowledge of which enables us to ascertain whether the latter is working correctly, and, if not, where the fault lies.

Without the help of these diagrams it would be impossible to tell in the majority of instances what are the faults and shortcomings of engines at work ; and it is no exaggeration to state that all the improvements which have been effected in the economical working of the steam-engine, considered as a means of turning heat into mechanical work, have been the result of the knowledge obtained from them.

Steam-Engine Indicator.—An instrument for automatically drawing these diagrams is called a Steam-Engine Indicator, and the figures which it draws are called indicator diagrams. The indicator which is at present in most common use is illustrated in fig. 133. It is called, after its inventor, Richards' indicator. The part A is a steam cylinder containing a piston, B. The part of the cylinder below the piston can be placed in connection with either end of the engine cylinder. Above the piston is a spiral spring, C. Whenever steam is admitted below the piston it presses against the spring, which is so constructed that the distance through which it is compressed is proportional to the pressure on the piston. Thus the height to which the piston ascends under the action of the steam is a measure of the pressure of the steam. When the steam is allowed to escape, or its pressure is diminished, the piston is immediately forced downwards by the spring. D is a piston rod, to the head of which, E, it would be easy to fasten a pencil, the point of which could be made to press against a piece of paper, and by its upward and downward motion register the steam pressure beneath the piston. This was, in fact, the mode of action of the older forms of indicators. It is, however, very desirable to limit the range of motion of the piston to something like three-quarters of an inch ; while, on the other hand, the indications traced by the pencil may,

with advantage, be three or four inches long. The reason why the range of the piston is limited in practice is that its velocity, and consequently its momentum, are thus reduced. In the older indicators the momentum was often so con-

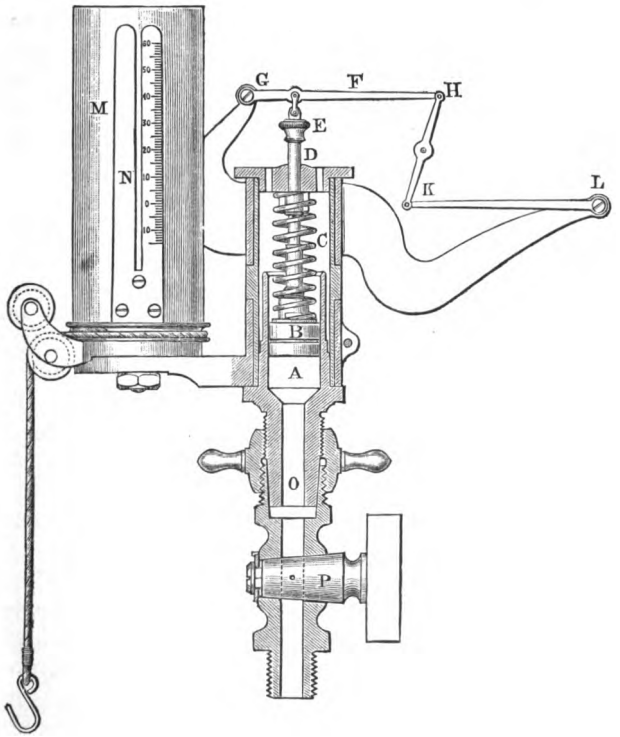

Fig. 133.

siderable that the work stored up in the moving parts enabled the resistance of the spring to be overcome beyond the point which represented the true pressure of the steam, and thus wavy lines were generated and the indications were untrustworthy. In order to obtain a long range of the pencil,

combined with a small motion of the piston, the piston rod is attached, not directly to the pencil, but to the arm of a lever, F, the fulcrum of which is at G, while the pencil is fixed to the end, H. This lever is so proportioned that the total length, GH, is three or four times as long as the portion between the fulcrum and the head of the piston rod, and consequently the pencil sweeps through three to four times the range of the piston. The pencil would now, however, describe circular arcs instead of straight lines up and down. To prevent this a parallel motion is made use of, which consists of another lever arm, KL, equal in length to the lever GH. The ends H and K are connected together by a link, in the centre of which the pencil is fixed. Whatever effect the lever GH may have in deflecting the part which holds the pencil from the vertical straight line, it is clear that the other lever exercises the opposite effect, and consequently the line of motion of the pencil keeps straight.

The function of the indicator, however, is not merely to register pressures, but to register them in relation to the position of the piston of the steam-engine. In order to effect this object, a drum, M, is provided, capable of revolution on its axis. Round the lower end of the drum is coiled a piece of string, the free end of which is attached to some one of the moving parts of the engine, the motion of which is proportional to the motion of the piston. Round the drum is fastened, by means of the clip N, the piece of paper on which the diagram is to be drawn. The pencil is brought to bear lightly against the paper by moving round the framework which carries the parallel motion and lever. When the piston of the engine begins to move forward, the string on the barrel M is uncoiled, causing the barrel to revolve; the pencil at the same time moving up or down under the varying pressure of the steam traces a line on the paper, which shows the pressure at each successive position of the piston. When the latter has reached the end of its stroke the barrel revolves in the opposite direction by means

of an internal spring, as fast as the tension of the string will permit, the pencil meantime continuing its indications. If the movement of the part of the mechanism chosen for attaching the string is greater than the circumference of the barrel, and if no part can be found having the requisite movement, then some special means must be made use of for reducing the motion of the piston to that of the circumference of the barrel.

General Character of Indicator Diagrams.—Suppose that the end, O, of the indicator is screwed into one end of the engine cylinder, or into a pipe which can be connected at will with either end of it. The cock, P, is furnished with a three-way plug, formed thus ⊣, and is also provided with a small hole communicating with the outside air. By means of the three-way plug the under side of the indicator piston can be placed either in communication with the atmosphere, or with the engine cylinder, or can be shut off from either. Firstly, let it be placed in communication with the atmosphere ; the spring above the indicator piston is then neither in tension nor compression. Let the pencil now be pressed against the paper on the barrel, and draw the string by hand so as to make the barrel revolve ; the pencil will then trace a horizontal straight line (*aa'*, fig. 134), which is the line of atmospheric pressure. Next draw the line *fg* parallel to *aa'* at a distance below it to represent 14·7 lbs. pressure to the scale corresponding to the spring above the piston. This will be the line of perfect vacuum when the barometer stands at its usual height. The atmospheric pressure is frequently spoken of as 15 lbs. to the square inch.

Let the cock now be turned so that the indicator piston shall be placed in communication with the steam cylinder at the commencement of a stroke. The piston will immediately rise, compressing the spiral spring above it, through a space proportional to the steam pressure. Suppose the pressure in the engine cylinder, when the steam is first admitted, to be 65 lbs. by the gauge = 65 + 15 = 80 lbs. above the line

of perfect vacuum or zero of pressure. The pencil will, when steam is first admitted to the engine cylinder, rise vertically upwards, tracing the line to represent 80 lbs. on the same scale that fa represents 15 lbs.

As the piston of the engine moves forward, let the steam continue to enter at undiminished pressure, say for one-third of the stroke, and let it then be sharply cut off. During this period the pencil will remain at the same height and will, consequently, as the drum of the indicator revolves beneath it, trace out the horizontal straight line AB parallel to fg, and proportional in length to one-third of the stroke of the piston of the engine.

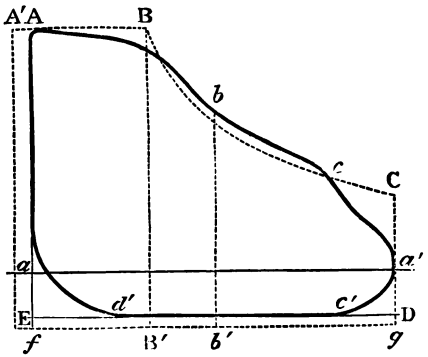

Fig. 134.

When the steam is cut off it will commence to expand, and the pressure will consequently drop. We will suppose, for the sake of simplicity, that the expansion takes place according to Boyle's law, and at the end of the stroke it will be $\frac{80}{3} = 26\frac{2}{3}$ lbs. absolute, and at intermediate points will (see p. 45) be represented by the ordinates of a rectangular hyperbola. The point of the pencil drops as the pressure diminishes, and as at the same time the drum revolves, a curve, BC, will be traced, which will be a portion of a rectangular hyperbola. Erect, therefore, at the point g, which represents the end of the stroke, the perpendicular gC, in length equal to one-third of Af; and through the points BC draw the curve BC, such that any ordinate, $b'b$, multiplied by its corresponding abscissa fb', shall be equal to the product BB' $\times$ fB'.

When the piston has reached the end of its stroke, let the exhaust valve open fully and sharply ; then, if there be no condenser, the pressure will fall to that of the atmosphere, and the pencil will descend, describing the straight line C*a'*. The piston will now commence its return stroke, during which there is no pressure of steam, but only the pressure of the atmosphere opposing its return, and the pencil will consequently trace the horizontal straight line *a'a*, till the piston reaches its original position, when the same series of operations recommences. If, however, the engine is provided with a condenser, when communication with the exhaust is opened the pressure falls below the atmospheric line to a point D, the position of which depends upon the amount of vacuum maintained in the condenser. Suppose, for instance, that twelve pounds was the amount of vacuum, then the pencil will continue moving in the direction C*a'* till the point D is reached, such that $a'D = \frac{2}{15} = \frac{4}{5}$ of *af*. If, during the return stroke, the vacuum is maintained at this figure, the pencil will trace the horizontal straight line DE, parallel to *aa'* ; and when the piston has regained its original position and steam is readmitted, the pencil will rise vertically upwards to A, and the above series of operations will be repeated.

How to measure the Power exerted during the Stroke of the Piston.—It must be remembered that all distances measured along *aa'* represent spaces traversed by the piston, while all distances measured at right angles to this direction represent pressures of steam in pounds per square inch. And consequently the work done, or pressure multiplied by space traversed under that pressure, is represented by an area. If the pressure were uniform, as it is, for instance, along the line AB, it would be easy to calculate the area, which would in this case, if the engine were of the condensing type $= AB \times AE$. As, however, the pressure varies at different parts of the stroke, it will be necessary to find the length of line representing the mean or average pressure, which, when

multiplied into the length of the stroke, will give an area equal to the irregular figure ABCDE. The usual method of finding the mean pressure is as follows. The line ED, representing the length of the stroke in feet to scale, is divided into ten equal parts, and from each point of division a perpendicular is erected intersecting the bounding lines of the diagram. The mean pressure in each of the ten divisions of the diagram must next be computed. This is easy to do by eye if the diagram is not very irregular. In the latter event it will, for the sake of accuracy, be better to divide the diagram into twenty instead of ten equal divisions. The ten mean pressures must next be added together, and their sum divided by ten, the quotient being the mean pressure for the entire diagram, which, being multiplied by the length of stroke in feet, gives the work in foot-pounds exerted during each stroke for every square inch of area of the piston.

To ascertain from the Diagram the Horse-power at which the Engine is working.—A horse-power is, as has been before explained, an expression used to denote the amount of work done in raising 33,000 pounds one foot high in a minute. Having ascertained the mean pressure from the diagram, we multiply it by the number of square inches of area of the piston; the product gives the total average pressure on the piston. This number, multiplied by the length of the stroke in feet, gives the number of foot-pounds of work per stroke, which, multiplied by the number of strokes per minute and divided by 33,000, gives the indicated horse-power.

EXAMPLE.

The cylinder of an engine is $14\frac{1}{2}$ inches diameter, the stroke is 3 feet, and the number of strokes per minute 88; the mean pressure in the cylinder is ascertained from an indicator diagram to be 40 lbs. per square inch: what horse-power is being exerted?

The area of a cylinder of $14\frac{1}{2}$ inches diameter is 165 square inches.

$$\frac{40 \times 165 \times 3 \times 88}{33000} = 52{\cdot}8 \text{ H.P.}$$

How peculiarities and defects are revealed by the indicator diagram.—The indicator diagrams of most engines differ in many important respects from the ideal diagram of fig. 134. In a fairly well-constructed engine the following modifications of form will probably take place. When the cylinder is fitted with an ordinary slide valve, the pressure during the portion of the stroke represented by AB will not be uniform, for two principal reasons. In the first place, the steam ports are, even when fully open, necessarily of limited area. At the beginning of the stroke the piston is moving comparatively slowly, while the valve, on the contrary, is at the quickest portion of its stroke, and is rapidly uncovering the port, and the steam pressure is consequently at first fairly well maintained. As the point of cut-off is approached, the piston is, however, moving nearly at its maximum velocity, while the port is being gradually closed by the valve, and the area available for the admission of the steam is gradually diminished. It results that the steam cannot enter fast enough to follow up the piston. It is, in fact, throttled at its entrance into the cylinder, and the pressure falls in consequence, so that the line AB is no longer horizontal, but droops, as represented by the full line.

Another cause of loss of pressure at the commencement of the stroke when the steam is worked expansively is the partial condensation of the entering steam, which takes place in consequence of its coming in contact with the sides of the port and walls of the cylinder, which have been previously cooled down by contact with the exhaust steam of the preceding stroke. This condensation of the fresh steam causes a very serious loss of efficiency in the steam engine, and will be referred to again in Chap. XI. It is sought to minimise the loss by keeping the cylinder hot by surrounding it with a jacket or outer cylinder, which ought always to be filled with fresh steam from the boiler.

The point of cut-off, B, instead of being sharply defined, as in fig. 134, is usually a rounded corner. This results from

the very gradual manner in which a simple slide valve cuts off the steam, causing excessive wire drawing and fall of pressure at this point.

The line of expansion.—The expansion curve BC will not, as a rule, follow the line shown in fig. 134. It may either fall above or below it. If the valves and piston are properly steam-tight, it will probably rise above BC, for the following reasons. In the first place, the line BC is calculated on the supposition that only the portion of the cylinder represented by the line AB is filled with steam. In reality this is not the case, because the piston never quite reaches the two covers of the cylinder ; there is always a vacant space between them, which, at the commencement of each stroke, is filled with steam, which expands with the remainder when the cut off takes place. Moreover, the steam port, which is often of very considerable cubic contents, is also filled with steam, which expands in the same manner. These extra steam spaces are called the *clearance*. The steam contained in them at each stroke would, under ordinary circumstances, but for expansion, be utterly wasted. The influence of clearance on the curve of expansion is, of course, to raise it ; for the clearance is virtually equivalent to an increase of the length of the cylinder, often of about five per cent. Consequently, instead of calculating the curve on the supposition that AB represents the volume of steam, we must also allow for the portion A′A, which in this case is represented as one-twentieth, or five per cent. of the stroke *aa′*.

The second principal cause of the raising of the expansion curve is the re-evaporation of some of the water which was condensed at the commencement of the stroke ; for this water, having the temperature of the incoming steam, is possessed of too much heat to remain in the aqueous condition when the pressure is lowered. A portion of it is consequently re-evaporated towards the end of the stroke. This effect is increased when the incoming steam, instead of being perfectly dry, contains a certain quantity of water. The waste

due to condensation and subsequent re-evaporation is sometimes very considerable, for though work is done to a certain extent during the re-evaporation, it is only done at the end of the stroke; whereas, had the steam not been condensed, it would have done work throughout the whole stroke. The only way to mitigate the evil is to jacket the cylinder with boiler steam, and thus, as far as possible, to prevent condensation.

A third cause of the raising of the expansion curve may, in the case of jacketed cylinders, be traced to the action of the jacket. The flow of heat from this latter to the contents of the cylinder is proportional to the difference of temperatures between these contents and the steam in the jacket. The temperature of the contents of the cylinder falls rapidly during the expansion, and consequently takes up most heat from the jacket towards the end of the stroke, the effect of the added heat being to raise the pressure of the steam above what it would otherwise be.

The true curve of expansion of dry saturated steam is not a real hyperbola like BC (fig. 134), but is a curve which falls slightly below BC. As the steam expands it does work. At the same time its pressure, temperature, and total heat diminish. The difference of heat between the states of higher and lower pressure is all converted into the mechanical work done during the expansion. The heat so supplied is, however, not nearly sufficient to account for the whole work done. In order to supply the total heat required some of the steam must be condensed, and yield up its internal or latent heat for conversion into mechanical work. As the expansion continues this condensation progresses, while, at the same time, some of the water condensed from the steam during the earlier period of the expansion is re-evaporated, because its temperature is too high to permit of its all remaining in the aqueous condition when the pressure is lowered. Thus we see that the actual curve of expansion of steam in the ordinary conditions of working of an engine

is most complex, being, in fact, the resultant of a series of opposing condensations and re-evaporations. The final form of the curve, for a given ratio of expansion, is determined by the amount of the clearance, by the original condition of the steam as to dryness, and the efficiency of the jacketing. The general result with steam of the usual degree of moisture is that the curve of expansion is often raised up to, and even beyond, the true hyperbolic curve, as shown by the full curve in fig. 134.

The release and back-pressure lines.—In cylinders provided with an ordinary slide valve the expansion is rarely carried to the end of the stroke. The steam port is generally opened to the exhaust a little before this latter point is reached, in order to allow the exhaust steam plenty of time to get away before the new stroke commences, and in order to allow of the port being pretty well opened at the end of the stroke. The result of this arrangement is that the expansion curve terminates at a point c (fig. 134) near the end of the stroke, and is succeeded by a rounded corner at c, which is caused by the exhaust being throttled at first by the smallness of the aperture of the port when it commences to open. After this point the curve falls rapidly to the end of the stroke. As the whole of the exhaust steam will probably not have escaped by the end of the stroke, instead of having a vertical line, a'D (fig. 134), bounding the end of the diagram, we shall have a curved line, $a'c'$, showing that the pressure gradually falls till the point of minimum pressure, c', is reached. The character of the exhaust curve, $ca'c'$, cannot be determined beforehand by theoretical considerations. It depends on the size of the exhaust opening, relatively to the size of the cylinder, on the efficiency of the condenser, and on the pressure and condition of the steam as to moisture at the point where the exhaust commences. If the steam contains much suspended moisture, it is more difficult to expel it from the cylinder than when in the dry condition ; and this is one of

those indirect causes of loss due to condensation in the cylinder and to priming, which is to a great extent corrected by the use of a jacket.

The height of the point c' above the line of perfect vacuum fg measures the minimum pressure which opposes the return motion of the piston. This opposing pressure is called the back pressure and, of course, the greater its amount, the greater the loss of efficiency of the engine. No matter how perfect the vacuum in the condenser, it is impossible to attain an equally good vacuum in the cylinder, on account of the resistance opposed to the passage of the exhaust steam by the ports, and the pipe connecting the cylinder with the condenser.

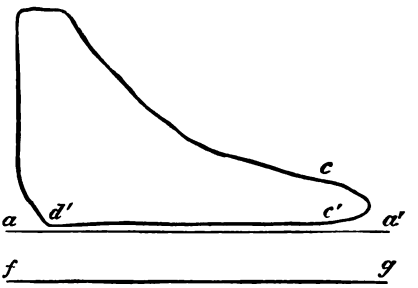

Fig. 135.

If the engine be non-condensing, the back pressure will at the least be equal to the pressure of the atmosphere, and in addition there will be an increment of pressure due to the difficulty of expelling the steam from the cylinder through the ports and exhaust-pipe. In the case of locomotives, where the exhaust passes through a blast-pipe, with a narrow orifice, up the chimney, for the purpose of promoting a draught, the increase of back pressure due to this cause may under certain circumstances (see p. 141) be very considerable. Fig. 135 shows the usual character line of exhaust and of back pressure for a non-condensing engine, the same letters being used as in fig. 134.

Reverting again to fig. 134, after the point c' of minimum back pressure is reached, the pencil will, if the vacuum be maintained uniformly, trace the horizontal line $c'd'$ till a

point d' is reached near the end of the stroke. At this point, if the engine be fitted with a slide valve, worked by an eccentric the exhaust will be closed; for just as the exhaust was made to open before the termination of the forward stroke, so it must close before the termination of the back stroke. The exact position of the point d' relatively to c is explained in the chapter on valve setting (see fig. 109). All the steam remaining in the cylinder and in the clearance spaces when the exhaust opening is closed, will be compressed as the piston advances, and consequently the pressure will rise, and the indicator pencil will describe a curve, which at some point or other, depending on the position of d' and on the pressure of the steam at d', will coalesce with the vertical line EA.

Cushioning, or compression of exhaust steam.—This compression of the steam at the end of the stroke—or *cushioning*, as it is usually called—by raising the mean value of the back pressure, detracts from the effective power of the engine, and would at first sight appear to be a source of waste. It must, however, be remembered that the steam which is compressed by the piston is, at the end of the stroke, in a condition to exert its pressure usefully on the piston when making the next stroke. It, moreover, fills the clearance spaces, which would otherwise have to be filled at each stroke with fresh steam from the boiler, the whole of which, excepting in so far as it would raise the curve of expansion, would be wasted. Consequently, cushioning, though it diminishes the effective power exerted, increases the efficiency, or the ratio of work done to steam expended. Some engineers recommend that the exhaust be closed at such a point, that the compressed steam, by the time the end of the stroke is reached, shall attain the boiler pressure, in which case the curve of compression would terminate at the point A, fig. 134. Cushioning is also very useful in engines having a high speed of piston, inasmuch as it assists in bringing the reciprocating masses, viz. the piston and the piston and

connecting rods, quietly to a state of rest. The lower diagram, fig. 146, is intended to illustrate a very marked curve of compression. It is taken from a locomotive engine working at a high grade of expansion.

We have now seen how the actual diagram in an ordinary steam engine may be expected to differ from the theoretical diagram of fig. 134. We have also seen how the diagram records the following facts and operations, viz.—

1. The exact point of the stroke at which steam is admitted.

2. The initial pressure of the steam in the cylinders, which being compared to the boiler pressure, shows us whether the steam pipes and passages are of the necessary dimensions.

3. The way in which the initial pressure is maintained or otherwise during the period of admission.

4. The point of cut-off.

5. The pressure during the whole period of expansion.

6. The point of release, *i.e.* when the exhaust is opened.

7. The rapidity with which the exhaust takes place, as shown by the nature of the exhaust curve.

8. The minimum back pressure, which in a condensing engine is also the test of the perfection of the vacuum, and in a non-condensing engine shows what the effect of the friction of the exhaust pipes and passages is in addition to the unavoidable pressure due to the atmosphere.

9. The period when the exhaust is closed.

10. The nature of the curve of compression.

11. The power which is being given off by the engine.

Principal causes affecting the forms of diagrams.—We have also seen that the principal causes which disturb the shape of the theoretical diagram are—

1. The friction of the steam pipes and ports.

2. The variable size of the opening of the steam ports as caused by the gradual motion of the slide valve.

3. The action of the sides of the cylinder in causing

condensation and partial re-evaporation of some of the entering steam.

4. The steam contained in the clearance spaces which affects the curve of expansion.

5. The gradual opening of the exhaust port, which makes it necessary to release the steam too early in the stroke.

6. The friction of the exhaust passages, which in the case of condensing engines prevents the attainment in the cylinder of the same degree of vacuum as in the condenser, and in the case of non-condensing engines adds to the back pressure.

7. The momentum of the moving parts, which, combined with cause 4, and also with the unavoidable nature of the simple slide valve driven by an eccentric, renders a curve of compression necessary.

Examples of diagrams from actual engines.—We shall afterwards see that the indicator possesses other important uses in addition to those named above, but it is first proposed to give a few examples of good and bad diagrams, taken from various engines, and to point out some of the peculiarities which they reveal.

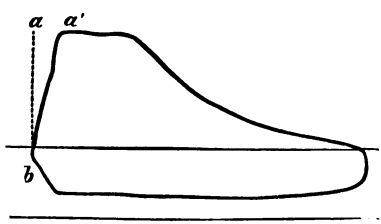

Fig. 136.

The diagram represented by fig. 136 shows a very late admission of steam, the maximum pressure not being attained till the piston has traversed a portion of the stroke represented by *aa'*. The proper position of the line *ba'* is the dotted line *ba*. The fault is evidently due to the valve having been badly set, either through becoming displaced, or else through the eccentric not having been given sufficient advance (see page 255).

Fig. 137 shows that the steam pressure during admission is injuriously affected by throttling, owing to insufficient opening or area of the port.

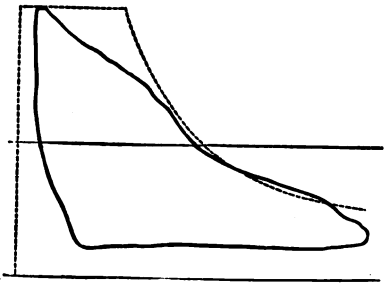

Fig. 137.

Fig. 138 represents a diagram which in addition to numerous other defects shows a very high back pressure. This was due to the fact that the steam, instead of being allowed to escape directly into the atmosphere, was passed first into a feed water heater. With some classes of feed heaters it happens that much more power is lost by the increase in the back pressure than is gained by raising the temperature of the feed.

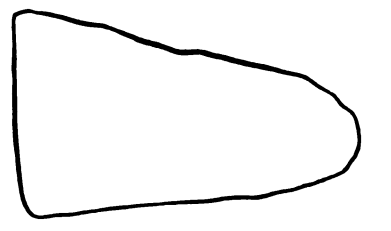

Fig. 138.

How to draw the hyperbolic curve of expansion.—The diagrams (figs. 139 to 142) are given to show the effects of condensation and re-evaporation on the curves of expansion. In order to be able to mark this effect more accurately it is advisable in all cases to lay down on the diagram the hyperbolic curve of expansion, which is the graphic representation of Boyle's law ; for though this curve represents neither the curve of expansion of steam nor the curve of its relative volumes, it is found, nevertheless, that it is the line to which the expansion of steam in the best types of engines most closely approximates, and

is for this reason the best curve to use as a standard of comparison.

In order to draw the curve of expansion for a given diagram, such as fig. 139, erect a perpendicular *ab*, to the line of perfect vacuum *ac*, the distance *aa'* representing the clearance reduced to an equivalent fraction of the stroke. The lines *ab*, *ac*, will be then the asymptotes of the hyperbola; and *ad*, drawn at an angle of 45° with *ab* and *ac*, will be the axis of the curve. We must now select some point in the expansion curve of the diagram from which to commence the hyperbolic curve. This latter will in general vary for every point which we may choose, for if there be condensation at the commencement and re-evaporation towards the end of the stroke, it is evident that there may be a higher pressure of steam in the cylinder at the end of the stroke than there should be if the true curve of saturated steam expanding and doing work were followed. It is usual to choose a point either at the commencement of the curve, such as *e* (fig. 139), when the steam port has been completely closed, or else a point *f*, just before the exhaust is opened. The co-ordinates, *eg* and *ek*, of the point *e* (or of *f* if we select the latter point) must then be measured and multiplied together. The points *e'*, *e''*, *e'''*, &c., corresponding to the positions of the piston, *g'*, *g''*, *g'''*, are such that the products of their co-ordinates equal the product of the co-ordinates of the original point, *e*.

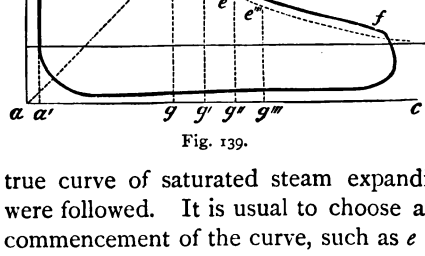

Fig. 139.

Thus $e'g' \times g'a = eg \times ga$. $\therefore e'g' = \frac{eg \times ga}{g'a}$.

Initial condensation and re-evaporation shown by diagrams.— In the diagram, fig. 139, we see that the actual

expansion curve, *ef*, of the steam lies throughout its whole length above the hyperbola line, showing a considerable re-evaporation of water, which has either been formed by condensation at the commencement of the stroke or carried over in the form of spray from the boiler.

Fig. 140 shows the same effect in a still more marked degree. In this case it was ascertained that a large quantity of water was carried over into the cylinder from the boiler, which was partially re-evaporated by the end of the stroke. That the water was not wholly re-evaporated, before the return stroke commenced, is shown by the bad vacuum line. The effect of water in the cylinder in increasing the back pressure is most marked, and is, no doubt, in part due to the re-evaporation which goes on during the exhaust, when the diminished pressure must enable considerable quantities of the highly heated water to burst into steam.

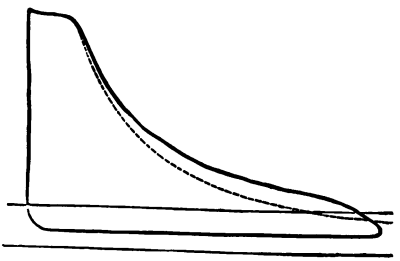

Fig. 140.

The diagram on fig. 141 is given to illustrate the case of condensation at the commencement and re-evaporation at the end of the stroke. Here, it will be noticed, the expansion curve falls below the hyperbola at the commencement, then crosses it, and for the remainder of the stroke lies above it.

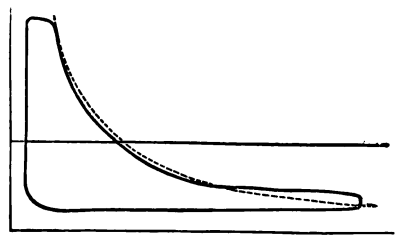

Fig. 141.

Leaky pistons and slide valves.—In engines which have been long at work, the expansion curve may be injuriously affected by leakage of steam through the valve or piston. The best method of ascertaining whether this is going on is to block the fly-wheel at any point in the stroke while the admission port is open, then to admit steam, and to open the lubricating cock at the other end of the cylinder. If steam continues to pour steadily forth from the cock, it shows that there is a leak. By blocking the fly-wheel when the valve is at mid-stroke, and consequently covering both ports, and then opening the lubricating cock, or looking at the mouth of the exhaust pipe, we can ascertain whether the leakage is through the valve.

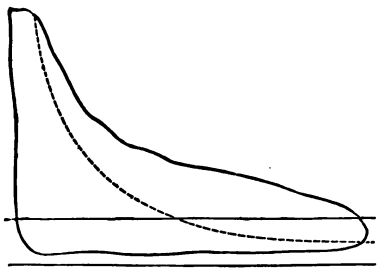

Fig. 142.

Fig. 142 is the diagram of an engine in which a very considerable leak took place past the valve. The initial pressure of steam in the cylinder was about 67 lbs. absolute, and the cut-off was supposed to take place at about one-tenth of the stroke. Hence the pressure at release should have been about seven pounds absolute, whereas it is shown by the indicator to have been nearly twenty pounds.

The diagrams on fig. 143 are intended to show differences of the exhaust line. In No. 1 the exhaust port is opened at *a* before the end of the stroke, and by the end of the stroke the steam pressure has fallen very low. In No. 2, which was taken from the same engine as No. 1, but with the eccentric badly set, so as to cause a late admission of the steam, the exhaust is not opened till just before the very end of the stroke, and the terminal pressure, is much higher

than in the case of No. 1, although the initial pressure is less, and the rate of expansion the same. The vacuum line is inferior to that of No. 1 at the commencement, as the steam has not had time to escape before the return stroke begins. It very often happens that what is gained by postponing the release till the stroke is finished is lost again through the increase in the back pressure. The terminal pressures are, however, very different in the two cases, and consequently, also, the twisting moments on the crank towards the end of the stroke.

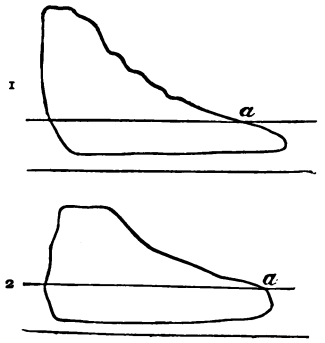

Fig. 143.

Fig. 144 is taken from a cylinder provided with exhaust valves and ports quite independent of the steam valves and ports. As will be seen from the sharp corner at *a* and the sudden fall of the pressure, the exhaust port opens sharply and fully, thus allowing the steam to escape very readily.

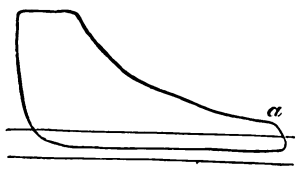

Fig. 144.

The diagram, fig. 145, shows as bad a distribution of the

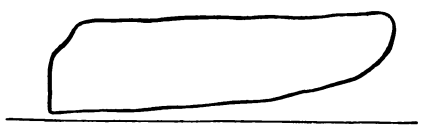
Fig. 145.

steam as it is possible to conceive. The valve is deficient both in lap and lead, consequently the admission of the

steam is late; the engine works with full steam during the whole stroke; the port is opened so gradually that the full pressure is not attained till the end of the stroke. The exhaust opens so late and so gradually that the pressure of the exhaust steam had not fallen to its proper point till nearly the end of the return stroke.

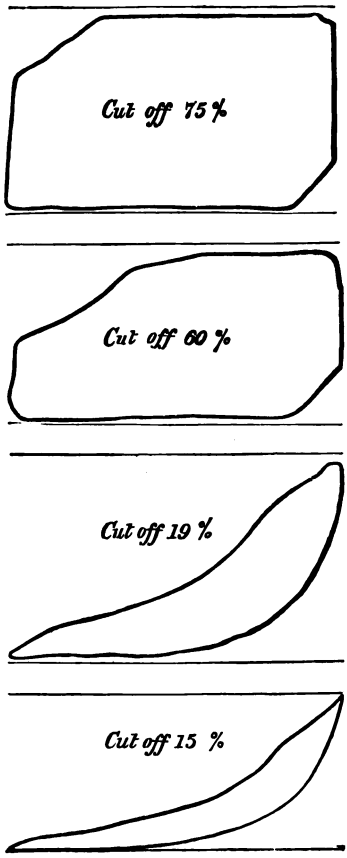

Fig. 146.

The diagrams on fig. 146 have reference to the line of compression. In an engine with a single slide valve to regulate the admission and exhaust of the steam from each end of the cylinder, the point of the stroke where the exhaust closes is dependent on the point where it opens — *i.e.* the point of release; and in proportion as the latter occurs early in the forward stroke, so will the former take place early during the return stroke. This effect is more particularly apparent in the diagrams of locomotives which are driven by a single slide, the rate of expansion being varied by means of the link motion. Nos. 1, 2, 3, and 4 are taken from the same loco-

motive engine, working at different rates of expansion, and with varying points of release and compression. In Nos. 3 and 4, the exhaust is closed so early that the curve of compression rises to a great height before the end of the return stroke, so much so that the back pressure at the end of the return stroke rises nearly to the initial pressure at the commencement of the forward stroke. The advantage of compressing the exhaust steam has already been explained.

It would be impossible to give in this chapter examples of all the peculiarities which may arise in diagrams from defective valve setting, or leaky valves and pistons, priming boilers, and unjacketed cylinders; but, as the nature of the best attainable diagram has been explained, and also the principal faults and peculiarities which occur in ordinary engines, enough has been said to enable the student to investigate the condition of most engines from an inspection of their indicator diagrams.

Gross and net indicated power.—The indicator diagram gives us, as we have seen, an exact account of the working of the engine and of the power which is being exerted and which is available for transmission, either to the working parts of the engine proper or to some external train of mechanism. It also gives the total power which is being exerted by the engine which includes the power which is being thrown away in overcoming back pressure. Take, for instance, the diagram, fig. 147, which is taken from a high-pressure expansive engine, the line AE being the line of absolute vacuum. It is evident that during the forward stroke the work done by

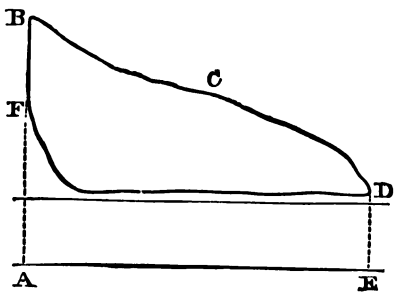

Fig. 147.

the steam on the piston is equal to the area ABCDE. During the back stroke the return of the piston is opposed by the pressure of the atmosphere and of the exhaust steam, on which it does work, measured by the area DEAF, so that the useful work available for transmission to the parts of the engine and external objects is represented by the difference between these two figures—*i.e.* BCDF.

The work represented by ABCDE is called the gross indicated power, and BCDF is called the net indicated power. This latter, again, is divisible into two parts, one being the work necessary to overcome the friction of the moving parts of the engine, and the other being the remainder, which is called the useful power, and which is all that is available for doing external work.

The above explanation will render clear the reason why the economy due to expansion in non-condensing engines is so very limited. For instance, taking fig. 148, it is evident that the power thrown away in overcoming the back pressure is about forty per cent. of the total power exerted. If a still greater degree of expansion were used, the gross power would be diminished, while the power wasted would remain the same, and the comparison between the useful and the gross power would be still more disadvantageous. Of course, condensation removes this evil to a great extent, but in condensing engines the theoretical gain due to high rates of expansion is also limited by causes which are explained in Chapter XI.

How to deduce from indicator diagrams the effective pressure on the piston.—The indicator diagram, though it shows the pressure at every point of the stroke, does not show the pressure which is available for transmission through the piston and connecting rods to the crank of the engine. This pressure is the difference between the total pressure, as shown by the diagram, on the driving side of the piston, and the simultaneous back pressure on the other side of the piston, which latter can only be obtained by taking a sepa-

rate diagram from the other end of the cylinder. In well-designed horizontal engines the diagrams from the two cylinder ends should in most cases be as nearly as possible alike, but in many engines, especially those in which the valve setting is defective, or in which the connecting rod is short compared to the length of the crank, the diagrams differ from each other very considerably, and consequently a pair of diagrams is often absolutely essential to enable us to compute the net pressure transmitted by the piston.

In order to obtain the pressure available for transmission externally at any point of the stroke we must construct a new diagram, which shall show the differences between the simultaneous forward and back pressures on the two sides of the piston.

Let ABCD, A′B′C′D′, fig. 148, represent diagrams taken from the opposite ends of a cylinder. The diagrams are arranged so as to overlap, the point D′ marking the end of the forward stroke, being in the same vertical straight line as the point A, which marks the commencement of the back stroke. This arrangement is convenient, as it enables the simultaneous forward and back pressures to be seen at a glance. Thus at the commencement of the forward stroke the pressure on the working face of the piston is measured by the distance of the point A from the line of absolute vacuum GG′. At the same moment the other side of the piston is acted on by a back pressure measured by

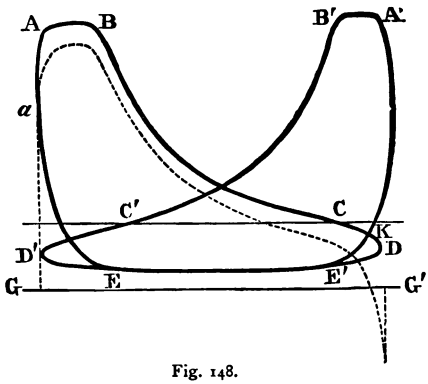

Fig. 148.

the distance of the point D′ from gg', and therefore the effective pressure is the difference between these two, and is measured by the line AD′. When the piston has reached the position K, being the place where the exhaust line of one diagram crosses the line of compression of the other diagram, the two pressures are equal, and consequently the piston is urged neither forwards nor backwards by the steam, and continues its forward motion only by reason of the energy stored up in the moving parts of the engine and the fly-wheel. From the point K to the end of the stroke D, the pressure urging the piston forward is actually less than the back pressure, which latter consequently tends to bring the piston to a state of rest.

The true diagram of resultant pressure on the piston is formed by setting off the differences between the forward and back pressures as ordinates, measuring from the line of absolute vacuum as a base and drawing a curve through the ends of the ordinates. Whenever the forward pressure is in excess, the ordinate is drawn above the line GG′, and whenever the back pressure is in excess it is drawn below. We thus get the dotted curve of fig. 148 which is reproduced for the sake of clearness as a separate diagram in the curve D′AB&DA′, fig. 149, which shows the resultant pressure on the piston at any point of the stroke.

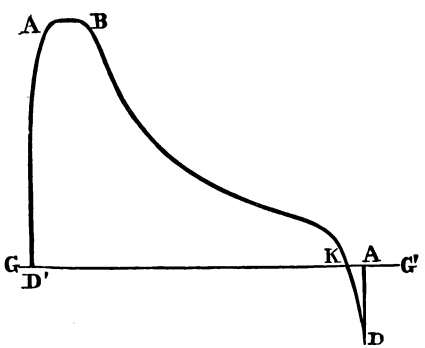

Fig. 149.

We are enabled by this method to see what important modifications of the resultant diagram may be caused by

alterations in the positions of the points of release C and compression E, fig. 148. If the release occurred at the end of the stroke instead of at the point C, the pressure between the points C and D would be greater than that shown in the figure. On the other hand, the vacuum at the commencement of the return stroke would not be so good, and consequently the back pressure between the points D and E′ would be greater than is represented. Moreover, if the steam distribution were controlled by an ordinary slide valve the points of release and of compression would be interdependent, and in the case under consideration there would be no appreciable compression. The general result would be that the positive pressure at the commencement and the negative pressure at the end of the stroke would each be diminished, and consequently the pressure on the piston would fluctuate between less wide limits.

The resultant diagram will be found of great use when we require to calculate the twisting moment on the crank-pin throughout a revolution (see Chapter V.).

How to ascertain the expenditure of steam accounted for by the diagram.—Another and most important use of the indicator diagram is to enable us to account for the expenditure of the steam and the heat supplied from the boiler to the engine. In order to measure, from the diagram, the steam consumed, we must ascertain the pressure at a point *a*, fig. 150, just before the release takes place. By reference to Col. 5 of Table I. we can deduce the weight of a cubic foot of steam of this pressure. We

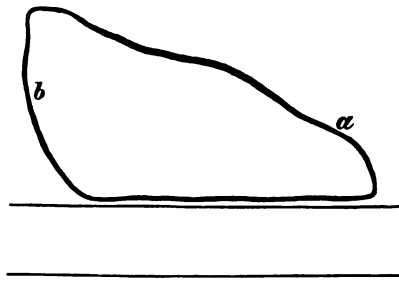

Fig. 150.

must ascertain the cubic contents of the cylinder, including the clearance, in cubic feet, up to the point *a*, and multiplying this number by the weight of the cubic foot of steam, we obtain the weight of steam present in the cylinder immediately before the release. When there is any compression, we must deduct the quantity of steam saved by the early closing of the exhaust. To do this we have only to measure the pressure at any point *b*, after the exhaust has closed, and to ascertain the weight of the contents of the cylinder up to the point *b*. The difference between the steam spent and saved is the quantity accounted for by the indicator. By comparing the quantity thus accounted for in an hour with the weight of water which leaves the boiler in the same time we can ascertain how much of it is lost by the combined effects of priming, condensation in the pipes, and condensation in the cylinder itself.

If we ascertain the pressure and weight of the steam contained in the cylinder at a point immediately after the admission is closed, we can, by the help of Table I., ascertain the number of thermal units contained in the steam at that point of the stroke. This number of thermal units will always be less than the total heat which has been supplied to the engine up to that point, for a certain number will have been expended in restoring the temperature of the ends of the cylinder and piston. Also during the expansion the steam loses the heat which has been converted into mechanical work, and consequently, if we were to measure the heat contained in the steam at any point, say *a*, fig. 150, before the release takes place, we should expect to find that the number of thermal units was less than that contained in the steam at the point of cut-off by the number converted into mechanical work As a matter of fact, however, the number is never less, and is often considerably greater, thus showing that a great deal of the heat which passes from the steam to

the sides of the cylinder during the early part of the stroke is, during the remainder of the stroke, re-transmitted from the cylinder to the steam, and passes out with the exhaust and is partly wasted.

The diagrams of compound engines will be considered in Chapter XI.

CHAPTER IX.

FUEL—COMBUSTION—THE GENERATION OF STEAM—BOILERS AND THEIR FITTINGS.

Combustion—Combination of oxygen with carbon—Combination of oxygen with compounds of carbon and hydrogen—Chemical symbols and atomic weights of constituents of fuel—Principal compounds of carbon, hydrogen, and oxygen—Constituents of fuel—Heat of combustion of carbon and hydrogen with oxygen—Description of fuels in common use—Table of the chemical constituents and evaporative power of various fuels—Weight and temperature of the products of combustion—Waste of fuel by splintering, distillation, insufficient air supply—Smoke forming—Draught creation, radiation, and conduction—Conduction of heat through the plates of furnaces—Importance of preventing an over-supply of air to fuel—The various types of boilers—Cylindrical boiler with external firing—Cornish boiler—Tubulous boilers—Lancashire boiler—Galloway tubes—The stiffening of internal flues—Locomotive boilers—Marine boilers for low-pressure steam—Marine boilers for high-pressure steam—Proportions of parts of boilers—Firegrate area—Evaporative power of fuel in various types of boilers—Consumption of fuel per square foot of firegrate area—Efficiency of heating surface—Cubic capacities of boilers of different types—Steam room—Strength of boilers—Hollow cylinder pressed from within: 1st, longitudinal strain; 2nd, transverse strain—Strength of riveted joints—Hollow cylinder pressed from without—Flat stayed surfaces—Effects of unequal heating in straining boilers—Materials of construction—Boiler fittings—Safety valves—Pressure gauges—Feed pumps—Injectors—Water gauges.

WE have hitherto considered chiefly the nature and the laws of heat, and the details of the engine which is employed to convert the heat into mechanical work; but of the source of heat—viz. the fuel, and the medium by means of which it is conveyed to the engine, viz. water, and the apparatus by which the heat of the fuel is transferred to the water, viz. the boiler—we have up till now said but little.

The source of heat which is always employed is fuel, such as coal, wood, peat, or mineral oil, the principal calorific

constituents of which are carbon and hydrogen. The chemical combination of these elements with oxygen produces intense heat, which, for the purposes of the steam-engine, is transmitted to the water contained in the boiler in a manner to be hereafter described.

The amount of heat which can be generated by the chemical combination of fuel and oxygen, commonly called combustion, depends upon the relative proportions of carbon and hydrogen which the fuel contains, as well as on the amount of oxygen which is supplied to it. It is well known that chemical elements combine with each other in certain definite proportions only; that is to say, for instance, a certain definite weight of carbon—viz. twelve units—will combine with a certain other definite weight of oxygen—viz. sixteen units—to form the compound called carbonic oxide, and these two elements will only combine in these proportions or in certain simple multiples of them. Thus, for example, no chemical combinations can be formed out of seven parts by weight of carbon to five parts by weight of oxygen; but, on the other hand, twelve parts of carbon will combine with twice sixteen parts of oxygen, forming the compound called carbonic acid or carbonic anhydride, which is the term applied to the product of the complete combustion of carbon in oxygen.

The numbers 12 and 16 applied to carbon and oxygen are called the atomic weights of these two substances, and it is supposed that the ultimate atoms of which they are composed have to each other the relative weights of 12 to 16. These numbers are also called the *chemical equivalents* of the substances.

If, instead of pure elementary substances, such as carbon and oxygen, we had to deal with the combination of a compound with a simple element, we should find that the chemical equivalent of the compound would be the sum of the equivalents of its constituents; thus, for example, a pound of olefiant gas, which is composed of carbon and

hydrogen in the proportion of six parts by weight of carbon to one of hydrogen, would, in order to effect its combustion, require a quantity of oxygen computed as follows. The carbon weighs $\frac{6}{7}$ of a pound, and would require $\frac{1}{12}^{6} \times 2$ of its own weight of oxygen in order to form carbonic acid. The hydrogen weighs $\frac{1}{7}$ of a pound ; its chemical equivalent or atomic weight is 1, and it combines with oxygen in the proportion of two parts by weight to sixteen of the oxygen. That is to say, the weight of oxygen required is $\frac{1}{7} \times \frac{1}{2}^{6}$ of a pound. Consequently for the pound of olefiant gas we shall want $(\frac{6}{7} \times \frac{1}{12}^{6} \times 2) + (\frac{1}{7} \times \frac{1}{2}^{6}) = 3\frac{3}{7}$ lbs. of oxygen.

The following are the chemical symbols and atomic weights of the principal elementary constituents of fuel.

Carbon . . .	C	. . 12
Hydrogen . .	H	. . 1
Oxygen . . .	O	. . 16

The following are the principal chemical combinations of the foregoing.

Name	Chemical composition	Chemical symbol
Carbonic oxide	Formed of carbon and oxygen, in the proportion of twelve parts by weight of the former to sixteen of the latter, or $C_{12} + O_{16}$.	CO
Carbonic acid or carbonic anhydride.	Formed of carbon and oxygen, in the proportion of twelve parts by weight of the former to thirty-two of the latter, or $C_{12} + O_{32}$.	CO_2

The above are the products of the combustion of carbon in oxygen, the former being the result of imperfect, the latter of perfect combustion.

Name	Chemical composition	Chemical symbol
Water . .	Formed of hydrogen and oxygen, in the proportion of two parts by weight of the former to sixteen of the latter, or $H_2 + O_{16}$.	H_2O

The above is the product of the combustion of hydrogen in oxygen.

Name	Chemical composition	Chemical symbol
Olefiant gas .	Formed of carbon and hydrogen, in the proportion of twelve parts by weight of carbon two of hydrogen, or $C_{12} + H_2$.	CH_2
Marsh gas .	Formed of carbon and hydrogen, in the proportion of twelve parts by weight of carbon to four of hydrogen, or $C_{12} + H_4$.	CH_4

The two above are gaseous forms of the large family of hydrocarbons, which exist largely in fuel in the solid and liquid states also. To this category belong the vegetable and mineral oils, animal fats, and the bituminous portions of coal.

Fuel.—Ordinary fuel is composed chiefly of carbon, hydrogen, oxygen, and mineral matters, or of chemical combinations of these three elements. Its heating power, as before mentioned, depends on the relative proportions of the two first elements, and on the manner in which they are supplied with oxygen.

The heat evolved by a pound of hydrogen when burnt with oxygen so as to form water is 62,032 thermal units, which is sufficient to evaporate 64·2 lbs. of water from and at 212°. It requires for its combustion 8 lbs. of oxygen.

The heat evolved by a pound of carbon when burnt completely is 14,500 thermal units, which is sufficient to evaporate 15 lbs. of water from and at 212°. The amount of oxygen required to effect the combustion is $2\frac{2}{3}$ lbs.

When imperfectly burned so as to form carbonic oxide the quantity of heat evolved is only 4,400 units, equivalent to an evaporative power of 4·55 lbs. of water from and at 212°. The amount of oxygen required is $1\frac{1}{3}$ lbs.

In every case the oxygen is obtained from the air, which is a mechanical mixture of nitrogen and oxygen in the

proportion of 77 parts by weight of the former to 23 of the latter. The nitrogen plays no part in the combustion, except that it mingles with the products of combustion, and reduces their temperature.

It will be noticed that the heat of combustion of hydrogen is about four times as great as that of carbon, and consequently those fuels which contain a relatively large quantity of hydrogen, such as the hydrocarbons, possess the greatest evaporative power; for the heating power of a compound of these two elements is *in most cases* nearly equal to the sum of the heating powers of the constituents.

In making an exact estimate of the calorific value of fuel, it is necessary to take account of the heat lost by breaking up any existing chemical compounds; for, just as the chemical union of carbon and oxygen, or of carbon and hydrogen, produces heat, so the separation of, say carbon from hydrogen, requires the expenditure of heat. If, then, a fuel contains hydrocarbons, which, during the combustion, are broken up into their constituent proportions of carbon and hydrogen, and each of these latter then combined with oxygen so as to form carbonic acid and water, we must not calculate the heat produced by the combination as being the same as if equal quantities of *free* carbon and *free* hydrogen were so combined. The proper way to proceed is to calculate first the heat that would be produced supposing the substances were all originally in the free or uncombined state, and then to subtract from this quantity the heat required in order to dissociate the hydrogen from the carbon. The latter quantity is always equal to the heat which would be produced by the combination of the equivalent quantities of free carbon and hydrogen. Thus, for example, one pound of marsh gas consists of three-quarters of a pound of carbon and one-quarter of a pound of hydrogen. If these constituents were in the uncombined state their combustion with oxygen would yield the following quantities of heat :—

Carbon, $\frac{3}{4}$ lb. . .	$14,500 \times \frac{3}{4} = 10,875$	thermal units.
Hydrogen, $\frac{1}{4}$ lb. . .	$62,032 \times \frac{1}{4} = 15,508$	,,
Total . . .	$= 26,383$	,,

But experiments prove that the heat developed by the combustion of one pound of marsh gas in oxygen is only 23,582 thermal units, thus leaving a deficiency of 2,801 units due to the heat absorbed in splitting up the chemical compound of carbon and hydrogen. In those compounds of carbon and hydrogen in which only two equivalents of hydrogen are combined with one of carbon, the heat of combination is so little that it is not necessary to take account of it in computing theoretically the calorific value of fuel. When a fuel contains oxygen, in addition to carbon and hydrogen, it is found that so much of the hydrogen as would be required to combine with the oxygen present in order to form water, must be left out of account in calculating the calorific effect. Any excess of hydrogen above this quantity must, however, be taken into consideration.

The fuels in most common use are coal, coke, peat, wood, and in some countries, such as South Russia, mineral oils. Of these coal is by far the most important; it is therefore the only kind of fuel which will be considered in detail in this chapter.

There are numerous varieties of coal found in this country, which differ from each other in appearance, in chemical constitution, and in their behaviour when undergoing combustion. Of these the principal varieties are anthracite, dry bituminous, and caking bituminous coals. Anthracite is found chiefly in South Wales. Chemically it consists almost entirely of pure carbon. It burns without flame or smoke. It is a very difficult coal to ignite, and unless gradually heated it splits up, when thrown on the fire, into small pieces.

Dry bituminous coal is the most useful fuel for steam generation. It consists chemically of carbon, hydrogen,

oxygen, and mineral matter which forms ash. It is lighter than anthracite, and burns easily with very little smoke.

Caking bituminous coal contains less carbon than the foregoing, and more hydrogen and oxygen. It is called caking because it softens when exposed to heat. It burns easily with a good deal of smoke. The following table[1] gives the chemical composition and theoretical heating power of various kinds of coal. The theoretical heating power is calculated in the manner already explained. The practical heating power differs very considerably from the

Name of fuel	Chemical constituents			Heat of combustion of fuel in thermal units	Evaporative power of one pound of the fuel in pounds of water from and at 212°
	C	H	O		
I. Charcoal from wood	0·93	—	—	13,500	14
,, peat.	—	—	—	11,600	12
II. Coke, good . .	0·94	—	—	13,620	14
,, middling .	0·88	—	—	12,760	13·2
,, bad . .	0·82	—	—	11,890	12·3
III. Coal:					
1. Anthracite . .	0·915	0·035	0·026	15,225	15·75
2. Dry bituminous .	0·90	0·04	0·02	15,370	15·9
3. ,, .	0·87	0·04	0·03	14,860	15·4
4. ,, .	0·80	0·054	0·016	14,790	15·3
5. ,, .	0·77	0·05	0·06	13,775	14·25
6. Caking . .	0·88	0·052	0·054	15,837	16·0
7. ,, . . .	0·81	0·052	0·04	14,645	15·15
8. Cannel. . .	0·84	0·056	0·08	15,080	15·6
9. Dry long flaming .	0·77	0·052	0·15	13,195	13·65
10. Lignite . .	0·70	0·05	0·20	11,745	12·15
IV. Peat, dry . .	0·58	0·06	0·31	9,660	10·0
Peat containing 25 per cent. moisture	—	—	—	7,000	7·25
V. Wood, dry . .	0·50	—	—	7,245	7·5
Wood containing 20 per cent. moisture	—	—	—	5,600	5·8
VI. Mineral oil from .	0·84	0·16	0·	21,930	22·7
,, to .	0·85	0·15	0·	21,735	22·5

[1] Taken from the *Journal of the Royal United Service Institution.* Vol. XI.

theoretical, and depends chiefly on the stoking, and on the furnace being suitably made to develope complete combustion. The proper method of designing furnaces will be explained later on, but it may here be mentioned that with the best of fuel and the most suitable of furnaces it is possible by bad stoking to obtain the most indifferent results. For instance, if the fuel is laid on in such a manner that air in sufficient quantities cannot reach it, the coal will be partly distilled instead of being burnt; the more volatile constituents, such as the hydrocarbons, will be driven off in the shape of unburnt gas, and a large proportion of the carbon proper will burn incompletely, forming carbonic oxide instead of carbonic acid, its heating power being thus reduced by more than 70 per cent.

The heating powers in the above table are calculated on the supposition that one pound of pure carbon is capable of evaporating fifteen pounds of water from 212°. This is an experimental result arrived at by chemists, and is greatly in excess of anything that has yet been realised in steam boilers.

Weight and temperature of the products of combustion.—The temperature of the products of combustion of fuel depends upon their weight and specific heat. The weight of the products of combustion depends upon the quantity of air which is supplied to the fuel. As stated above, one pound of carbon requires for its perfect combustion two and two-thirds pounds of oxygen, or about twelve pounds of air. When imperfectly burned it requires one and one-third pounds of oxygen, or six pounds of air. One pound of hydrogen gas consumes eight pounds of oxygen, or thirty-six pounds of air.

It is found, however, that in practice more air than the above quantities must be supplied to the fuel in order to effect complete combustion. The extra quantity required depends upon the nature of the draught. Thus when the draught is produced by a chimney it is usual to estimate

that twice the theoretical quantity is required, *i.e.* twenty-four pounds of air per pound of carbon. When the draught is artificial, such as that produced by a blower, or by a fan, or by the blast-pipe, one and a half times the theoretical quantity, or eighteen pounds of air per pound of carbon, is usually required. Although common coal is a complicated mixture of carbon, hydrogen, and oxygen, no serious error will be committed by estimating the quantity of air required for its combustion on the supposition that it is pure carbon.

From the above it will be evident that, even with the same fuel, the temperature of the products of combustion will vary according to the nature of the draught. Thus taking, again, the case of pure carbon, burnt under an artificial blast, and, therefore, requiring eighteen pounds of air per pound of fuel, we have the total weight of the products of combustion $=18+1=19$ pounds. The total heat of combustion is, as stated above, 14,500 units. The mean specific heat of the products is, according to Rankine, ·237 for constant pressure, and the temperature is found by dividing the total heat of combustion by the weight multiplied by the specific heat. Thus, in the present instance, the temperature $=\frac{14500}{19\times\cdot237}$

If the draught were produced by means of a chimney, so that twenty-four pounds of air would be required, instead of eighteen, the temperature would only be $\frac{14500}{25\times\cdot237}=2{,}440°$. On the other hand, if it were possible to burn the fuel completely with only the theoretical quantity of air necessary, viz. twelve pounds, the weight of the products of combustion would be only thirteen pounds, and the temperature $\frac{14500}{13\times\cdot237}=4{,}580°$, or very nearly double the temperature which is usually obtained in practice. It will be seen presently, when considering the waste of fuel in the

gaseous state, that this question of initial temperature and weight of the products of combustion assumes an important aspect.

In practice it is found that a pound of coal falls very far short of the evaporative power stated in the table. The reasons for this are twofold. First, the fuel is wasted in various ways, which will presently be enumerated; and second, the boiler is unable to abstract from the fuel all the heat which actually is generated and to convey it to the water, so the residue passes up the chimney unutilised.

Waste of fuel.—The ways in which fuel is wasted are various. Many kinds of coal, such as anthracite and dry steam coal, are extremely brittle when exposed suddenly to great heat, and small splinters are broken off which fall through between the bars of the grate into the ashpit. In the majority of cases, however, the great waste takes place not so much in the solid as in the gaseous state. In an ordinary coal fire, kindled from below, the upper layers of fuel are heated through long before they become incandescent. When thus warmed, the coal is partially distilled, instead of being burnt, and many of its most valuable constituents are driven off in a gaseous state, and escape up the chimney unburnt, unless special provision is made to mingle fresh air with the gases as they arise, and to burn them, as it were, above the fuel.

An insufficient supply of air to the fuel itself is often a source of very great waste. It has been stated before that if only enough oxygen be present to burn the carbon into carbonic oxide, instead of into carbonic acid, the units of heat so generated will be only 4,400 per pound of carbon, instead of 14,500. Carbonic oxide is a perfectly colourless gas, and its formation in very large quantities may easily escape detection. If mingled, however, with a sufficient supply of fresh air, and suitably ignited, it will burn into carbonic acid, and in so doing will give out the missing 10,100 units of heat.

The formation of smoke is also a most fruitful, as it is one of the most common sources of waste. Smoke is pure unburnt carbon in a finely divided state which floats about in the hot gases and air proceeding from the fire, so that whenever we see dense volumes of smoke escaping from a chimney, we know that it represents so much valuable fuel absolutely thrown away, beyond the reach of recovery. Smoke when once formed is extremely difficult to ignite, and the greatest art of good firing consists in its prevention. Coals which are rich in hydrocarbons are also the most fruitful smoke producers. It is supposed that these volatile hydrocarbons when driven off at a considerable temperature, in the manner described, evolve free carbon before they are mingled with the air above the fuel, and becoming cooled down by contact with the air, the suspended particles of carbon show themselves in the form of smoke. Many arrangements have been contrived for mingling fresh air with the gases arising from the fuel in order to effect their combustion. Some of these will be referred to hereafter, when the practical details of boilers come under consideration.

Fuel is often largely wasted in forming the draught which feeds the furnaces with air. The draught is produced either by means of a chimney or by some more artificial blower, such as the steam blast-pipe or the revolving fan. In the case of a chimney it is found that the best temperature for the ascending gases is about 600°, whereas the temperature of the fire is about 2,440° above that of the outside air. Consequently about one-fourth of the total heat of combustion is wasted in forming the draught. Hence it appears that a chimney is a most wasteful expedient, for it involves the necessity of supplying the fuel with a double allowance of air, and in order to maintain the draught efficiently it carries off this air at a high temperature. With a blast-pipe or fan it is not necessary, as far as the draught is concerned, that the escaping gases should have any higher temperature than that of the

atmosphere, and, moreover, the quantity of air which must be supplied to the fuel is one fourth less than when a chimney is employed.

Radiation and conduction are usually set down among the causes of waste of heat, but when the fire is properly enclosed, and the boiler surrounded with non-conducting materials, losses from this cause may be rendered extremely small.

The inability of the boiler to abstract all the heat which the fuel gives out is a consequence of the nature of the conduction of heat through the plates which separate the water in the boiler from the fire. The rate at which conduction takes place between the plates depends, first, upon the difference in temperature between the two sides of the plate ; the greater the difference the quicker being the rate of conduction ; second, upon the conductivity of the metal which forms the plate ; and third, upon the thickness of the plate.

As regards the difference in temperature between the sides of the plate, it is evident that when the temperature on each side is the same no conduction of heat can take place. If, for example, a boiler be used to form steam of 100 lbs. pressure to the square inch, the temperature of this steam and of the water from which it is formed is 337·5° ; consequently the hot gases coming from the fire cannot be cooled down below this point, and must at the least escape up the chimney at this temperature ; and therefore all the heat due to the difference between this temperature and that of the atmosphere is of necessity wasted. As a matter of fact, it is impossible to retain the hot gases long enough in contact with the plates to allow of their temperature dropping to that of the steam, and consequently the waste from this source is considerably greater than what has been stated above. This evil may be reduced, to a certain extent, by introducing the comparatively cold feed water at that part of the boiler where the gases are coldest, an arrangement which is always carried out in carefully designed boilers.

From the above it will be readily understood how important it is to reduce the supply of air to the fuel to the minimum which is consistent with perfect combustion. Any excess quantity of air, in the first place, reduces the temperature within the furnace, and thus diminishes the rate of conduction through the heating surface; and, in the next place, it augments the bulk of the gaseous products of combustion, and thus makes it more difficult than it otherwise would be for the heating surface to reduce the temperature of the products to that of the steam and water within the boiler; for it is self-evident that a given area of heating surface is more efficient in abstracting the heat from a small than from a large bulk of gases.

Description of various Types of Boilers.

The essential parts of all boilers are as follows:—

A furnace, which contains the fuel to be burnt; a water receptacle, which contains the water to be evaporated; a steam space to hold the steam when generated; heating surface to transmit the heat from the burning fuel to the water; a chimney, or other apparatus to cause a draught to the furnace and to carry away the products of combustion; and various fittings for supplying the boiler with water, for carrying away the steam, when formed, to the engine in which it is used; for allowing steam to escape into the open air when it forms faster than it can be used; for ascertaining the quantity of water in the boiler; for ascertaining the pressure of the steam, &c.

The forms of steam generators are most numerous, and depend chiefly upon the purposes for which they are required. They may all be divided into three principal categories; viz. stationary, locomotive, and marine boilers.

In stationary boilers the size and weight are of secondary consideration, whereas for locomotive purposes, they are paramount.

The great majority of stationary boliers are cylindrical in shape, because the cylinder is the best practical form for strength as against internal pressure; the ends are either flat or else segments of spheres. It would be impossible within the limits of this short work to describe all the varieties of stationary boilers which have been contrived in various countries, but a few representative types will be considered.

Stationary Land Boilers.

Cylindrical boiler with external firing.—The simplest of all steam generators, and one which is now but seldom used

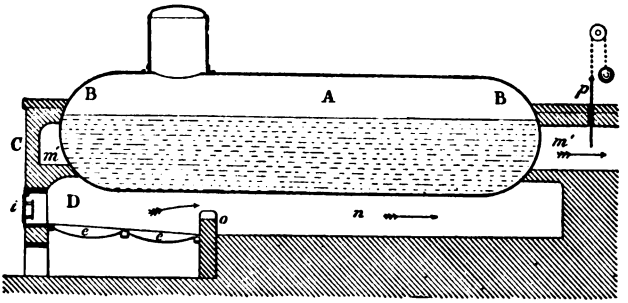

Fig. 151.

in this country, is illustrated in longitudinal and transverse section in figs. 151, 152. It consists of a cylinder A, formed of iron plate with hemispherical ends BB, set horizontally in brickwork C. The lower part of this cylinder contains the water, the upper part the steam. The furnace D is external to the cylinder, being underneath one end. It consists simply of a series

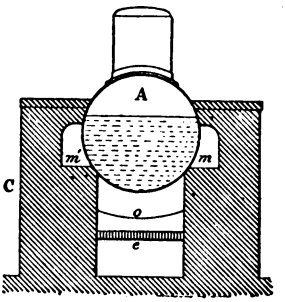

Fig. 152.

of grate bars *e e* set in the brickwork at a convenient distance below the bottom of the boiler.

The sides and front of the furnace are walls of brickwork, which, being continued upwards, support the end of the cylinder. The fuel is thrown on to the bars through the furnace door *i*, which is set in the front brickwork. The air enters between the grate bars from below. The portion beneath the bars is called the ashpit. The flame and hot gases, when formed, first impinge on the bottom of the boiler, and are then carried forward by the draught to the so-called bridge *o*, which is a projecting piece of brickwork, which contracts the area of the flue *n*, and forces all the products of combustion to keep close to the bottom of the boiler. Thence the gases pass along the flue *n*, and return past one side of the cylinder in the flue *m* (fig. 152), and back again by the other side flue *m'* to the far end of the boiler, whence they escape up the chimney. This latter is provided with a door or damper *p*, which can be closed or opened at will, so as to regulate the draught.

This boiler has two great defects. The first is that the area of heating surface, which is represented by those portions of the flues which are bounded by the surface of the cylindrical shell, is too small in proportion to the bulk of the boiler. The second is that if the water contains solid matter in solution, as nearly all water does to a greater or less extent, this matter becomes deposited on the bottom of the boiler, just where the greatest evaporation takes place. The deposit, being a non-conductor, prevents the heat of the fuel from reaching the water in sufficient quantities, thus rendering the heating surface inefficient; and further, by preventing the heat from escaping to the water, it causes the plates to become unduly heated, and quickly burnt out. This defect is sometimes obviated by placing within the shell of the cylinder a segment shaped trough, shown in transverse section in fig. 153. The trough *a* is fixed a few inches from the bottom of the boiler.

In the water space below it the ebullition is most violent, and the circulation of the water so rapid, that no deposit can take place; while within the trough the water is com-

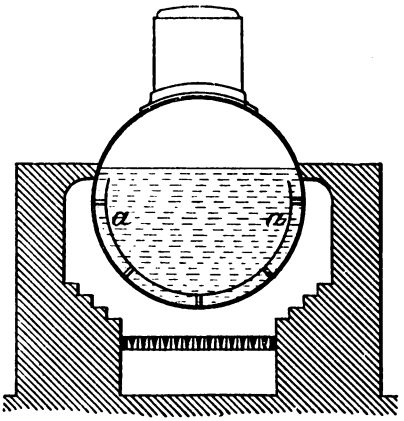

Fig. 153.

paratively quiet, and the mud consequently deposits itself here and does no harm, and can easily be removed periodically.

There is another defect belonging to this system of boiler to which many engineers attach great importance, viz. that the temperature in each of the three flues *n*, *m*, *m'* is very different, and consequently the metal of which the shell of the boiler is composed expands very unequally in each of the flues, and cracks are very likely to take place where the effects of the changes of temperature are most felt.

Cornish boiler.—The Cornish boiler obviates most of the defects of the system just described. It consists also of a cylindrical shell A (figs. 154, 155), with flat or semi-circular ends. The furnace, however, instead of being situated underneath the front end of the shell, is enclosed within it in a second cylinder B, having usually a diameter rather

greater than half that of the boiler shell. The arrangement of the grate and bridge is evident from the diagram. After passing the bridge the flame and gases travel along through the internal cylinder B, till they reach the back end of the boiler ; they then return to the front again by the two side flues *m*, *m'*, and thence back again to the chimney by the bottom flue *n*.

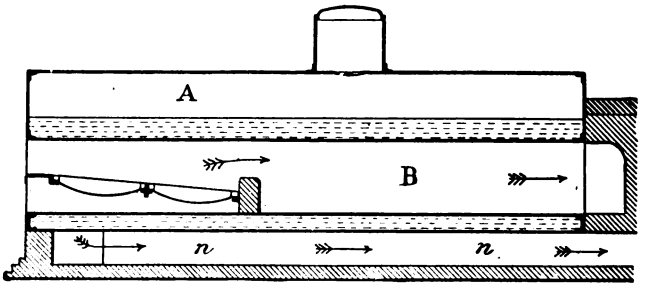

Fig. 154.

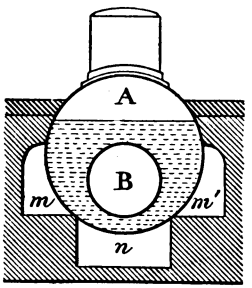

Fig. 155.

In this form of boiler the heating surface exceeds that of the last described by an amount equal to the area of the internal flue, while the internal capacity is diminished by its cubic contents ; hence for boilers of equal external dimensions the ratio of heating surface to mass of water to be heated is greatly increased. Boilers of this sort can, however, never be made of as small diameters as the plain cylindrical sort on account of the necessity of finding room inside, below the water level, for the furnace and flue. The disadvantage attending the deposits in the plain cylindrical type is, to a great extent, got over in the Cornish boiler ; for the bottom, where the deposit chiefly takes place, is the coolest, instead of being the hottest, part of the heating surface.

The internal flue in the Cornish system is the hottest portion of the boiler, and consequently undergoes a greater linear expansion than the outside shell. The result is a tendency to bulge out the ends, and when the boiler is out of use the flue returns to its normal size, and thus has a tendency to work loose from the ends to which it is riveted. If the ends are too rigid to move, a very serious strain comes on the joints of the flue. To remedy this the latter is often made up of a number of short cylindrical lengths jointed together by being riveted to a ring, the section of which is shown in fig. 156. This ring is intended to serve as a spring, and to allow the ends *a*, *a'* to approach or recede from each other, when undergoing change of temperature. It also serves to stiffen the flue against the pressure from the outside, which tends to collapse it. Another way of effecting the same end is shown in the section of the Lancashire boiler, fig 159.

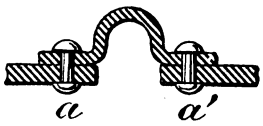

Fig. 156.

Even while in use, the flue of a Cornish boiler is liable to undergo great changes in temperature, according to the state of the fire. When this latter is very low, or when fresh fuel has been thrown on, the temperature is a minimum, and reaches a maximum again when the fresh fuel commences to burn fiercely.

Lancashire boiler.—To remedy this inconvenience, and also in order to attain a more perfect combustion, Fairbairn contrived the double-flued, or Lancashire boiler, the arrangement of the furnaces of which is shown in transverse section in fig. 160. It will be observed that there are two internal furnaces instead of one, as in the Cornish type. These furnaces are sometimes each continued as a separate flue to the other end of the boiler, as shown in fig. 160 ; but as a rule they merge into one internal flue. They are supposed to be fired alternately, and the smoke

and unburnt gases issuing from the fresh fuel are ignited in the flue by the hot air proceeding from the other furnace, the fuel in which is in a state of incandescence. Thus all violent changes of temperature in the flue are avoided, and the waste of fuel due to unburnt smoke is avoided, if the firing is properly conducted.

The disadvantage of the Lancashire boiler is the difficulty of finding adequate room for the two furnaces without unduly increasing the diameter of the shell. Low furnaces are extremely unfavourable to complete combustion, the comparatively cold crown plates, where they are in contact with the water of the boiler, extinguishing the flames from the fuel when they are just formed, while the narrow space between the fuel and the crown does not permit of the proper quantity of air being supplied above the fuel to complete the combustion of the gases as they arise. On the other hand, though this type of boiler favours the distillation of the fuel and the formation of smoke, it supplies the means of completing the combustion afterwards by means of the hot air from the second furnace.

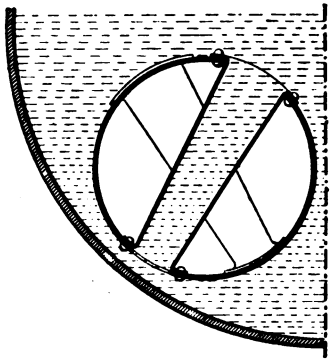

Fig. 157.

Another disadvantage which the Lancashire boiler has in common with all steam generators having circular internal furnaces, is the danger to which the flues are exposed of collapsing, because of the pressure which they have to sustain from without. There are many ways of getting over this evil.

In the Galloway form of boiler the flue is sustained and stiffened by the introduction of numerous conical tubes,

flanged at the two ends and riveted across the flue. These tubes, a sketch of two of which is given in fig. 157, are in free communication with the water of the boiler, and, besides acting as stiffeners, they also serve to increase the heating surface and to promote circulation. Other methods of strengthening the flues have already been described under the head of Cornish boilers.

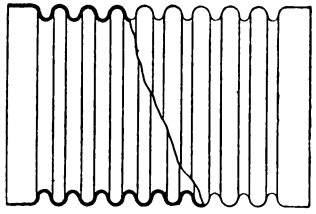

Fig. 158.

Mr. Fox of Leeds corrugates the flues in the manner shown in fig. 158. This plan increases the resisting power of the flues enormously, and, moreover, increases the heating surface and provides for the contraction and expansion of the flue.

The annexed illustrations give all the principal details of a Lancashire boiler fitted with Galloway tubes. Fig. 159 represents a longitudinal section, and fig. 160 shows to a larger scale, an end view of the front of the boiler with its fittings, and also a transverse section. The arrangement of the furnaces, flues, and the Galloway tubes *a a a* is sufficiently obvious from the drawings. The usual length of these boilers is 27 feet, though they are occasionally made as short as 21 feet.

The minimum diameter of the furnaces is 33 inches, and in order to contain these comfortably the diameter of the boiler should not be less than 7 feet. The ends of the boiler are flat, and are prevented from bulging outwards by being held in place by the furnaces and flues, which stay the two ends together, and also by the so-called gusset stays, *e*, fig. 159, which are explained in greater detail on page 397. Great care should be taken to keep the lower ends of the gusset stays 8 or 9 inches away from the nearest points of furnaces and flues, otherwise when the latter expand under

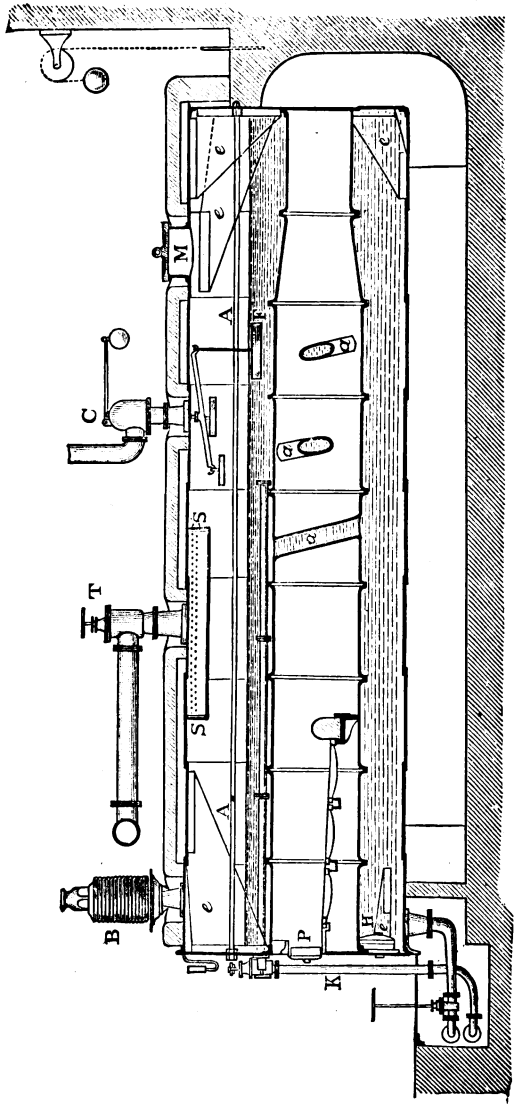

Fig. 159.

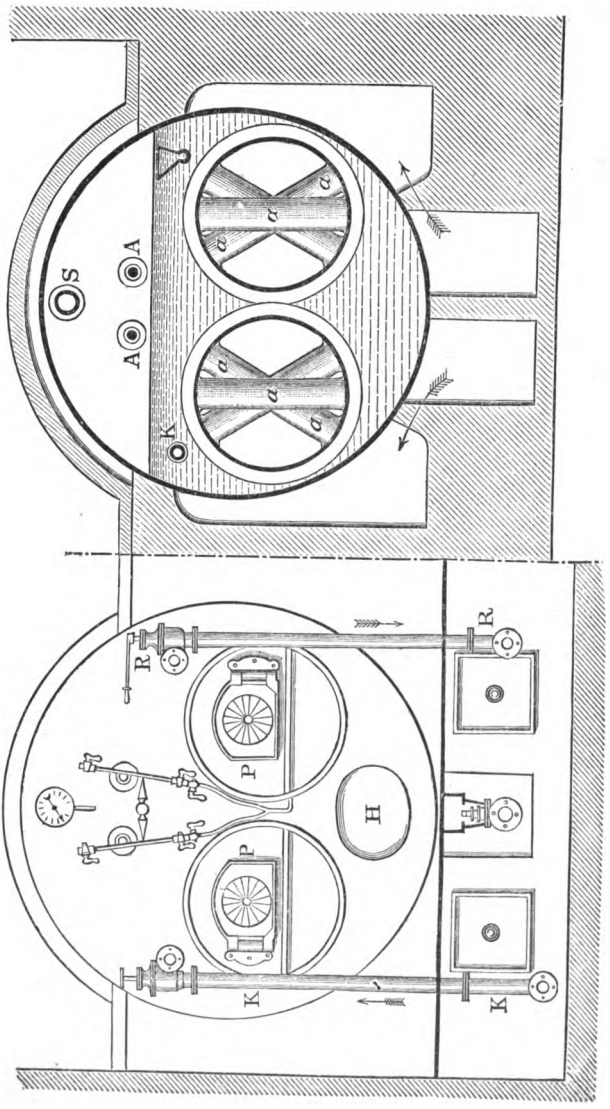

Fig. 160.

the influence of the heat the boiler ends will be so stiff, that instead of slightly bulging out to accommodate themselves to the increased length of the flues, they will be very severely strained in the neighbourhood of the angle irons by which they are fastened to the flues. The result of this local straining will be the opening of the grain of the iron, and its subsequent rapid corrosion. For the same reason the thickness of the end plates should not exceed a half inch for pressures up to 75 lbs. per square inch. In addition to the gusset stays, the flat ends of the boilers frequently have longitudinal rods to tie them together. One of these is shown at AA, fig. 159.

The steam is collected in the pipe S, which is perforated all along the top so as to admit the steam, and exclude the water spray which may rise from the surface during ebullition. The steam thence passes to the stop-valve T, outside the boiler, and thence by the steam pipes to the engines.

There are two safety valves on the top of the boiler, one, B (fig. 159), being of the dead weight type explained on page 404, and the other, C, being a so-called low-water safety valve. It is attached by means of a lever and rod to the float F, which ordinarily rests on the surface of the water. When, through any neglect, the water sinks below its proper level, the float sinks also, and causes the valve to open, thus allowing steam to escape and giving an alarm.

M is the man-hole, with its covering plate, which admits of access to the interior of the boiler, and H is the mud-hole by which the sediment which accumulates all along the bottom is raked out. Below the front end and underneath are shown the pipe and stop-valve by which the boiler can be emptied or blown off.

On the front of the boiler (fig. 160) are shown the pressure gauge, the water gauges, and the furnace doors, which are described in detail on pages 401, 407, 413. K is the seed-pipe; RR, a pipe and cock for blowing off scum. In the front of the setting, fig. 160, are shown two iron doors by which

access may be gained to the lower external flues for clearing purposes.

In the Lancashire boiler it is considered advisable to take the products of combustion, after they leave the internal flues, along the bottom of the boiler, and then back to the chimney by the sides. When this plan is adopted the bottom is kept hotter than would otherwise be the case, and circulation is promoted, which prevents the coldest water from accumulating at the bottom. If this precaution be neglected, the boiler is very apt to strain locally, from the fact of the top and sides being hotter, and consequently expanding more than the bottom. The result is that the lower portions of the transverse seams of rivets give way.

The principal dimensions and other particulars of the boiler shown in figs. 159 and 160 are as follows :—

Steam pressure	75 lbs. per sq. inch
Length	27 feet
Diameter	7 ,,
Weight (total)	15½ tons
Shell plates	$\frac{7}{16}$ inch
Furnace diameter	33 inches
Furnace plates	$\frac{3}{8}$ inch
Grate area	33 sq. feet
Heating surface :—	
In furnace and flues	450 ,,
In Galloway pipes	30 ,,
In external flues	370 ,,
Total	850 ,,

Fuel consumption, 17 to 23 lbs. of coal per sq. ft. of grate per hour

Water evaporated per lb. of coal, at and from 212°, from 10½ to 11 lbs. with the help of a feed-heater.

Tubulous Boilers.

The above term is applied to a class of boilers in which the water is contained in a series of tubes, of comparatively small diameter, which communicate with one another and with a common steam-chamber. The flame and hot gases

from the furnace circulate between the tubes, and are usually guided by baffle plates or partitions, so as to act equally on all portions of the tubes. There are many varieties of this type of boiler. Fig. 161 illustrates Root's patent boiler. Each tube is screwed at either end into a square cast-iron head, and each of these heads has two

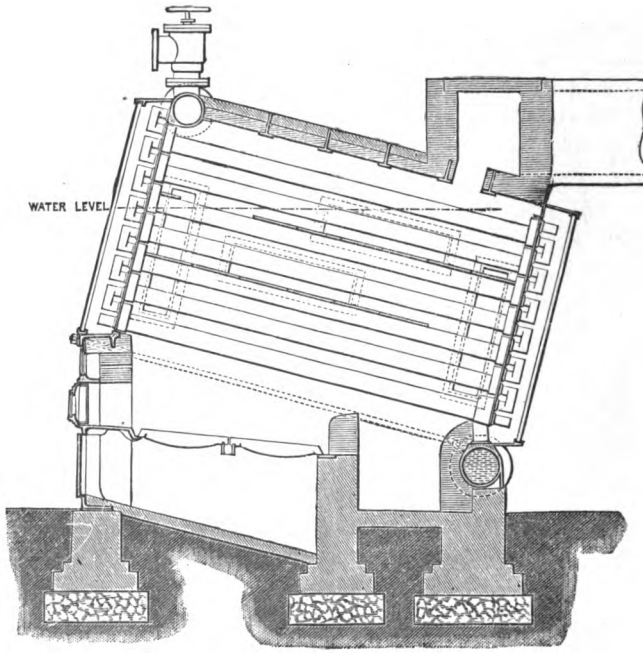

Fig. 161.

openings, one communicating with the tube below, and the other with the tube above. The communication is effected by means of hollow cast-iron caps shown at the ends of the tubes. The caps have openings in them corresponding with the openings in the tube heads to which they are bolted, the joints being made by india-rubber washers.

Locomotive Boilers.

The essential features of locomotive boilers are dictated by the duties which they have to perform under peculiar conditions. The size and weight are limited by the fact that the boiler has to be transported rapidly from place to place, and also that it has to fit in between the frames of the locomotive; while, at the same time, the pressure of the steam has to be very great, in order that with comparatively small cylinders the engine may develope great power; moreover, the quantity of water which has to be evaporated in a given time is very considerable. To fulfil these latter conditions a large quantity of coal must be burned on a fire-grate of limited area; hence intense combustion is necessary under a forced blast. To utilise advantageously the heat thus generated, a large heating surface must be provided, and this can only be obtained by passing the products of combustion through a great number of tubes of small diameter. The manner in which these conditions are carried out in practice will be best understood by reference to the accompanying illustrations (figs. 162 to 165). Fig. 162 is a longitudinal vertical section of a boiler of a modern locomotive. Fig. 163 is a vertical transverse section, half through the fire-box, and half through the cylindrical body of the boiler showing the tubes in section. Fig. 164 is half a section through the smoke box and funnel, and half an elevation of the front end of the locomotive, showing the smoke-box door. Fig. 165 is a horizontal section through the fire-box, showing some of the fire-bars in plan. The furnace (figs. 162, 163, 165), or fire-box as it is called, is a box shaped casing of copper. In horizontal plan the fire-box is also rectangular, the width in this example being 40 inches, and the length from front to back 60 inches, and the height from top of fire-bars to crown $64\frac{1}{2}$ inches. The fire-bars B usually form the bottom of the box, though in some boilers there is a water

space provided with two openings for the admission of air below the bars. The lower edge of the furnace door is about

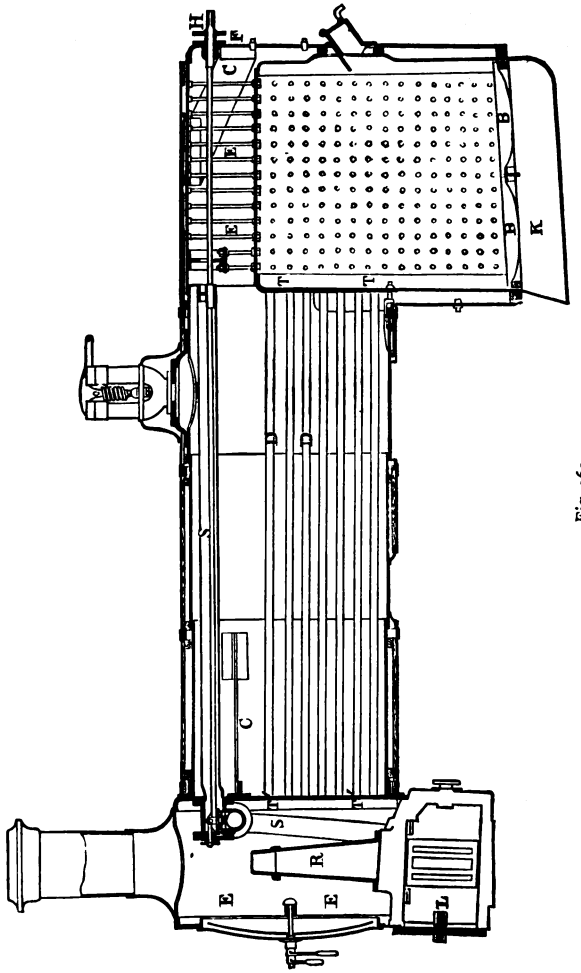

Fig. 162.

30 inches from the grate. Within the fire-box and below the tubes a bridge or arch of fire-brick is often built, which serves

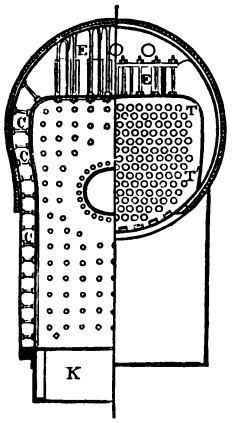

Fig. 163.

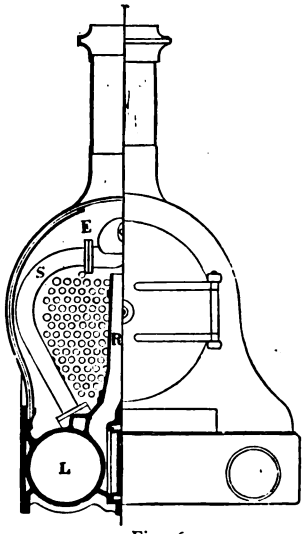

Fig. 164.

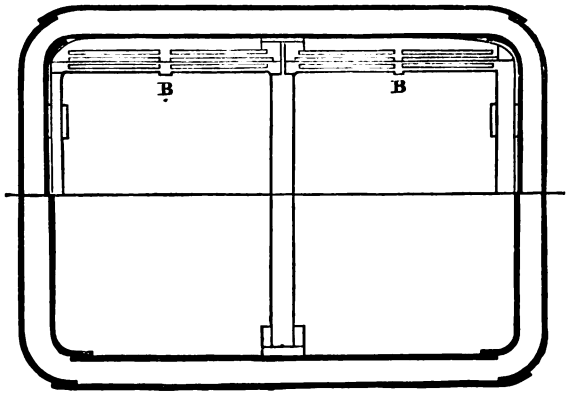

Fig. 165.

to deflect the products of combustion, which would otherwise rush direct into the tubes, and causes them to impinge on the sides and crown. The fire-box is enclosed completely within the body of the boiler, and consequently the four sides, and also the top or crown, are available as heating surface. The sides and top, being flat, would quickly collapse under the pressure of the steam unless special provision were made to stiffen them. The plan invariably adopted is to connect the sides of the fire-box with the outside shell of the boiler by a number of short bolts or stays CCC, screwed and riveted into each, as shown in fig. 163. The shell of the boiler exterior to the fire-box is also in plan a rectangular box made of wrought iron plates. The tendency of the steam being to bulge the shell outwards, and the sides of the fire-box inwards, the two pressures neutralise each other in the stays, which latter are of course put into a state of tension. They are clearly shown in the drawing, and are spaced about four inches apart all over the flat surfaces. The portion of the shell immediately over the crown of the fire-box is not flat, but semi-circular. It would consequently be often inconvenient to stay these two surfaces together in the manner described. The crown of the fire-box is therefore stiffened in a different manner, and usually by means of stout girders, though in the case of the boiler under description, the crown is supported by stays EE hanging from the shell plate.

The remainder of the shell of the boiler consists of a cylindrical barrel united to the rectangular portion surrounding the fire-box. This barrel is 4 feet $2\frac{3}{16}$ inches in diameter and 10 feet 11 inches long. The products of combustion from the furnace are conveyed through the barrel to the smoke-box E, figs. 162 and 164, by means of 195 thin tubes, made of brass, $1\frac{3}{4}$ inch in external diameter, and 10 feet $11\frac{1}{8}$ inches long. In this manner a very large heating surface is obtained. Care must be taken in designing these tubes to make their diameter sufficiently large to carry off the products of combustion with ease. In some of the earlier

locomotive boilers, the diameter was made too small, with the object of getting as many tubes as possible into the available space, and thus increasing the heating surface; but it was found that the narrow tubes offered a most serious impediment to the escape of the smoke and gases, and they were moreover, on account of their small diameter, very liable to become choked by soot, so that this plan had to be avoided. It was formerly the custom to make the tubes much longer than shown in fig. 162, with the object of gaining heating surface; but modern experience has shown that the last three or four feet next the smoke-box were of little or no use, because, by the time the products of combustion reached this part of the heating surface, their temperature was so reduced that but little additional heat could be abstracted from them. The tubes, in addition to acting as flues and heating surface, fulfil also the function of stays to the flat end of the barrel of the boiler, and the portion of the fire-box opposite to it. They always tend to work loose and consequently to leak at the tube plates (T,T, fig. 162), because they expand and contract more than the outside shell. They must therefore be very securely fastened. This is accomplished either by riveting the ends over the tube-plates, and driving in ring ferrules, or else by expanding the tube immediately behind the tube plates by an instrument specially made for this purpose. In addition to the staying power derived from the tubes, the smoke-box tube plate and the front shell plate F are stayed together by several long rods, or else the ends are strengthened by gusset stays CCC. In the boiler under consideration the heating surface given by the tubes is 964 square feet in area, while the sides and crown of the fire-box, or the direct heating surface, as it is called, is 101 square feet. The grate area is 16 square feet.

The forced draught in a locomotive boiler is obtained by causing the steam from the cylinders after it has done its work to be discharged into the chimney by means of a pipe

R (figs. 162, 164) called the blast-pipe. The lower portion of the blast-pipe consists of two branches, one in communication with the exhaust port of each cylinder. One of these branches and one cylinder L are shown in section, fig. 164. The most advantageous position for the mouth of the blast-pipe is some few inches below the base of the funnel. As each puff of steam from the blast-pipe escapes up the chimney, it forces the air out in front of it, causing a partial vacuum which can only be supplied by the air rushing through the furnace and tubes. The greater the body of steam escaping at each puff, and the more rapid the succession of puffs, the more violent is the action of the blast-pipe in producing a draught, and consequently this contrivance regulates the consumption of fuel and the evaporation of water, to a certain extent automatically, because when the engine is working its hardest, and using most steam, the blast is at the same time most efficacious. The blast-pipe is perhaps the most distinctive feature of the locomotive boiler, and the one which alone has rendered it possible to obtain large quantities of steam from so small a generator.

The chamber E (figs. 162, 164) into which the smoke and other products of combustion issue on leaving the tubes is called the smoke-box. It is provided with a door in front, for giving access to the interior and to the tubes. The chimney is placed on the top of the smoke-box. The steam is either collected in a dome, on the top of the barrel, and which contains the mouth of the steam pipe S, leading to the cylinders ; or else, as in the case of fig. 162, a perforated pipe, S, is used, which runs along the top of the steam space in the barrel of the boiler. Unless these precautions were taken, the steam would carry over quantities of spray into the cylinders ; in other words, the boiler would prime. Priming, besides being a great inconvenience, is also a source of waste of heat, for the hot water carried over into the cylinders is incapable of doing work itself, and, moreover, lowers

the temperature of the steam in contact with it, and in this way may indirectly become a most prolific source of waste.

On account of the oscillations to which the boilers of locomotive engines are subject, weighted safety valves are inadmissible, and springs are used instead to hold the valves in place. A spring safety valve is described on p. 406.

Marine Boilers.

The boilers used on board steamships are of two principal types. The older sort used for steam of comparatively low pressure, viz. up to 35 lbs. per square inch, is usually made of flat plates stayed together, after the manner of the exterior and interior fire-boxes of a locomotive boiler. Modern high-pressure marine boilers, constructed for steam of 60 to 150 lbs. per square inch, are circular or oval in cross section, and are fitted with cylindrical interior furnaces and flues like land boilers.

Figs. 166, 167 represent the general arrangement of the older type of marine boiler, in longitudinal and transverse sections. A A, fig. 167, are the grate bars; B, the furnace door; D, the ashpit. After passing the bridge the hot air and flame enter a large chamber E, called the back up-take, thence they return through the tubes *eee*, to the front up-take F and the chimney. The heating surface consists of the sides and crown of the furnace, the sides of the back up-take and the tubes. The front up-take F is provided with doors G for giving access to the tubes and chimney for cleaning and repairs. The outside shell of this type of boiler is rectangular box-shaped. Some of the stays are represented at *a a a* in both views.

The general arrangements and construction of this type of boiler are rendered clear by the illustrations. As the form of the boiler contributes nothing to its strength, the latter is maintained by staying all the opposite surfaces together in

the manner already described for the flat fire-boxes of locomotive boilers.

The use of high-pressure steam of from 60 to 150 lbs.

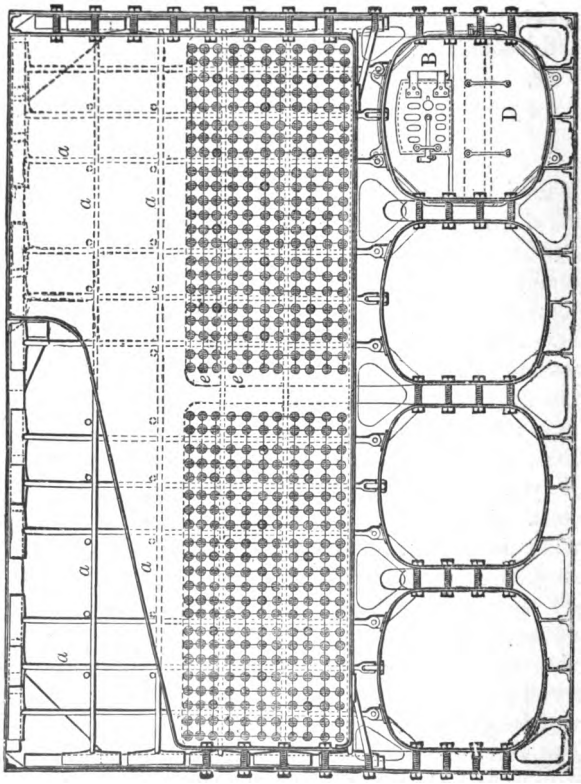

Fig. 166.[1]

per square inch in modern marine engines has necessitated the abandonment of the type of boiler just described, as it would not be safe when dealing with such great pressures

[1] Figs. 166 and 167 are taken by permission from Mr. R. Sennett's work on the *Marine Steam Engine.*

and large surfaces to depend solely on the strength of the stays. Accordingly we find that modern marine boilers are circular, or nearly so, in cross section, with flat ends.

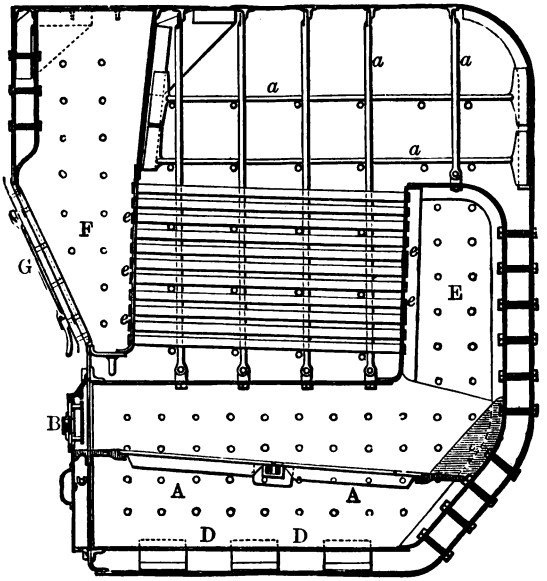

Fig. 167.

Fig. 168 shows a front elevation and partial sections of a pair of such boilers, together with their up-takes, steam-chest, and other fittings; and fig. 169 shows one of them in longitudinal vertical section. It will be seen from these drawings that there are three internal cylindrical furnaces in each end of these boilers, making in all six furnaces per boiler. The firing takes place at both ends. The flame and hot gases from each furnace, after passing over the bridge, enter a flat-sided rectangular combustion chamber, and thence travel through tubes to the front up-take, and so on to the chimney. The sides of the combustion chambers are stayed to each other and to the shell plate of the boiler. The tops

are strengthened in the same manner as the crowns of locomotive fire-boxes already described. The flat-end plates of the boiler shell are stayed together by means of long bolts,

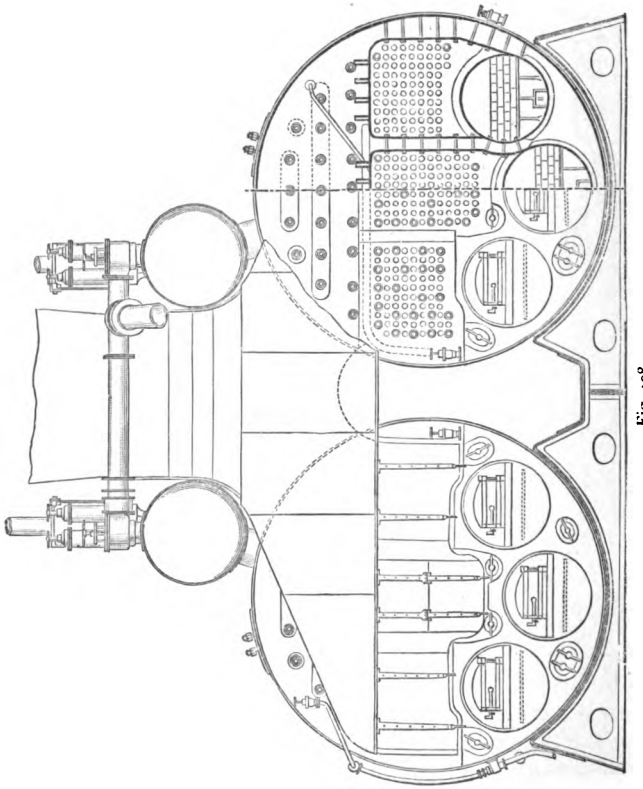

Fig. 108.

which can be tightened up by means of nuts at their ends. Access is gained to the up-takes for purposes of cleaning, repair of tubes, &c. by means of doors on their fronts, just above the furnace doors. The steam is collected in the

large cylindrical receivers shown above each boiler. The material of construction is mild steel.

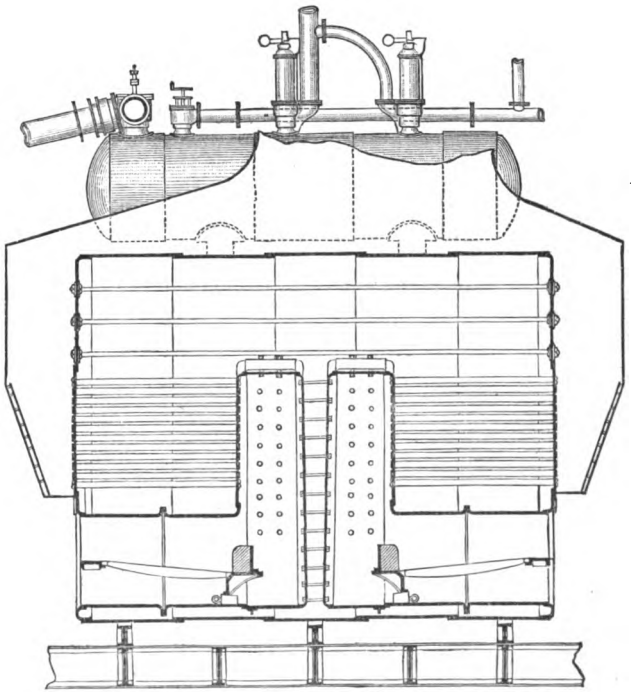

Fig. 169.

The following are the principal dimensions and other particulars of one of these boilers :—

Length from front to back, 20 ft.
Diameter of shell, 15 ft. 6 in.
Length of furnace, 6 ft. 10 in.
Diameter of furnace, 3 ft. 10 in.
Length of tubes, 6 ft. 9 in.
Diameter, $3\frac{1}{2}$ in.

No. of tubes 516.
Thickness of shell plates, $\frac{15}{16}$.
Thickness of tube plates, $\frac{3}{4}$.
Grate area, 126·5 sq. ft.
Heating surface, 4015 sq. ft.
Steam pressure, 80 lbs. per. sq. in.

There are many varieties of marine boilers, adapted to suit special circumstances. Fig. 170 for instance is a sketch of a modern boiler, which is only fired from one end, and is in consequence much shorter in proportion to its diameter than the type illustrated in fig. 168. The cross section is often not circular. The sides are sometimes flat, and are prevented from bulging by being stayed to each other. The top and bottom are semicircular in shape. This form of section has been adopted in order to save some of the space which is wasted when the true circular shape is adopted, and which can be ill spared on board ship. It will have been noticed that the boiler illustrated in fig. 168 has a separate combustion chamber for each of the six furnaces. This arrangement is very good, because if a tube gives way in one chamber the fires in the other furnaces are not affected by it, but it is nevertheless not always adopted. Sometimes, even in double-ended boilers, all the furnaces have only one chamber in common. The disadvantages of this plan are so serious that it is now but seldom adopted. Very often the two opposite furnaces of a double-ended boiler have a chamber in common, and in single-ended boilers with two furnaces we frequently find the same arrangement. The number of furnaces depends upon the diameter of the boiler shell, and the very confined natural limits, which are set to the diameter of furnaces. If the latter are less than 36 inches, the crown of the furnace is so low that a large proportion of the heating value of the fuel is lost by the process of distillation. On

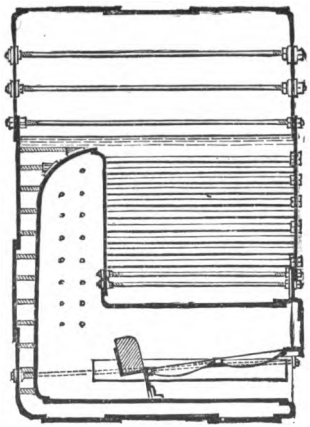

Fig. 170.

the other hand, if over 48 inches, the thickness of plates necessary to give sufficient strength to the structure is so great, that the metal would be liable to be burnt, and its heat-transmitting powers would be greatly diminished. Boilers over nine feet in diameter have generally two furnaces, those over thirteen to fourteen feet three, while the very largest boilers used on first-class mail steamers, and which often exceed fifteen feet in diameter, have four furnaces.

The Proportions of the Parts of Boilers.

In designing a boiler of a given type to furnish a certain amount of steam in a given time, it is requisite to know the following things :—

The size of fire-grate necessary to burn the requisite quantity of fuel in a given time with various kinds of blast.

The capacity of different sorts of heating surface to transmit heat.

The area of heating surface required to evaporate the given quantity of water in the given time.

The relative proportions of the cubic contents of the boiler, which should be occupied by steam and water respectively.

The quantity of water which a pound of fuel will convert into steam of a given pressure depends upon the pressure of the steam, the nature of the coal, and the efficiency of the type of boiler. For the purposes of comparison, all rates of evaporation at various pressures and various temperatures of feed are reduced to the corresponding rates at and from 212°. When the above data are known, it is easy to fix the size of fire-grate necessary in order to effect the combustion of the fuel.

Evaporative power of fuel in different types of boilers.— As a general rule, with fair average coal, it may be stated that the following rates of evaporation are obtained with the different types of boilers named :—

	Per lb. of coal
Lancashire boiler using feed heater	9 to 10·5 lbs. of water at and from 212°
Marine boiler of old type . .	8·7 lbs. of water at and from 212°
Marine boiler of new type . .	8·1 lbs. of water at and from 212°
Locomotive boiler . . .	9 to 12 lbs. of water at and from 212°

With bad fuel, such as steamers have often to take in at foreign ports, these figures will have to be reduced about twenty per cent.; on the other hand, with picked fuel they may all be increased about fifteen per cent.

Fire grate area.—A given grate area will burn very different weights of fuel in a given time according to the nature of the draught. Where the size of the boiler is a matter of no importance, as in most land boilers, a slow rate of combustion is maintained, with a natural draught, on account of the saving in wear and tear of the furnaces. In such cases a common rate is from ten to twenty pounds of fuel per square foot of grate area per hour. On the other hand, when the size of the boiler is limited by the circumstances, as in the case of locomotives and torpedo boats, a rate of from forty to one hundred and twenty pounds per square foot can be maintained by using a forced draught. Most of the following figures are given in Rankine's 'Manual of the Steam Engine.'

	Per sq. ft. per hour
Slowest rate of combustion in Cornish boilers .	4 lbs.
Ordinary rate of combustion in Cornish boilers .	10 lbs.
,, ,, in factory boilers .	12 to 16 lbs.
,, ,, in marine boilers .	15 to 24 lbs.
Quickest rate of complete combustion of dry coal, air coming through grate alone . . .	20 to 23 lbs.
Rate in locomotive boilers with blast-pipe . .	40 to 120 lbs.
Ordinary rate of locomotive boilers with blast-pipe	65 lbs.

The length of fire-grate is limited by the distance to which a stoker can throw the coals back with accuracy. It

is usual to fix six feet as the utmost limit. The breadth of the grate depends chiefly on the breadth or diameter of the boiler, and on the arrangement of the furnaces. In Lancashire boilers with two internal flues the breadth is extremely limited. Narrow furnaces have the great disadvantage that they allow the fire, which is necessarily of small bulk, to be chilled by the proximity of the cold sides of the furnace, and allow but little room above the fuel for the introduction of air to complete the combustion.

Grates of large area are difficult to cover evenly with the fuel. As a consequence the fire is apt to burn through too quickly in the thin places, and the air rushing in most where it finds the easiest entrance, causes the imperfect and slow combustion of the fuel wherever it is piled on too thickly.

Efficiency of heating surface.—The capacity of the heating surface to transmit heat to the water depends on the conductivity and the thickness of the metal, also on the position of the surface, and the difference in temperature between the water in the boiler and the hot gases in the furnace. The metals most commonly used to separate the water from the fuel are wrought iron and steel. These materials are, however, inferior to copper in conducting power, in the ratio of about one to three, and for this reason the latter metal is used to form the sides of furnaces in all cases where it is necessary to obtain a high rate of evaporation from a boiler of limited size. The inner fire-boxes in locomotives form a case in point. For the tubes of locomotive and many marine boilers brass is very generally used, both on account of its high conducting power, and also because of the facility with which tubes can be drawn from this material; but tubes of steel and wrought iron are also used, especially in the boilers of the mercantile marine. The most effective portions of the heating surface are the sides, and especially the crown of the furnace and combustion chamber, and the first foot or two of the flues or tubes. The reason of this is that the products of combustion are much

hotter at these parts than elsewhere, and the effects of radiation are also most strongly felt in these portions of the boiler.

In a boiler with horizontal flues and tubes the lower portions of these latter are considered of no value as heating surface, because of the difficulty with which the steam escapes from them. For this reason the effective value of tube-heating surface is usually estimated to be only three-fourths of the total area of the tubes. In estimating the amount of heating surface in a boiler the surfaces of the furnace below the fire-bars, and of the combustion chamber below the bridge, and also of the tube-plate, farthest from the flame are altogether omitted.

It is impossible to lay down general rules for the evaporating power of a given area of heating surface; for, as has been stated above, so much depends on the temperature which is maintained in the furnace, and also on the position of the surface relatively to the hottest part of the fire. For these reasons the effects of different portions of the heating surface in evaporating the water are widely different, and nothing but an average of effect can be taken. In designing boilers it will, therefore, be safest to follow the proportions of heating surface to grate area, in the various types, which experience has proved to give the best results. The following are the proportions in a few representative cases:—

Lancashire boiler, ratio of grate area to heating surface	1 : 26
Lancashire boiler, including surface of feed-water heater, ratio of grate area to heating surface .	1 : 44
Marine boiler, ratio of grate area to heating surface .	1 : 22 to 35
Locomotive boiler, ratio of grate area to heating surface	1 : 60 to 90

Roughly speaking, it may be said that for every foot of heating surface in a Lancashire boiler 6·8 to 9 lbs. of water can be evaporated per hour, excluding the surface of the feed-heater; in marine boilers from 6 to 8 lbs.; and in locomotives from 10 to 15, for good average coal and ordinary

conditions. The higher figures apply to the case of the boilers being forced. With the locomotive type of boiler applied to torpedo boats 18 lbs. of water have been evaporated per square foot of heating surface, with a forced draught equivalent to six inches of water. The ratio of grate area to heating surface was, however, only 1 : 34, and the water evaporated per pound of coal was consequently very low, having been only about 6 lbs.

As a general rule it may be stated that the proportion of heating surface to fuel burnt is as follows :

	Per lb. of coal burnt per hour
For Lancashire boilers . . .	1·1 to 1·5 sq. ft. of surface
For modern marine boilers . .	1 to 1·5 sq. ft. of surface
For modern locomotive boilers .	·9 to 1·5 sq. ft. of surface

In order to obtain good evaporative results the higher figure should be chosen, but little is gained by any further increase beyond the allowance of 1·5 square ft. per lb. of coal per hour. When the allowance is less than ·7 square ft. per lb. of coal the results are distinctly uneconomical.

Cubic capacities of boilers of different types.—The absolute cubic capacity and the relative capacities of the water and steam rooms in a boiler are determined very much by the nature of the work which is expected to be done. Of course, in the case of boilers of a portable nature the circumstances limit the absolute capacity, but in the case of land boilers, where the bulk is of no particular account, the cubic contents are determined solely by the nature of the work to be done. Thus in cases where steam is only required occasionally, and where, when wanted, it must be raised with great rapidity, the capacity of the boiler is necessarily small, and the heating surface and grate area large, relatively to the cubic contents ; but where steady continuous work is required the capacity is always large.

In boilers of small capacity the greatest care must necessarily be bestowed on the feed ; otherwise it would be impossible to maintain a uniform steam pressure, and more-

over, on account of the rapidity of evaporation, the upper portions of the heating surface are liable to be denuded of water, in which case serious damage is likely to ensue. In boilers of small capacity and great evaporative power it is usual to put a lead plug into the crown of the furnace, in order that, if the water has been allowed to sink below the level of this portion, the plug may melt, and allow the steam to enter the furnace and extinguish the fire.

The absolute capacity of steam and water room in cubic feet, per pound of water evaporated per hour in the boiler, varies greatly in different types of boilers. The following figures give the proportions adopted in a few cases of the best examples.

Lancashire boilers .	1 cubic foot capacity for every 3 lbs. of water to be evaporated per hour
Marine boilers . .	1 cubic foot capacity for every 7 to 10 lbs. of water to be evaporated per hour

In the case of locomotive boilers the rate of evaporation varies within such wide limits in the same boiler according to the work that has to be done, that it is impossible to give any general rule.

The relative cubic capacities of the steam and water rooms also vary very considerably in the different types of boilers. For instance, where high pressure is used, and small quantities of steam are very frequently withdrawn from the boiler, as in the case of locomotives, the steam room need not be relatively so great as when large volumes of steam are slowly withdrawn, in the case, for example, of paddle engines. The effect of withdrawing a large volume of steam from a confined space is to lower the pressure considerably; at the same time the water in the boiler has the temperature due to the higher pressure; consequently, when the pressure falls, the surplus heat in the water at once generates immense volumes of steam, which rushing to the surface carry large quantities of spray with them, and thus give rise to serious priming.

In ordinary practice we find that in Lancashire boilers the water occupies about three-fourths of the diameter of the boiler.

In marine boilers the proportion varies from $\frac{1}{6}$ to $\frac{1}{3}$ of the total shell capacity, according as they supply quick-running short-stroke screw engines, or slow and long stroke paddle engines. It should be borne in mind that when very steady running is required, and comparatively unskilled stoking is all that can be had, it is imperative that there should be not only a large steam room, but also a large water room to back it up with. The importance of a large cubic capacity of water as an equaliser of pressure lies in the fact that the specific heat of water is so high, that whenever the pressure tends to drop through the neglect of the firing, the immense store of heat in the water, at the temperature due to the higher pressure, is at once available for the immediate generation of steam.

The Strength of Boilers.

Hollow cylinder pressed from within.—In considering the strength of boilers the first point to be examined is the case of a hollow cylinder pressed from within. Let the circle (fig. 171) represent the transverse section of such a cylinder, which is supposed to be filled with steam of a pressure of P lbs. to the square inch. It is required to find the stress on the shell of the cylinder at any two points AA, diametrically opposite. It is evident that every inch of the circumference of the shell is subjected to a pressure acting radially outwards from the centre C, so that the total pressure acting on the semi-circumference of a ring one inch wide$=\pi \cdot r \cdot$ P, where $r=$

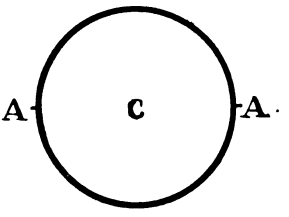

Fig. 171.

the radius in inches. The force, however, which tends to separate the metal at AA is not the whole radial pressure $\pi . r . P$, but the sum of the components of this force resolved in a direction at right angles to the diameter AA. The sum of these components on a ring one inch in width may be proved to be equal to the pressure on an area equal to the diameter of the circle ($2r$) multiplied by the width (one inch). Hence the sum $= 2rP$. The cross section of the metal at AA has, therefore, to sustain a tension $= 2rP$; or at either of these points separately, the tension $= rP$.

Let the thickness of the shell at A be t inches, and let the tensile strength of the metal be W lbs. per square inch; then the total strength of the ring at A per inch run of length of cylinder $= Wt$ lbs.; and when the boiler is on the point of bursting we must have $Wt = rP$, or $P = \frac{Wt}{r}$. In other words, the pressure P, which a cylindrical boiler will support, is directly proportional to the strength and thickness of the metal, and inversely proportional to the diameter. This reasoning applies only to the case of the thickness of the shell being small compared with the diameter; had the thickness at AA been considerable, as in the case of hydraulic presses and heavy guns, we should have had to argue in a different manner.

As an example, take the case of a cylindrical boiler of wrought iron of 6 feet diameter, the plates being $\frac{1}{2}$ an inch thick, and the steam pressure 80 lbs. per square inch, the tensile strength of the iron being 48,000 lbs. per square inch. We have rP = the tension at any point in the circumference of the shell $= 3 \times 12 \times 80 = 2{,}880$ lbs. per inch of length. On the other hand, the strength of a ring one inch in length and half an inch thick is $\frac{48{,}000}{2} = 24{,}000$ lbs.; that is to say, the strength of the boiler in this case is about eight and a half times greater than the stress brought to bear upon it.

In actual practice, however, we could not take the full

strength of the metal, because the riveted joints are much weaker than the solid plate. The tensile strength of a double riveted joint, for such a boiler, would be about 34,000 lbs. per square inch, and consequently the strength of the boiler at the weakest part would be $\frac{34,000}{2} = 17,000$ lbs. per inch of length; that is to say, the strength would be about six times greater than the stress.

Factors of safety.—The number which expresses the ratio of the strength of a boiler to the working strain is called the factor of safety. Thus in the above example the factor of safety is six. The proper factor of safety is a point not yet fully settled. The number adopted by the Board of Trade for the shells of marine boilers, subject to their inspection, is five for the most favourable cases; that is to say, when materials, construction, and workmanship are all of the best. In Lancashire boilers the factor four is considered sufficient for the weakest strip in the boiler. The French Government has fixed three as the factor in land boilers, and this low number has been found to give perfect security.

In addition to the strength of the longitudinal section of a boiler we must also consider the case of the transverse or ring-shaped section. The stress in this instance is brought to bear by means of the pressure of the steam on the two ends, and tends to pull the shell out like a telescope.

No matter what the shape of the end, the pressure on it tending to pull the boiler in two is exactly the same as if the ends were flat. Taking, therefore, flat ends, and using the same symbols as before,

we have πr^2 = the area of the end,

$\pi r^2 P$ = the total pressure on the area,

$2\pi r$ = the circumference of any transverse section,

and $2\pi rt$ = the area of such section.

The area $2\pi rt$ has, therefore, to sustain the pressure $\pi r^2 P$.

When the boiler is on the point of bursting in this manner we must have, therefore,

$\pi r^2 P = 2\pi r t W$. $\therefore P = \frac{2tW}{r}$, or $W = \frac{Pr}{2t}$, whereas in the former case we had $W = \frac{Pr}{t}$; that is to say, W, or the tensile strength of the metal, requires to be only half as great to resist the transverse stress as the longitudinal.

Taking the same example as before, we have the area of the end $= 3{\cdot}14159 \times (3 \times 12)^2 = 4071{\cdot}5$ square inches; while the pressure on the end $= 4071{\cdot}5 \times 80 = 325{,}720$ lbs. Now the area of the transverse section of the boiler to resist this stress $= 3{\cdot}14159 \times 6 \times 12 \times \frac{1}{2} = 113{\cdot}1$ square inches, the tensile strength of which is 5,428,800 lbs.; that is to say, the strength is about sixteen and three quarter times greater than the stress brought to bear on it. As before, however, we cannot in practice consider the full strength of the plate, because all boilers are made up of two or more rings riveted together. On account, however, of the comparatively light load which the joints have to bear, it is not considered necessary to double rivet the joints. The strength of a single riveted joint in the above example would be only about 26,000 lbs. per square inch, and consequently the factor of safety would be between eight and nine. The transverse strength of a cylindrical boiler with internal furnaces is of course much greater than in the above example, for while the area of the ends is diminished by the transverse area of the furnace or flue, the section of metal which resists the stress is increased by the area of metal contained in a transverse section of the flue. The way in which the strength is calculated is so apparent from what has gone before that it is unnecessary to give another example. See also pp. 398–9.

Strength of riveted joints.—The principles of the construction of riveted joints are fully explained in the treatise on the 'Elements of Machine Design,' published in this series of text-books.[1] It is here only necessary to state that,

[1] *Elements of Machine Design.* By W. C. Unwin.

according to Fairbairn's experiments, the strength of a single riveted joint when properly proportioned is 56 per cent. of the strength of the plate, and that of a double riveted joint 70 per cent. Fairbairn's experiments were made on plates $\frac{1}{4}$ inch in thickness, and it seems highly probable that with the much thicker plates now in use, and the consequent alteration in the pitch and proportions of the rivets, his figures can no longer be accepted as correct. They appear to err in representing the strength of the joint as being greater than it really is.

Single Riveted Joints.

Iron Plates, iron Rivets				Steel Plates, iron Rivets			
Thickness of Plates	Diameter of Rivets	Pitch of Rivets	Efficiency of Joints	Thickness of Plates	Diameter of Rivets	Pitch of Rivets	Efficiency of Joints
$\frac{5}{16}$	·670 = $\frac{11}{16}$	1·82 = $1\frac{13}{16}$	·621	$\frac{5}{16}$	$\frac{11}{16}$	1·54 = $1\frac{9}{16}$	·552
$\frac{3}{8}$	·735 = $\frac{3}{4}$	1·87 = $1\frac{7}{8}$	·606	$\frac{3}{8}$	$\frac{3}{4}$	1·58 = $1\frac{9}{16}$	·538
$\frac{7}{16}$	·790 = $\frac{13}{16}$	1·94 = $1\frac{15}{16}$	·598	$\frac{7}{16}$	$\frac{13}{16}$	1·66 = $1\frac{11}{16}$	·512
$\frac{1}{2}$	·849 = $\frac{7}{8}$	1·98 = 2	·571	$\frac{1}{2}$	$\frac{7}{8}$	1·70 = $1\frac{11}{16}$	·501
$\frac{5}{8}$	·949 = $\frac{15}{16}$	2·08 = $2\frac{1}{16}$	·543	$\frac{5}{8}$	$\frac{15}{16}$	1·80 = $1\frac{13}{16}$	·472
$\frac{3}{4}$	1·04 = $1\frac{1}{16}$	2·17 = $2\frac{3}{16}$	·521	$\frac{3}{4}$	$1\frac{1}{16}$	1·89 = $1\frac{7}{8}$	·450
$\frac{7}{8}$	1·12 = $1\frac{1}{8}$	2·25 = $2\frac{1}{4}$	·500	$\frac{7}{8}$	$1\frac{1}{8}$	1·97 = 2	·431
1	1·20 = $1\frac{1}{4}$	2·33 = $2\frac{5}{16}$	·485	1	$1\frac{1}{4}$	2·05 = $2\frac{1}{16}$	·415
Double Riveted Joints.							
$\frac{3}{8}$	$\frac{3}{4}$	3	·75	$\frac{3}{8}$	$\frac{3}{4}$	$2\frac{7}{16}$	·69
$\frac{7}{16}$	$\frac{13}{16}$	$3\frac{1}{16}$	·73	$\frac{7}{16}$	$\frac{13}{16}$	$2\frac{1}{2}$	·67
$\frac{1}{2}$	$\frac{7}{8}$	$3\frac{1}{8}$	·72	$\frac{1}{2}$	$\frac{7}{8}$	$2\frac{9}{16}$	·66
$\frac{9}{16}$	$\frac{7}{8}$	$3\frac{1}{8}$	·72	$\frac{9}{16}$	$\frac{7}{8}$	$2\frac{9}{16}$	·66
$\frac{5}{8}$	$\frac{15}{16}$	$3\frac{3}{16}$	·71	$\frac{5}{8}$	$\frac{15}{16}$	$2\frac{5}{8}$	·64
$\frac{3}{4}$	$1\frac{1}{16}$	$3\frac{5}{16}$	·69	$\frac{3}{4}$	$1\frac{1}{16}$	$2\frac{3}{4}$	·61
$\frac{7}{8}$	$1\frac{1}{8}$	$3\frac{3}{8}$	·66	$\frac{7}{8}$	$1\frac{1}{8}$	$2\frac{13}{16}$	·59
1	$1\frac{1}{4}$	$3\frac{1}{2}$	·64	1	$1\frac{1}{4}$	$2\frac{15}{16}$	·57

The above tables, and figs. 172 to 174, extracted from Mr. Unwin's work, give the proportions, together with the efficiency of riveted joints—that is to say, the ratio of their strength to that of the solid plate, for iron and steel plates, and for single and double riveting. Fig. 172 illustrates a single riveted lap joint; fig. 173 a similar butt joint, in both single and double shear—

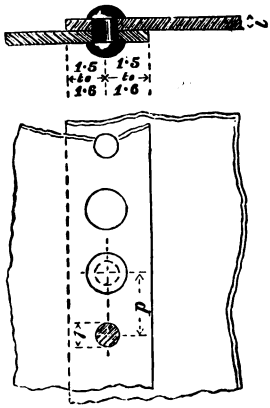

Fig. 172.

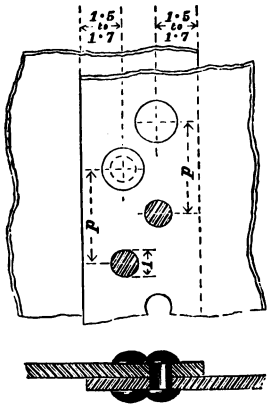

Fig. 174.

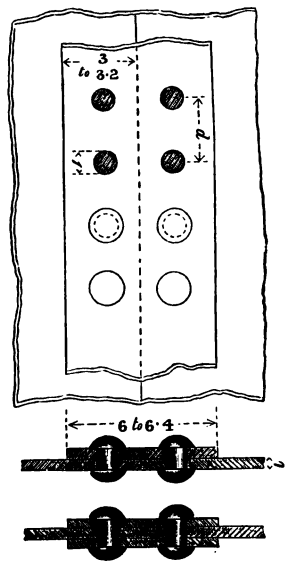

Fig. 173.

that is to say, with single and double cover plates. Fig. 174 shows a double riveted lap joint of the usual dimen-

sions. All dimensions are given in terms of the diameter of the rivet as unit. See also App., Examples 140 *et seqq.*

Hollow cylinder pressed from without.—The strength of cylinders pressed from without is much more difficult to determine than when they are pressed from within. Theoretically the metal would for similar pressures be in a state of compression equal to the tension as determined above. There is, however, a great practical difference between the two cases. When a cylinder is pressed from without, unless it is of a mathematically perfect shape, and perfectly homogeneous in strength, the pressure tends to change its shape, so that it may yield by deformation long before the limit of the crushing strength of the metal is approached. Thus it is frequently found that internal furnace flues give way by collapsing. On the other hand, when pressed from within, the tendency of the pressure is to keep the cylinder in shape, so that it can only give way when the metal yields. The standard experiments on the strength of cylinders to resist external pressure were those made by Fairbairn thirty-three years ago, under conditions widely different from those now common. The results arrived at by Fairbairn were as follows: The strength varies inversely as the length of the cylinder, inversely also as the diameter, and directly as the square of the thickness of the metal. The following formula is given by Rankine to determine the collapsing pressure of such cylinders :—

$$P = 806000\frac{t^2}{Ld},$$

when P is the pressure in pounds per square inch, t the thickness of the sides in inches, d the diameter in inches, and L the length in feet.

In order to obviate the weakness of long flues, rings of angle or tee iron are riveted round them at fixed intervals, as shown in fig. 175, or else the joints are made as represented in fig. 156, or as shown in the furnace of the boiler in fig. 159. The strength in these cases, according to Fair-

bairn, is to be calculated on the supposition that the length is equal to the distance between two consecutive rings.

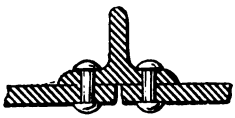

Fig. 175.

Those portions of cylindrical flues which do not contain the furnace are very successfully strengthened by means of Galloway's tubes, described on p. 364. The method of strengthening the furnaces themselves which appears to be most successful is the plan of corrugating the plates introduced by Mr. Fox (fig. 158), and now much used in the furnaces of high-pressure marine boilers. This system has been already described, see p. 365.

Staying of flat surfaces.—When boilers are formed principally of flat plates, like low-pressure marine boilers, or the fire-boxes of locomotive boilers, the form contributes nothing to the strength, which must, therefore, be provided for by staying the opposite surfaces together. Fig. 176 shows the arrangement of the stays in a locomotive fire-box. They are usually pitched about 4 inches from centre to centre, and are fastened into the opposite plates by screwing, as shown, the heads being riveted over. Each stay has to bear the pressure of steam on a square *aa*, and the sectional area of the stay must be so chosen that the tensile strength will be sufficient to bear this strain with the proper factor of safety. Thus if a be the area of the section of the stay in inches, and a' that of the square of plate which it supports, P the pressure of the steam per square inch, and t the tensile strength of the stay, we must have—

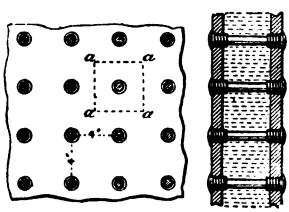

Fig. 176.

$$a'\mathrm{P} = at, \text{ or } a = \frac{a'\mathrm{P}}{t}.$$

It is usual to allow a factor of safety of eight for locomotive boilers, while in marine boilers the factor is from nine to ten, a large margin of strength being necessary on account of the liability of the stays to corrosion.

If the spaces between the stays are too great, or the plate too thin, there is a danger of the structure yielding through the plate bulging outwards between the points of attachment of the stays, thus allowing the latter to draw through the screwed holes made in the plates. Rankine recommends that if the material of the plate is equal in strength to that of the stay, the thickness of the plate should equal half the diameter of the stay; and that if the material of the plate be weaker, its thickness should be proportionately increased.

The flat ends of cylindrical boilers are usually stayed to the cylindrical portions by triangular plates of iron, called gusset stays (see figs. 159, 162, 177). Gusset stays should never be brought too close to any internal flues riveted to the flat ends for the reasons explained on p. 365. The two opposite ends are also stayed together by long bar stays, running the whole length of the boiler. It is dangerous, however, to trust too much to the latter class of stays; for, in consequence of the alternate expansion and contraction which takes place every time the boiler is heated and cooled, they have a tendency to work loose at the joints; and if the portion of the boiler in which they are situated should happen to be hotter than the outside shell, they have a tendency to droop, and are then perfectly useless.

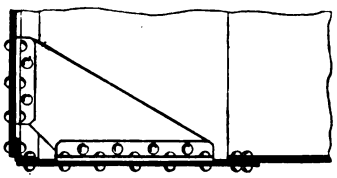

Fig. 177.

In designing boilers with stayed surfaces care should be taken that the opposite plates connected by any system of stays should, as far as possible, be of equal area, otherwise

there is sure to be an unequal distribution of load in the stays, some receiving more than their proper share, and, moreover, the least supported plate is exposed to the danger of buckling.

The Effects of Unequal Expansion and Contraction in Straining a Boiler.

If every portion of a boiler were, when heated, raised to exactly the same temperature, and if the same description of metal were used throughout the entire structure, there would of course be no strains set up by the change of temperature, because all the parts would expand and contract proportionately to their dimensions. In the majority of boilers, however, the various portions are at very different temperatures, and the more highly heated parts expand to a greater extent than the remainder, thus distorting the shape of the boiler, and inducing sometimes very serious strains. Take, for instance, the internal furnace and flue of a Cornish boiler: this portion, containing, as it does, the fire, is considerably hotter than the outside shell, and consequently expands more. One of two things must then happen: either the flat ends of the boiler must bulge out, or if these are too rigid to yield, or are stayed too stiffly, the whole of the metal of the flue will be put into a state of compression, the effects of which are sometimes most conspicuously seen in the joints.[1] Again, the flue, though hotter on the whole than the rest of the boiler, is not itself uniformly heated, the upper portions above the fire-bars being at a higher temperature than the lower. The result of this is to twist the flue out of shape, provided the ends of the boiler can yield, the flue cambering up towards the top of the shell. If the ends were quite rigid the top of the flue would be put into compression, and the bottom in tension.

The outside shells are also subject to considerable dif-

[1] See also page 363.

ferences of temperature, caused by the top of the boiler being filled with hot steam, while the bottom contains water, often not much warmer than the feed. In the case of Cornish and Lancashire boilers with external return flues, this difference in temperature is compensated for, and sometimes more than compensated for, by the high temperature of the hot gases circulating underneath, but in the case of boilers having no external heating surface there may be a considerable difference in temperature, unless means are taken to circulate the water.

In estimating the intensity of the strains due to temperature it should be borne in mind that one degree of rise in temperature elongates a bar of ordinary boiler iron by the same amount as would a tensile stress of the intensity of about 190 lbs. per square inch. Hence such a bar, if held rigidly at the ends, so that these could not move, and then heated ten degrees, would be subjected to a force of compression equal to 1900 lbs. per square inch. Similarly, if cooled ten degrees below the normal temperature, it would be subjected to a tensile stress of the same amount.

Materials of Construction.

The metals principally used in the construction of boilers are wrought iron, mild steel, copper, and brass. Copper is used almost exclusively for the inner fire-boxes of locomotive furnaces, on account of its great conductivity, and the property which it possesses of resisting the intense temperature of combustion usual in this class of boiler. The use of brass is limited to the tubes, but even these are now often made of steel or iron.

Wrought iron has till lately been the principal metal used in the structure of boilers, but it is now rapidly being superseded by mild steel. The advantages of mild steel are very great. Its strength to resist strains of tension and compression is considerably greater than that of iron, thus

enabling lesser scantlings to do the work. Its ductility is greater, its structure more homogeneous, and its quality more uniform; while its power to resist corrosion, when proper precautions are taken, is reported on most favourably by those who have had the best practical opportunities of watching its behaviour in use. The only drawback which retarded its general introduction was a certain difficulty which the boiler-makers experienced in working the new metal safely into shape, especially when at a black heat; but this difficulty has to a great extent been got over with increased experience.

The following table gives the approximate numerical value of the tensile strength of the three metals. It must be understood that samples of the same metal vary so much that nothing but approximate or rough average values can be given.

Name of Metal	Tensile Strength, lbs. per sq. inch	
	With grain	Across grain
Best Lowmoor plate	58,487	55,033
Ordinary wrought-iron boiler plate	50,000	46,500
Mild Siemen's steel plates, average	64,600	64,500
Brass tubes	80,000	—
Copper plates	30,000	—
Copper bolts	36,000	—

Lloyd's rules for marine boilers require that when the material of the shell plates is mild steel it shall have a tensile strength of not less than 26 tons and not more than 30 tons per square inch of section, and the ultimate elongation of a test piece 8 inches in length after fracture, must be not less than 20 per cent. of the original length.

The Board of Trade rules for steel marine boilers require that the tensile strength of plates not exposed to flame should be about 28 tons and should not exceed 32 tons per square inch of section. The tensile strength of furnace, flanging

and combustion box plates should range from 26 tons to 30 tons per square inch.

Fittings of Boilers.

The principal parts of boilers which now remain to be considered are furnace doors and grates, safety valves, pressure and water gauges, feeding apparatus, and feed heaters.

Furnace Doors.—The chief points to be considered in the design of furnace doors are to prevent the radiation of heat through them, and to provide for the admission of air above the burning fuel in order to aid in the consumption of smoke and unburnt gases. In all cases where the doors are exposed to very rough usage—such, for instance, as in locomotive and marine boilers—the means for admitting air must be of the simplest, and consist generally of simple perforations, as shown in fig. 178, which represents a front view and section of the furnace door of a locomotive boiler. The heat from the burning fuel is prevented from radiating through the perforations in the outer door by attaching to it a second or baffle plate *a*, at a distance of about $1\frac{1}{2}$ inch, the holes in which do not coincide in direction with those of the door proper. By the constant entry of cold air from the outside the greater part of any heat which may be communicated to the door by radiation or conduction is returned to the furnace.

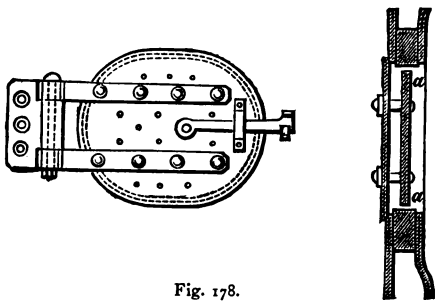

Fig. 178.

Doors similar to the above provide for the constant

admission of limited quantities of fresh air above the fuel. In actual practice, however, air is only needed above the fire for a few minutes after fresh fuel has been thrown on the grate, and is then required in considerable quantities. In the case of land boilers, the furnace doors of which undergo comparatively mild treatment, it is possible to introduce the necessary complications for effecting the above objects. Fig. 160 shows an arrangement in common use in Cornish and Lancashire boilers, and consists of a number of radial slits in the outer door plate, which can be closed or opened at will in the same manner as an ordinary window ventilator. Other and more complicated arrangements have been frequently devised which work admirably so long as they remain in order, but the frequent banging to which furnace doors are subjected, even in factory boilers, soon deranges delicate mechanism.

Furnace doors should be kept as small as is compatible with the proper distribution of the fuel over the grate area, as otherwise the great rush of cold air, when the door is opened, rapidly cools down the flues, and does considerable injury to tube plates, crowns of furnaces, &c. For this reason it is desirable, when grates are over forty inches in width, to have two doors to each furnace, which can be fired alternately.

Dead-plate and Fire-bars.—The dead-plate is a flat plate of iron immediately inside the furnace door, and which is used in many boilers in order to insure the combustion of the volatile portions of bituminous coal. When the fresh fuel is laid on it is placed on the dead-plate instead of on the grate. In this position the coal is coked, the volatile hydrocarbons being driven off by the radiated heat from the incandescent fuel, and ignited as they pass over the latter by the surplus air coming through the grate, or by a special admission through the furnace door. As soon as the coking process is complete the fuel is pushed forward from the dead-plate over the fire-bars. Dead-plates are also frequently

used where anthracite coal is burned, as this fuel is apt to crack and splinter into small pieces if thrown fresh on to the grate without having been previously warmed through.

The grate consists of a number of cast-iron bars, called fire-bars, which are supported on wrought-iron bearers. Innumerable forms of fire-bars have been contrived to meet the cases of special kinds of fuel. The type in common use is represented in fig. 179, which shows a side view and a section of a single bar, and a plan of three bars in position Each bar is, in fact, a small girder, the top surface of which is wider than the bottom. On each bar are cast lugs, the width of which determines the size of the interstices for the

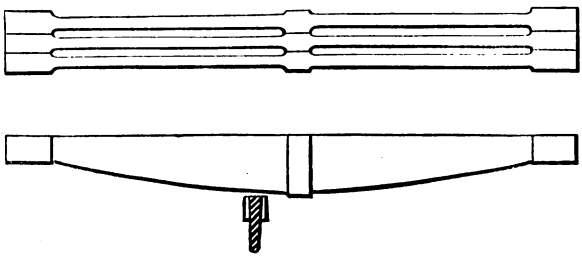

Fig. 179.

passage of air. In marine boilers the usual width of the bar on the top surface is $1\frac{1}{4}$ inch, tapering down to one-third of this size at the bottom. The interstice varies in width according to the character of the fuel. For anthracite $\frac{1}{2}$ inch is a maximum, while for caking coals $\frac{3}{4}$ inch is often used. For long furnaces the bars are usually made in two lengths, with a bearer in the middle of the grate. In the Lancashire boiler, illustrated in fig. 159, the bars are in three lengths of two feet each. They are $\frac{3}{4}$ inch wide on the top, and spaced $\frac{3}{8}$ inch apart. In locomotive boilers the bars are generally in one length. As a rule long grates are set with a considerable slope towards the bridge, in order to facilitate the distribution of the fuel. A slope of an inch

to the foot is the rule. The grates of locomotive engines are nearly always flat.

Safety valves.—The safety valve is a circular valve seated on the outside of the boiler, and weighted to such an extent that when the pressure of the steam exceeds a certain point, the valve is lifted from its seating and allows the steam to escape. Safety valves can be loaded directly with weights, in which case they are called dead-weight valves, or the load can be transmitted to the valve by a lever. Again, the end of the lever is sometimes held down by a simple weight attached to it, a plan commonly adopted in land boilers; while sometimes, as in the case of locomotive and marine boilers, the lever is weighted by means of a spring, the tension of which can be adjusted.

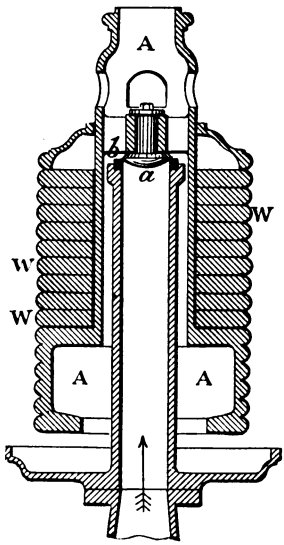

FIG. 180.

Fig. 180 shows a form of dead-weight safety valve, where *a* is the valve which rests on the seating *b*.

The valve is attached to the circular casting AAA, so that both rise and fall together. The weights WW, &c., are disposed on the casting in rings, which can be adjusted to the desired blow-off pressure. Owing to the centre of gravity of the casting and weights being below the valve, the latter requires no guides to keep it in position. This is a great advantage, as guides frequently stick, and prevent the valve from acting. Another advantage of this form of valve is that it is difficult to tamper with. For instance, a four-inch

valve, intended to blow off at 100 lbs. per square inch, would require weights of 11 cwt., which occupy a considerable bulk. An unauthorised addition of a few pounds to such a mass would make no appreciable addition to the blowing-off pressure, while any effectual increment of weight would be immediately noticed. It is quite different with the lever safety valve, about to be described. A small addition to the weight at the end of the lever is multiplied several times at the valve.

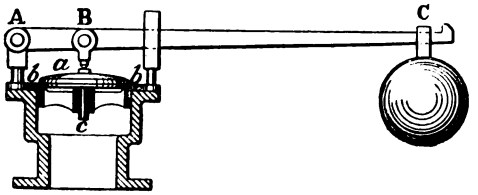

Fig. 181.

The second form of safety valve is shown in fig. 181 Here the load is attached to the end C of the lever ABC, the fulcrum of which is at A.

Calling the weight W,

the weight of the lever w,

the weight of the valve, w',

the distance of the centre of gravity of the lever from A, l,

$$w' + W\frac{AC}{AB} + w\frac{l}{AB}$$

is the pressure brought to bear on the seat bb of the valve a. The effective pressure on the valve, and consequently the blowing-off pressure in the boiler, can be regulated, within certain limits, by sliding the weight W along the arm of the lever. In locomotive engines the weight would, on account of the oscillations, be inadmissible, and a spring is used to hold down the end of the lever. The pressure on the valve can be regulated by altering the tension of the spring.

A valve much used in locomotives is shown in fig. 182. It is called, after the name of its inventor, Ramsbottom's patent safety valve. It consists really of two valves AA, placed side by side, at a little distance apart. A cross-piece B bears upon each valve, and to the cross-piece is attached a powerful spiral spring D, the lower end of which is so fixed at C that its tension can be adjusted by means of a set screw at E which is out of reach of the engine driver. Before the valves can rise they have to overcome the resistance of the spring, to which the pressure is communicated by means of the cross-piece B. The spring is attached to the cross-piece below the bearing points of the cross-piece on the valves. If one of the valves should rise from its seating before the other, the spring leans a little towards this latter, easing the pressure on it, and allowing it to open. The rise of the valves from the seating is much greater with these directly loaded valves than when the pressure is transmitted through a lever, and thus the steam escapes with much greater rapidity.

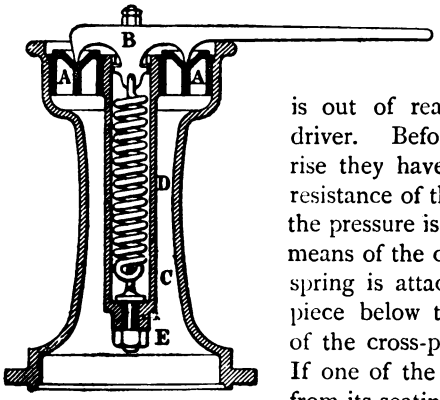

Fig. 182.

Every boiler should be provided with two safety valves, one of which should be put beyond the control of the attendant. The size of the opening depends of course upon the steam-producing power of the boiler, the object to be attained being to reduce the pressure within the boiler to its normal point as quickly as possible. The following rule is given by Rankine for valves having a lift of one-twentieth of their own diameter. Let a = area of valve; A = area of heating surface in square

feet; P = pressure of steam in pounds per square inch. Then—

$$a = \frac{A}{3P}.$$

The Board of Trade rule for marine boilers is to allow half a square inch of safety valve for every square foot of fire-grate area.

Pressure Gauges.—These instruments are used for showing the pressure at which the steam happens to be within the boiler. The one in most common use is Bourdon's, and is illustrated in fig. 183. It consists of a bent metal tube *aa*, which is put in connection with the interior of the boiler by means of the pipe *b*, which is provided with a stopcock. The tube *aa* is elliptical in cross section, as shown at A. The effect of internal pressure on the tube is to tend to transform the elliptical into a circular cross section. This, however, cannot be done without partially unbending or straightening the tube *aa*; that is to say, the effect of internal pressure is ultimately to straighten the tube, and the greater the pressure the more the tube is unbent, and consequently the more the free end *c* is moved from its normal position. The free end is connected by means of a link with an index like the hand of a watch, either directly, or else through the medium of a small rack and pinion, which multiplies the motion of the index; and when the free end of the tube moves under the influence of pressure, the end of the index describes an arc of a circle. By placing a dial behind the

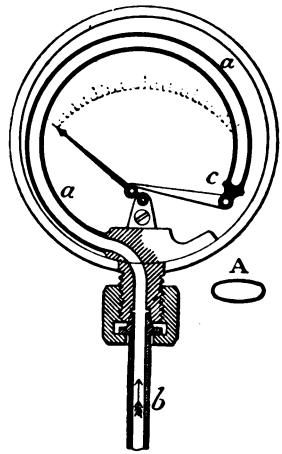

Fig 183.

index, and graduating the former experimentally, so that a given position of the needle corresponds with a given pressure in the tube, we obtain an exact pressure gauge. The experimental division of the circumference of the dial is made by connecting the Bourdon gauge with a mercurial syphon gauge and a force-pump. The force-pump is then worked, so that the syphon gauge registers successive increments of pressure of one pound per square inch, and at each of these a mark is made on the dial of the Bourdon gauge opposite the position of the index finger. These gauges should be tested from time to time by a mercurial gauge, as they are apt to get out of order, in consequence of water lodging in the end of the bent tube and corroding the latter. It may easily be known when they are out of order by raising the pressure of the steam in the boiler, and watching till it commences to blow off at the safety valve, and then noting the position of the index finger. The pressure registered by the finger should of course then correspond with the known blow-off pressure of the valves; if it does not, one or other or both of these instruments must be out of order: but the safety valve is usually kept in order; therefore when this is the case, and a disagreement occurs, the Bourdon gauge may be presumed to need correction.

Feeding Apparatus.—The water of a boiler is replenished by means of force-pumps or injectors, or by both. For safety's sake every boiler ought to have two feeds, in order to avoid accidents when one of them gets out of order. Pumps for feeding are of two principal kinds, viz. those driven by a crank or eccentric on the main axle of the engine, and those which are connected direct to a separate small engine, which is only employed for pumping purposes: these latter are called donkey engines. The feed-pumps of land boilers are usually made large enough to supply, if kept continuously at work, from two to two and a half times the quantity of water actually consumed by the engine.

In old-fashioned marine boilers, where the engine is not provided with a surface condenser, the pumps had to be made still larger, in order to allow for the waste occasioned by the discharge of brine from the boilers. The pumps themselves, being ordinary force-pumps, require no special description.

Injectors.—The injector, which was invented by Giffard, is in many respects the most peculiar and interesting apparatus connected with the steam engine. It is an instrument which converts the energy of the heat in the steam into mechanical work without the aid of any moving mechanism whatever. Before describing it, it is necessary to notice the difference between the velocity of steam escaping from a boiler, and water escaping from the same vessel under the same pressure of steam. The velocity of the water is, in accordance with a well-known law of hydrodynamics, and neglecting the effect of friction, the same as it would acquire by falling down a height equal to the length of the column of water which would produce the same pressure as the steam. Thus, let the pressure of the steam in the boiler be five atmospheres above that of the external air; the pressure of one atmosphere will balance the weight of a column of water 33·9 feet in height; therefore five atmospheres will balance a column of 169·5 feet. The velocity acquired by falling down this height would be about 104 feet per second. This, therefore, would be approximately the velocity of efflux of the *water* from the boiler.

The velocity of efflux of the steam is much greater, although the pressure is the same. It would be impossible in the limits of this chapter to give an account of the theory of the flow of gases and of saturated steam. It must be enough to mention that for the pressures usual in land boilers the velocity of the steam is from 16 to 18 times greater than that of the water.

Suppose now that some of the steam were discharged from a boiler through a pipe at this high velocity, and that while in the act of discharge it were condensed suddenly by

passing through an intensely cold medium; the resulting water would travel forward with the same velocity which it had already acquired when in the state of steam; and if the various particles of water could by any means be gathered together into a continuous stream, they would be more than able to overcome and to force back into the boiler any opposing stream of water of the same size directed against them from the water-room of the boiler. Now the velocity of the condensed steam is so great that it possesses not only energy enough to re-enter the boiler in the face of an opposing stream of water of its own size, but it can also impart energy to a much larger mass of water, so that this larger mass can also enter the boiler. The injector is simply an instrument for allowing steam to rush from a boiler, and to suck up and mix with itself a stream of cold water, by which it is condensed, and to which it imparts so much of its own velocity that the combined mass of cold water and condensed steam enters into and feeds the boiler.

Fig. 184 shows an elementary form of such an injector. A is the section of a boiler, B a pipe leading from the steam space and terminating in a nozzle, C is the cold water pipe leading from the tank, and terminating in a hollow cone surrounding the steam nozzle. When the steam is turned on, and escapes from the lower edge E of the hollow cone, it creates a partial vacuum in the cone and in the pipe C. The water then rushes up the pipe and into the cone sur-

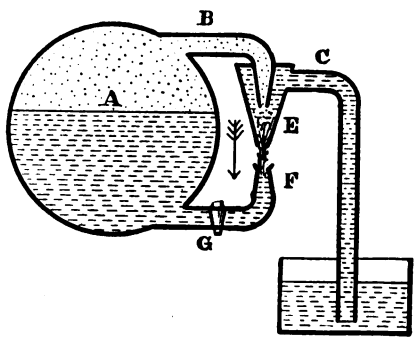

Fig. 184.

rounding the nozzle, where it meets with the escaping steam, which it condenses. The particles of condensed steam, impinging on the water surrounding them, communicate their motion to the latter, and the combined mass is delivered at a high velocity into the feed-pipe F, and through the valve at G into the boiler. Such an injector, if properly proportioned, would work well for a fixed pressure of steam in the boiler, and for a fixed temperature of the feed water. In practice, however, these quantities vary, and injectors must be made to suit all such contingencies. For instance, when the pressure of the steam increases, the area of the opening in the steam nozzle must be increased, and *vice versâ*. There are very many forms of injectors. Fig. 185 illustrates one which is in common use in this country. The steam and water supply pipes, nozzle, and cone are rendered sufficiently clear by the drawing. The steam supply is varied by altering the position of the conical spindle *a*, which can be screwed towards or away from the mouth of the nozzle.

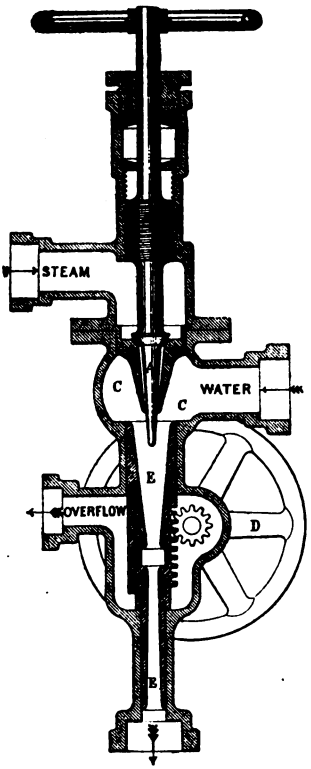

Fig. 185.

The water chamber CC is so arranged that it completely surrounds the steam nozzle. The supply of the water is varied by contracting or expanding the conical aperture

below the mouth of the steam nozzle. This is accomplished by moving the conical sliding tube E backwards or forwards by means of the hand-wheel D and the rack and pinion.

If the supply of steam is not properly adjusted to the water, some of the latter will escape at the aperture made in the sliding tube E into the overflow pipe. For instance, if the supply of steam be too small, the current will not have sufficient energy to enter the boiler, and part of it will choke up the sliding tube and escape by the aperture. When this occurs it is only necessary to turn on more steam, or shut off some of the water.

The efficiency of the injector is measured by the temperature of the current of feed water as it enters the boiler, compared to its temperature before it enters the injector. The less the rise in temperature, the more the energy of the steam is utilised. Theoretically speaking, if we measure the units of heat in the feed water as it enters the boiler over and above the heat before it enters the injector, and subtract the amount from the total heat of the steam used, the result ought to give the useful work which the injector does. A great deal of power is wasted in these instruments, as at present constructed, by the formation of eddies. When used for feeding boilers, all the heat represented by the rise of the temperature of the feed water is of course restored to the boiler. As might be expected, the efficiency of an injector increases as the original temperature of the feed water diminishes. These instruments are also used for other purposes besides the feeding of boilers. They have even been employed on a large scale to drain a mine. In this case the work done was represented by about 80 gallons per minute raised through a vertical height of 240 feet. This probably is the greatest amount of work which has ever been accomplished with an injector, and could of course only be undertaken where expenditure of fuel was no consideration.

Water Gauges.—These are used to ascertain the level

of the water in the boiler. The simplest sort consist of three cocks screwed into the face of the boiler at different levels, one being usually at the normal level of the water, one above this in the steam space, and a third lower down at a level below which it is dangerous to allow the water to sink. By opening these cocks in succession the position of the water level can be approximately ascertained.

Another variety in common use consists of a straight glass tube, so fixed that its upper end communicates with the steam, and the lower end with the water space. Cocks are provided for cutting off the connection at either end, and for allowing steam or water to be blown through the glass tube. The latter is fixed at the ends in metal sockets which allow of its being removed and replaced when broken. With this form of gauge the water level is always visible. It is usual to provide a boiler with both forms of gauge, in order that if one gets out of order the other may be available.

Feed-water Heaters.—It is very desirable, whenever it is possible, to feed the boiler with water of the temperature of or about 212°. There are three good reasons for this practice. In the first place, the introduction of cold water into the hot boiler tends to produce the strains due to unequal temperature which have been already commented on. In the next place, it has been observed that water which has been previously heated, otherwise than by surface condensers, exercises a far less corrosive effect on the boiler than cold water, the corrosive action taking place in the heater instead, where its injurious effects are not nearly so important. Lastly, there is of course a very considerable saving of fuel effected by utilising waste heat to raise the temperature of the fuel. Supposing, for instance, the water were raised from 60° to 212°, there would be a saving of 152 units of heat for every pound of water, which is equivalent to about one-seventh of the total heat required to evaporate the water at 212° from the temperature of 60°.

There are three distinct methods in use of heating feed

water. In modern marine engines fitted with surface condensers the steam condensed from the engines is used over and over again in the boilers. The temperature of water coming from surface condensers should be about 130°. Unfortunately such water is generally more or less charged with fatty acids, generated by the decomposition of the oils used for lubricating the cylinders, and consequently great care has to be exercised to prevent the rapid corrosion of the boilers. Sometimes, also, the lubricant is carried back into the boiler in the shape of a gelatinous, non-conducting substance, which settles on the crowns of furnaces, and prevents the transmission of heat through the plates. The consequence is, the furnace crowns become over-heated and collapse.

With high-pressure non-condensing engines the exhaust steam is frequently used to raise the temperature of the feed. When the apparatus for utilising the heat of the exhaust steam is properly designed, very excellent results may be obtained with this class of heater, but not unfrequently the steam is *forced* through a series of pipes surrounded with cold water, the result being that the back pressure in the cylinder is unduly raised, and much more heat is thus often lost in the engine than is gained by raising the temperature of the feed.

The third class of feed heater utilises the waste heat from the furnaces before it passes up the chimney, and is admirably adapted to factories where room can be spared. The apparatus usually consists of a series of vertical pipes connected together, through which the feed water is forced. The hot air and gases proceeding from the boiler flues circulate between these pipes, and heat the water contained in them. As, however, the tubes rapidly get covered with non-conducting soot, it is necessary to provide each of them with a scraper driven by machinery, which is constantly travelling up and down the tube as long as the apparatus is at work. A feed heater of this description,

applied to the Lancashire boiler illustrated in fig. 159, has sixty tubes, exposing a total heating surface of 600 square feet. It is situated in the base of the chimney. All heaters of this type are, in reality, low-pressure tubulous boilers, and, as such, should invariably be provided with safety valves.

Chimneys and other Means of producing the Draught.

A chimney promotes a flow of air through a furnace, because the hot air contained in the chimney is lighter than the surrounding atmosphere, which consequently endeavours to force its way into the chimney from below in order to restore the balance of pressure. The only way into the chimney is through the fire-bars and furnace, and in passing through these the air maintains the combustion, and at the same time becoming itself heated, makes the action of the chimney continuous.

In estimating the action of a chimney of a given size in producing a draught, the density, temperature, and volume of the products of combustion must be considered.

The nitrogen which passes through undergoes no chemical change, and consequently its density and volume are unaltered except by the change of temperature. The oxygen combines partly with the carbon and partly with the hydrogen contained in the fuel, while a great portion goes through unchanged like the nitrogen. The portion which combines with the carbon so as to form carbonic acid undergoes no change of volume except so far as it is affected by temperature, for the volume of the carbonic acid gas is the same as that of the oxygen from which it is formed, but its density is of course increased by the weight of carbon taken up. The portion of the oxygen which combines with the hydrogen forms steam, the volume of which is greater than that of the oxygen, but the proportion of utilisable hydrogen in fuel is so small that it is usually left out of account.

Consequently the mixed air and products of combustion which escape from a furnace may be considered as approximately of the same volume as the air which is supplied to the furnace when at the same temperature. One pound of air at 32° has a volume of $12\frac{1}{2}$ cubic feet; and as we have seen that when the blast is produced by a chimney 24 lbs. of air are necessary to consume a pound of coal, the volume of furnace gases for every pound of fuel consumed will be, when reduced to $32° = 12\frac{1}{2} \times 24 = 300$ cubic feet. At any other temperature the volume will equal the volume at 32° multiplied by the ratio of the absolute temperature of the new temperature to the absolute temperature of 32° Thus, taking 2000° as the temperature of the furnace, the volume in the above case

$$= 300 \times \frac{2000° + 461°}{32° + 461} = 1497 \text{ cubic feet;}$$

and, generally, $V = V_{32} \times \frac{\tau}{\tau_{32}}$, where V_{32} is the volume at 32°, and τ and τ_{32} the absolute temperatures of the gas when at the heat of the furnace and at 32°.

The density of the current depends on the quantity of air supplied per pound of fuel, and on the final temperature of the products of combustion. Thus, if 24 lbs. of air be supplied per pound of fuel, the volume of this quantity of air at $32° = 24 \times 12\frac{1}{2} = 300$ cubic feet. The weight of the mixture of air and fuel is $24 + 1 = 25$ lbs. and the volume at the temperature τ of the furnace gas is $= 300 \times \frac{\tau}{\tau_{32}}$.

The density, or weight of a cub. ft. $= \frac{25}{300 \times \frac{\tau}{\tau_{32}}} = \cdot 083 \times \frac{\tau_{32}}{\tau}$.

In the general case let w lbs. be the weight of fuel burned; V_{32} the volume of air supplied at 32°.

Then the total volume of the products of combustion $= w\, V_{32} \frac{\tau}{\tau_{32}}$,

the weight of the products $= \frac{w V_{32}}{12\frac{1}{2}} + w$ lbs.;

the density or weight per cubic foot equals the total weight in lbs. divided by the total volume

$$= \frac{w V_{32} + 12\frac{1}{2} w}{12\frac{1}{2} w V_{32}} \times \frac{\tau_{32}}{\tau} = \left(\frac{1}{12\frac{1}{2}} + \frac{1}{V_{32}}\right)\frac{\tau_{32}}{\tau}.$$

The quantity $\frac{1}{V_{32}}$ varies in value according to the air supply.

The Effect of Height in a Chimney.—The difference between the weight of a column of outside air of the height of the chimney above the fire-bars, and standing on a base equal in area to the cross section of the chimney, and that of the column of hot air within the chimney is the measure of the force which produces the draught. Let τ_1 be the outside temperature (absolute measure), and H be the height of the chimney in feet.

Then, since one cubic foot of air at 32°, or τ_{32}, weighs $\frac{1}{12\frac{1}{2}}$ = ·08 lb., therefore $H\left(\cdot 08\frac{\tau_{32}}{\tau_1}\right)$ = weight of column of outside air of height of chimney standing on an area of one square foot. The corresponding column within the chimney weighs $H\left(\cdot 08 + \frac{1}{V_{32}}\right)\frac{\tau_{32}}{\tau}$, and the difference between these two weights is the pressure in lbs. per square foot of chimney section which produces the draught. A column of the hot gas equal in weight to this difference is called the head of the chimney; and just as in hydraulics the velocity of discharge of water from the bottom of a full vertical pipe is proportional to the square root of the height of the pipe, so, in the case of a chimney, the velocity with which air would flow, if unimpeded, into the bottom of the chimney is also proportional to the square root of the height of the head. The height of the head, reckoned in feet of hot gas, is found by dividing the weight of a column of external air as high

as the chimney, as found above, by the weight of one cubic foot of the hot gas (this gives the height of a column of the hot gas weighing as much as the column of the external air). If we subtract from this the height of the chimney, the difference is the height of the head.

In actual chimneys the velocity of the discharge of the gas is greatly diminished by the resistance opposed by the fire-grate and layer of fuel to the entrance of the air, and also by the friction of the sides of flues, tubes, &c., and of the internal surface of the chimney itself.

Peclet gives the following formula for the height of head necessary to produce a given velocity of the gas in the chimney :—

Let l = the length of the chimney + that of the flue leading to it.

v = area of section of chimney divided by its circumference.

f = co-efficient of friction of sides of flues and chimney, which depends for value on the condition of the surfaces.

G = co-efficient of resistance of grate and layer of fuel to entrance of air.

u = velocity of gases in chimney.

g = acceleration due to gravity ; and

h = height of head in feet.

Then, according to Peclet,[1] $h = \frac{u^2}{2g}\left(1 + G + \frac{fl}{v}\right)$.

The value of the co-efficients varies according to circumstances. When the surfaces of the flues are sooty, $f = .012$. With ordinary grates, burning from 20 to 24 lbs. of fuel per square foot of grate surface per hour, $G = 12$, and the formula then becomes—

$$h = \frac{u^2}{2g}\left(13 + \frac{.012l}{v}\right).$$

[1] See Rankine's *Manual of the Steam Engine*, p. 287. Ninth edition.

If the head is given, then the velocity of the gas can be calculated from the same formula; and when this is ascertained, the weight of fuel which can be consumed in a given time may be calculated on the supposition that each pound of coal requires 24 lbs. of air = 316 cubic feet at the ordinary temperature (60°) of the atmosphere.

The use of very high chimneys is in many situations a necessity, not in order to create the draught, but in order to discharge the noxious products of combustion at a considerable distance above animal and vegetable life; otherwise a forced blast might often be more advantageously employed. It is considered that a chimney is most efficacious in producing a draught when the temperature inside it is about 600°, and at this temperature about one-fourth of the available heat of combustion is wasted in creating the draught.

When a forced blast is produced by means of a fan, blast-pipe, or air injector, the products of combustion may be cooled down as far as is found practicable and convenient; and as much less air is required to effect combustion, the saving of heat may be very considerable. On the other hand, heat must be expended in order to produce a forced draught. Thus in the case of the blast-pipe the heat expended is represented by the excess of back pressure in the cylinder.[1] In the case of a fan, the heat consumed in driving the fan must be taken into account; and when an

[1] According to Mr. D. K. Clark's experiments the excess of back pressure over and above the pressure of the atmosphere, caused by the use of the blast-pipe, varies approximately (1) as the square of the speed of piston; (2) as the pressure of the steam at the commencement of the exhaust; (3) inversely as the square of the area of the nozzle of the blast-pipe. He also found that the back pressure was largely increased by the presence of liquid water in the spray. As to the amount of the back pressure in certain cases the reader can consult the examples of diagrams from locomotive cylinders on page 338. Mr. Clark also states the vacuum in the smoke-box to be about 70 per cent. of the blast pressure, while the vacuum in the fire-box is from one-third to one-fourth of the same pressure, and the rate of evaporation varies about as the square root of the vacuum in the smoke-box.

air injector is used, the heat expended in producing the draught is represented by the total heat of formation of the steam used in the injector.

Forced draught.—A forced draught is now frequently applied to the furnaces of marine boilers, especially in ships of war. In torpedo boats it is necessary to develope immense power out of a comparatively small boiler, and some sort of artificial draught becomes an absolute necessity. A blast-pipe is of course impossible, as the engines are condensing, and would, moreover, be quite inapplicable on account of the noise and shape of the funnel. The method which has been adopted is to force the air into the furnace by means of a rotary fan, driven either from the main machinery, or else by a separate engine. If the blast were directed solely beneath the fire by means of the device of a closed ash-pit, it would always be leaking outwards through the furnace door, and would, whenever the latter were opened for fresh fuel, cause the smoke and flame to fly out in the face of the stokers. To obviate this, the plan has been adopted of closing the stoke-hold so as to make it air-tight, and then forcing the air into the closed chamber, which can only escape through and over the fuel and boiler tubes to the funnel. When the furnace door is opened, the compressed air in the stoke-hold rushes through it to the tubes, thus preventing the escape of flame. In this way most remarkable results have been obtained. The defect of the system is that it is difficult, in a boiler of moderate size, to provide sufficient surface to absorb the heat generated by the large amount of fuel which can be burnt on the grate. The result is that the water evaporated per pound of fuel is necessarily low.

The application of forced draught is by no means limited to the boilers of torpedo boats. In large men-of-war the system is also applied for the purpose of obtaining a large additional supply of steam when extra speed is required. The general result attained may be stated as follows. With

natural draught about 10¼ horse-power are indicated per square foot of fire-grate at full power; while with forced draught between 16 and 17 horse-power are obtained. The pressure of air being about two inches.

In the mercantile marine forced draught is beginning to receive considerable attention, for two reasons. First, its application will enable a considerable saving to be effected in the weight of boilers, provided always that the evaporation per pound of coal burnt be not seriously diminished. Second, it enables very inferior and cheap fuel to be utilised.

In the mercantile marine, hitherto, the closed ash-pit has been used instead of the closed stoke-hold. Fig. 186 illustrates the system applied by Mr. Howden successfully

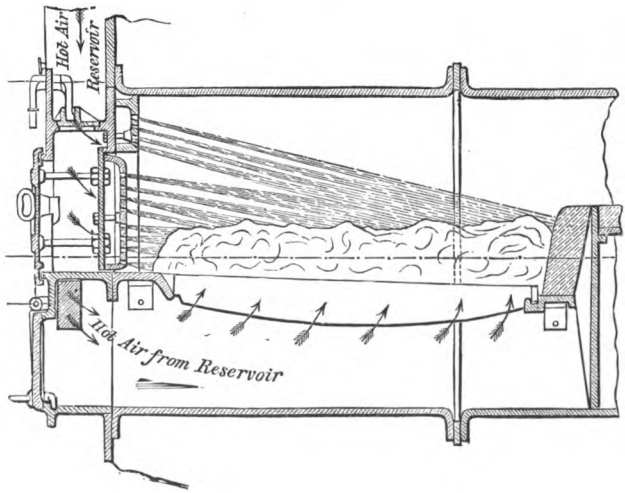

Fig. 186.

to the boiler of a merchant steamer. The ash-pit is closed in front by a door, and the furnace door is double with a hollow chamber between its outer and inner faces. Air

under pressure is supplied beneath the fuel through the ash-pit, and above the fuel through the holes in the inner furnace door, the air issuing through these holes being under considerably higher pressure than that in the ash-pit. The object of this double admission is to secure the complete combustion of the fuel with a very moderate supply of air, and thus to increase the temperature of the products of combustion. The air supply is heated by the waste gases from 180° to 200° above its ordinary temperature, and whenever the furnace door is opened the current of air is cut off for the moment so as to prevent the sudden cooling down of the furnace.

At first sight it would appear that the forced draught system is of no special advantage where economy of fuel is a primary consideration ; for, although by its use the grate area may be diminished, nevertheless, the heating surface and the total weight of boiler cannot be reduced. It must however be borne in mind that if the supply of air be properly regulated, the volume of the products of combustion will be greatly diminished, and their temperature increased; consequently, a less heating surface is required to produce the same evaporative results per pound of fuel than when natural draught is employed. Also with forced draught it is possible to make use of tubes of comparatively small diameter, and consequently a considerably increased heating surface can be obtained without increasing the dimensions of the boiler.

CHAPTER X.

CONDENSATION AND CONDENSERS.

The object and advantages of condensing steam—General description of condensers—Quantity of water required to condense steam—Objects of surface condensation for marine engines—Description of a jet condenser for a stationary engine—Description of a marine surface condenser—Air-pumps—Ejector condensers—Method of indicating the vacuum.

THE condenser may, in a certain sense, be described as having the inverse functions of the boiler; for, whereas the latter is employed to raise the medium with which the engine works to the superior limit of temperature, the purpose of the latter is to reduce the inferior limit of temperature as far as possible. The boiler fulfils its purpose by converting the feed water into steam, and the condenser by re-converting that steam after it has done its work into water.

The advantages from the thermal point of view of condensing the steam, instead of allowing it to escape into the open air at a little above the atmospheric pressure, are very easily explained by reference to the principles which enable us to calculate the maximum efficiency of heat engines (see p. 88). Suppose, for instance, that we have two precisely similar engines working with steam of 50 lbs. pressure per square inch absolute, and one provided with a condenser, while the other discharges the exhaust into the open air. Suppose, also, that the former expands down to a pressure of 3 lbs. absolute, and the latter down to a pressure of about 3 lbs. above the atmosphere, say 18 lbs. absolute.

The relative theoretical efficiencies of the two engines may be expressed as follows. The temperature of steam of 50 lbs. absolute is 280·5°, while that of steam of 18 lbs. absolute is 222°·5, and of 3 lbs. absolute is 141·5°. Then, according to the principles of the efficiency of heat engines, the maximum efficiency of the condensing engine is—

$$\frac{280{\cdot}5-141{\cdot}5}{461+280{\cdot}5}=\frac{139}{741{\cdot}5};$$

while that of the non-condensing engine is—

$$\frac{280{\cdot}5-222{\cdot}5}{461+280{\cdot}5}=\frac{58}{741{\cdot}5}.$$

Thus the condensing engine is theoretically the more efficient of the two in the ratio of 139 to 58·5, or 2·37 to 1.

From the mechanical point of view, as illustrated by the indicator diagram, the advantages of condensation are most apparent, for it enables the back pressure to be reduced from some three pounds above the atmosphere to, say, ten or eleven pounds below it. Also, as in non-condensing engines it is practically impossible to expand the steam below the atmospheric pressure, it is evident that condensation enables us to make use of much higher grades of expansion than would otherwise be possible. This latter is merely another mode of expressing the advantage explained above by reference to the principles of thermodynamics.

The condenser is an apparatus into which the steam is discharged when it has done its work, and where it comes in contact either with a jet of cold water, or else with a large area of metallic surface, one side of which is kept cool by contact with cold water. The steam on entering this chamber is instantly condensed, giving up its heat to the water; and the result would be, if a sufficient quantity of water were used, the formation of a practically perfect vacuum, were it not for the fact that the feed water usually contains a large quantity of air, which passes over with the

exhaust steam into the condenser, and exerts a back pressure against the piston. In order to get rid of this air, an air-pump, driven by the engine, is fitted to the condenser, and is also made use of to pump away the water into which the steam condenses. Various types of condensers, together with their fittings, are described and illustrated on pages 429 to 437.

Quantity of water required to effect condensation.—Suppose the steam is expanded in the cylinder down to a pressure of say 4·5 lbs. per square inch, the temperature corresponding to which is 158°; and suppose, further, that the temperature of the final mixture of condensed steam, and of condensing or injection water is to be 110°, then for every pound of steam which enters the condenser the injection water will have to absorb the total heat of the steam of 158° above the water of 110°.

The total heat of steam of 158° is 1,130 thermal units; subtracting from this the heat of water at 110°, which is approximately 110 thermal units, we have 1,020 units, which have to be absorbed by the injection water. The quantity of the latter required obviously depends upon its initial temperature. Suppose the latter to be 50°, each pound of it can absorb $110 - 50 = 60$ thermal units by rising in temperature to 110°. Therefore the total quantity of water required is $\frac{1020}{60} = 17$ lbs. This, therefore, is the *minimum* quantity of water required under the given circumstances.

It is obvious from the foregoing that the quantity of injection water required in any given case depends upon the final pressure of the steam, the initial temperature of the injection water, and the temperature at which it is required that the mixture of injection water and condensed steam should be maintained.

Let T_1 = the temperature of the steam when the exhaust opens.

Let L = the latent heat of the steam at this temperature.

Then $T_1 + L$ = the total heat in thermal units of 1 lb. of the steam.

Let T_2 = the temperature of the injection water.

Let T_3 = „ „ final mixture.

Let W = the weight in pounds of the injection water per pound weight of steam.

The injection water in rising from T_2 to T_3 absorbs—

$(T_3 - T_2)$ W thermal units.

The pound of steam in falling from the condition of steam at T_1 to that of water at T_3 gives out—

$T_1 + L - T_3$ thermal units.

Now the heat lost by the steam must equal that gained by the injection water. Hence we have—

$$(T_3 - T_2)W = T_1 + L - T_3.$$

$$\therefore \quad W = \frac{T_1 + L - T_3}{T_3 - T_2}.$$

The numerator of this fraction is the expression for the total heat of steam of the temperature T_1 over and above the heat contained in water at the temperature T_3, which (see p. 100)

$= 885{,}200 + 235{\cdot}46\,(T_1 - 212°) - 772\,(T_3 - 32)$ foot-lbs.

Reducing and dividing by 772, so as to obtain thermal units, and substituting the result in the above formula for W, we get—

$$W = \frac{1114 + {\cdot}3T_1 - T_3}{T_3 - T_2}.$$

EXAMPLE.

Find the amount of injection water required when the exhaust steam has a pressure of 19·5 lbs. absolute, the injection water a temperature of 60°, and the required temperature of the mixture 110°. The temperature T_1 of steam of the above pressure is 227°. Hence—

$$W = \frac{1114 + {\cdot}3 \times 227 - 110}{110 - 60} = 12{\cdot}4 \text{ lbs.}$$

Surface condensation.—In former times, when the pressure of steam rarely exceeded 35 lbs. per square inch, jet condensers were universally used for marine engines. The boilers were fed from the hot mixture of condensed steam and injection water, which contained nearly as large a per-centage of salt and other solid matters as the sea-water itself. The necessity of making marine engines more economical of fuel led to the abandonment of jet condensers at sea, and the substitution for them of surface condensers, in which the steam is condensed by contact with a cold metallic surface, all mixture of the condensed steam and the injection water being avoided.

There were two principal reasons for this change. The first was that when jet condensers were used the boilers could only be fed with salt water, which during the process of evaporation became constantly more and more saturated with salt, and which would, unless special measures were taken, eventually deposit its solid contents in large masses on the heating surface, and thus destroy the boiler, or render it useless. The only way to avoid this was from time to time to blow off large quantities of the brine in the boilers, and to supply its place with corresponding quantities of feed water. The hot water blown away from the boiler involved, of course, a large loss of heat, the amount of which depended on the state of saturation which the water was allowed to reach before blowing off. If the latter be allowed to reach three times the density of sea-water, the loss of heat in blowing out would be 7·4 per cent. of the total heat supplied to the boiler; and if less densities were permitted, the loss was considerably greater. The maximum density permissible was three times that of sea-water. The average loss of heat due to blowing off may fairly be set down as equal to from 12 to 15 per cent. of the total fuel supply. In order to save this loss it was necessary to feed the boilers with fresh water, and to use the condensed steam over and over again as feed water. This of course could only be

effected by keeping the steam separate from the condensing water, and hence the introduction of surface condensers.

The second reason which led to the abandonment of jet condensers was that, in order to effect any considerable economy in the engine as distinguished from the boiler, it was necessary to resort to higher pressures of steam. Now if the temperature of the steam be raised above 280°, which is that due to 35 lbs. per square inch above the atmosphere, the sulphate of lime contained in the sea-water is deposited in hard and insoluble layers all over the boiler surface, and destroys the efficiency of the heating surface. Hence for this reason also the use of fresh water in the boilers, and consequently of surface condensation, became a necessity.

The amount of water required to condense the steam when surface condensers are used depends upon the efficiency of the cooling surface in abstracting the heat, and this again depends upon the thickness and conductivity of the metallic surfaces, their condition as to cleanliness, and the difference in temperature between the two sides.

In spite of the fact that the difference between the temperatures is much less than in the case of boilers, the efficiency of the cooling surface of the condenser in abstracting heat is far greater than that of the heating surface of the boiler in transmitting it. This is due in part to the fact that the condenser surfaces are much thinner than are the tubes and plates of a boiler, and they are also as a rule much cleaner. For these reasons condensers have usually less than half the surface found necessary for the boilers. Peclet found experimentally that sheet copper backed by water of the temperature of from 68° to 77° was capable of condensing 21 lbs. of steam per square foot per hour; while Joule, adopting special means, condensed as much as 100 lbs. per square foot in the same time; but in practice it is usual to allow one square foot for every 13 lbs. of steam of the terminal pressure usual in compound marine engines. And even with this large allowance of surface the amount

of cooling water required is about forty per cent. greater than with jet condensers. The usual allowance is about 30 lbs. of water per pound of steam for vessels which run in the temperate zone, and about 35 lbs. for the tropics.

In order to realise the advantage due to maintaining the greatest possible difference of temperature between the two sides of the metallic surfaces, the cooling water must be kept in a constant state of circulation through the condenser by means of a special pump, which removes the water which has been warmed by the condensing steam from contact with the plates. As the air-pump of a surface condenser is only required to pump the condensed steam and air, it may be considerably smaller than the pump of a jet condenser; but in spite of this advantage surface condensers are much larger, heavier, and more costly than those in which the steam comes into direct contact with the cooling water.

Examples of condensers.—Fig. 187 gives a transverse and longitudinal section of a jet condenser, applied to the quick-running Allen stationary engines. The plunger of the pump

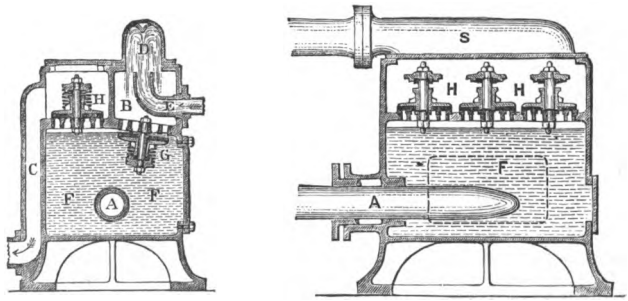

Fig. 187.

A is worked direct by the engine piston-rod prolonged backward. The exhaust steam enters the condensing chamber B showed in transverse section, by means of the pipe S. It there meets with the jet of water D, which enters by the pipe E, and is condensed. Every time the plunger is with-

drawn, the pressure in the condensing chamber predominates over that in the pump chamber F, the valves, of which one

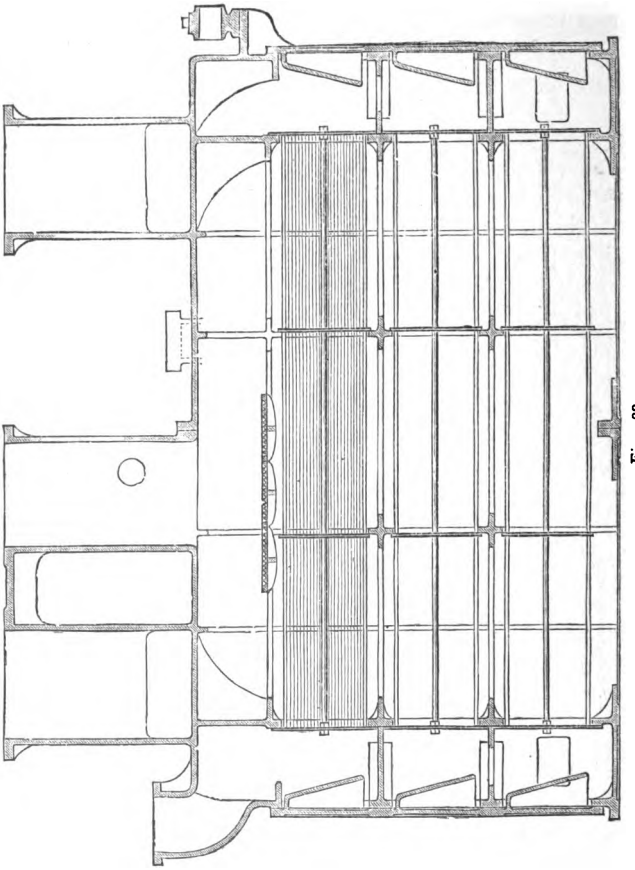

Fig. 188.

is shown at G, open, and the mixed air, vapour, and water enter F. When the plunger returns, it displaces its own volume of water, the level of the water in F rises, and forces

out the air and vapour, and a certain amount of water, through the valves HH, the hot water flowing away through the pipe C to a receptacle called the hot well. The valves G and H, which are made of india-rubber discs, are closed

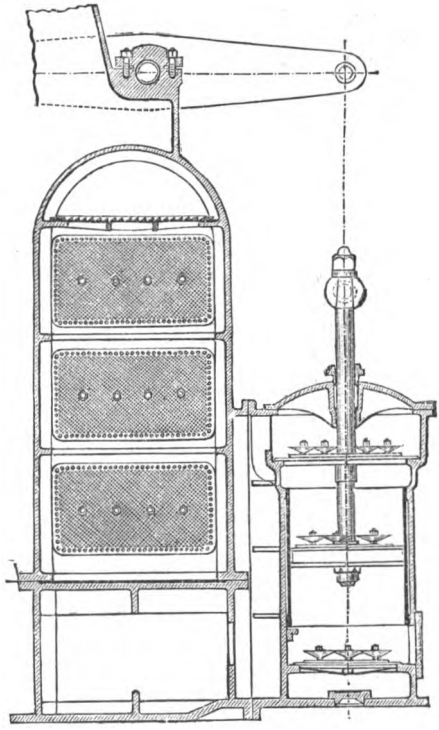

Fig. 189.

by spiral springs which exert a pressure equivalent to a quarter of a pound per square inch. With very quick-running engines these springs are found preferable to so arranging the valves that they may close by their own weight. It will be noticed that the valves G G are seated on a slope.

This is to allow the air and vapour as they come through to escape easily to the surface of the water in F, and so avoid the possibility of the plunger working in a mixture of air and water, which would injure its efficiency. The surface of the water in this type of condenser is the real air-pump, for it is by its rising and falling that the air is expelled and admitted. The velocity with which the surface rises and

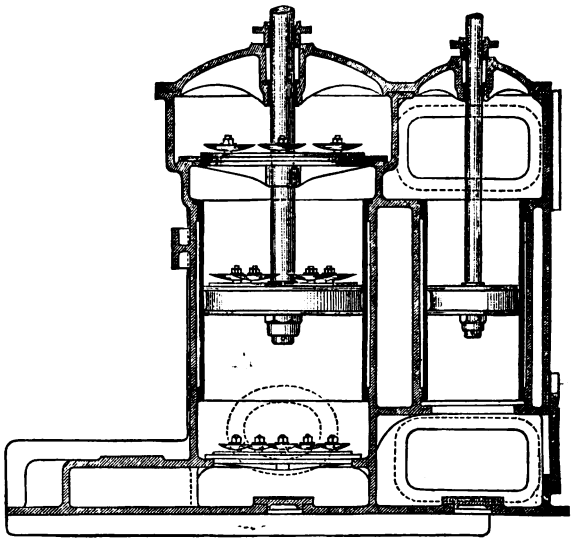

Fig. 190.

falls is, on the average, about 30 feet a second, while the velocity of the plunger is 800 feet.

Figs. 188, 189 illustrate longitudinal and transverse sections of a modern marine surface condenser for a compound engine capable of working up to 3430 horse-power. It contains 3402 brass tubes of $\frac{3}{4}$ inch diameter external and 15 feet long. The total cooling surface is 10,018 square feet, being at the rate of a square foot of surface to ·342 horse-power. The tubes are arranged in three horizontal nests, and in them

the cooling water circulates, entering the bottom and being discharged after passing through the top nest. The steam enters at the top, passes round the outside of the tubes, and is distributed evenly by means of the perforated plates, of which some are shown in section above the top tubes in fig. 188.

The air-pump which is shown in section in fig. 189 is worked by a lever from the cross head of the main engine. It is of the single-acting vertical type, and is similar in principle to the ordinary lift-pump. When the piston or bucket ascends, it draws the condensed steam, air, and vapour through the lower or foot valves, and at the same time lifts whatever has passed through the piston-valves on the down stroke through the head valves shown at the top of the pump, after passing which the water flows away to the hot well. There are two of these air-pumps to the condenser in question, each having a stroke of 3 feet, a diameter of 34 inches, and a combined discharging capacity of about $\frac{1}{21}$ of the volume of the low-pressure cylinder. The barrels are of gun-metal, and the valves are indiarubber discs of the type shown in fig. 191, where the full lines represent the disc, above which is a curved metal guard plate which prevents the valve rising too high, and by its shape ensures a quick return of the valve to its seat, when the pressure which causes it to open is removed.

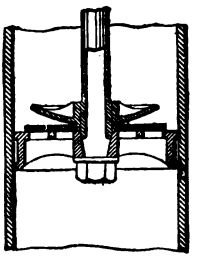

Fig. 191.

Fig. 190 shows another section of the air-pump, together with the circulating pump which forces the cold water through the tubes. The two are cast together in one piece. The circulating pump is a double action force-pump of 3 feet stroke and 20 inches diameter. When the engines are making 55 revolutions, it is capable of discharging nearly 9 lbs. of water per square foot of cooling surface per minute.

The circulating, like the air pump, is worked by a lever from the cross head of the main engine. Very often centrifugal pumps are used for circulating the water in surface condensers, and not infrequently they are driven by a separate engine.

Condenser tubes are almost invariably made of brass, which is sometimes tinned on both surfaces. Copper, though a better conductor, is never used, as the fatty acids formed in the condenser from the lubricating materials carried over by the steam from the cylinders attack the metal and form salts of copper, which, becoming dissolved in the condensed steam, are carried back into the boiler where they act most injuriously on the iron plates.

The packing of the ends of the tubes, so as to make a steam-tight joint, is a troublesome and expensive operation. The method in most general use is shown in fig. 192. In the thickness of the tube plates small stuffing boxes are formed, the tape packings at the bottom of which are tightened up by means of screwed ferrules. When the tubes are se- vertically, the bottom ferrule is flanged so as to overlap the end of the tube to prevent the latter from dropping out, should the packing become loose.

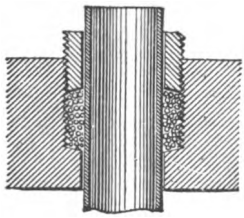

Fig. 192

Air pumps.—Nothing is more important in a condenser than the design of the air-pump. If the condenser is of the old-fashioned type the pump has to discharge not only the condensed steam, and the condensing water, but also the large amount of air which is always present in sea water, and which of course expands in volume when raised to the temperature of the condenser. It may be stated that, on an average, when the cooling water has a temperature of 60° and the condenser 120°, the discharging capacity of the air-pump should be from thirty-six to forty times the volume

of the water into which the steam condenses. Hence its theoretical capacity may be calculated when we know this latter quantity, and the number of revolutions made by the engine per minute, and also the nature of the pump, whether double or single acting. The actual dimensions are determined when we know the efficiency of the pump—that is to say, the ratio which its actual bears to its theoretical discharging capacity. Vertical single acting air-pumps are by far the most efficient. They are almost always used with vertical engines, but when the latter are of the horizontal type, the use of a double-acting horizontal pump is often unavoidable. The actual efficiency of a single-acting vertical pump is, in the most favourable circumstances, about 60 per cent. of the theoretical, but should not in general be taken as more than 50 per cent. The average efficiency of horizontal double-acting pumps is about 35 per cent., or in other words the actual size of such a pump should be about three times greater than is theoretically necessary.

In surface condensers the pumps have only to discharge the condensed steam, and any small quantity of air which comes over by leakage ; but as surface condensers are generally arranged to act with a jet in case of necessity, it is usual to make the pumps much larger than is ordinarily necessary, though not so large as if jet injection were the rule.

Ejector condensers.—Fig. 193 illustrates a type of condenser which has no air-pump nor other moving parts. It is similar in principle to the injector described on page 409. Water having a pressure due to a few feet of head enters by the pipe A and flows through the nozzle B. Exhaust steam from one cylinder enters by the pipe C at a velocity varying with its pressure. If the pressure is 5 lbs. absolute, the velocity of the steam entering a vacuum of 25 inches of mercury is about 1200 feet per second. The entering steam surrounds the nozzle B, and is condensed on coming in contact with the cold jet issuing from B, and increases ten

energy of the stream, just as the boiler steam imparts energy to the feed water of an ordinary injector. When the condenser is used with a two-cylinder engine, the exhaust steam from the second cylinder is arranged to enter by the pipe M, and is condensed by the combined jet issuing from the nozzle F. The steam from the second cylinder further increases the energy of the jet which passes through the nozzle N and into the trumpet-shaped pipe P, where its velocity is gradually reduced. With a steam pressure equal to 14·7 lbs. per square inch absolute at the commencement of exhaust, and a double-cylinder engine, the energy imparted to the jet is sufficient to raise the discharged water to a height of 6 or 8 feet above the condenser; or, if there is no head of condensing water to start with, the apparatus can raise the water from a level of 6 to 8 feet below the condenser. In the latter cases, however, the apparatus must be started by means of a jet of boiler steam, introduced through the pipe D and down the hollow spindle E. The rate at which the condensing water flows to the nozzle B is regulated by shifting the spindle E up or down by means of a hand-wheel and screw.

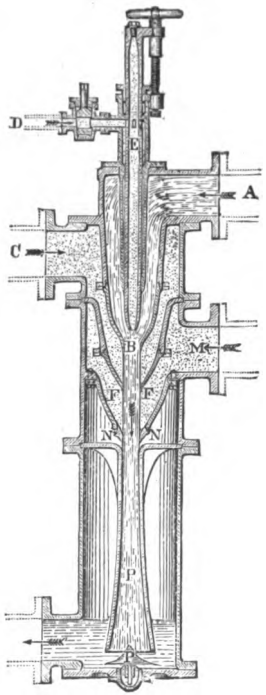

Fig. 193.

By means of these ejector condensers a perfectly steady vacuum can be maintained of 24 to 25 inches of mercury. The initial temperature of the cooling water should not, in general, exceed 60°, and the quantity of water supplied

should be such that its temperature, on issuing from the condenser, would not be raised more than 20° to 25°. Experiments have however been made in which the rise of temperature was 64° and the final temperature 120°; but the vacuum at this temperature fell about two pounds per square inch. It must be borne in mind that, with condensers of this type, the whole of the power usually employed in driving the air-pump is saved, the energy of the exhaust steam being alone sufficient for the purpose of discharging the condensing water and the condensed steam.

Vacuum indications.—The amount of vacuum is indicated by a Bourdon pressure gauge, graduated in inches of mercury. It does not show the actual pressure in the condenser, but the difference between that pressure and the external atmosphere. For instance, if the barometer stood at 30 inches, and the indication of the vacuum gauge were 26 inches, the actual pressure would be $30-26=4$ inches of mercury. If the pressure in the condenser remained constant, and the outside barometer varied, the indications of the vacuum gauge would vary correspondingly.

In order to know if the vacuum is good or bad, notice must always be taken of the state of the barometer. The maximum attainable vacuum depends on the temperature at which the condenser is maintained. Thus if the latter be 120°, the pressure of vapour formed at this temperature is 1·68 lb. per square inch; then if the barometer stand at 30 inches of mercury, or 14·7 lbs. per square inch, the maximum vacuum attainable will be $14{\cdot}7-1{\cdot}68=12{\cdot}02$ lbs. per square inch.

CHAPTER XI.

ON SOME OF THE PRINCIPAL CAUSES OF LOSS OF EFFICIENCY IN STEAM ENGINES, AND THE METHODS EMPLOYED FOR REDUCING THE LOSS—SUPERHEATING —STEAM JACKETING—COMPOUNDING.

Early improvement in the steam engine consisted in the separation of the functions of steam generating, steam using, and condensing—The cylinder, even in modern engines still acts as a generator and condenser—Experimental confirmation of the foregoing—Cause of condensation and re-evaporation in engine cylinders—Hypothetical example showing the successive stages of condensation and re-evaporation—Injurious effects of presence of water in cylinders—Four principal causes of the presence of water in cylinders: 1, priming; 2, excess of condensation over re-evaporation; 3. liquefaction due to work done; 4, loss of heat by radiation—Influence of dimensions of cylinder, pressure of steam, and rate of expansion on the initial condensation—Experiments on steam consumption in conducting and non-conducting cylinders—Means employed to diminish the loss due to liquefaction: 1, superheating the steam; 2, steam jacketing—The benefits derived from the use of steam jackets—Cases in which jackets are useless—Precautions to be observed in jacketing—Experiments showing the use of jackets in simple and compound engines—3, cushioning, or compressing the exhaust steam; 4, compounding—Variations in temperature of cylinders due to working high-pressure steam expansively—Compounding reduces the variation of temperature in each cylinder—Tandem compound engines—Two-cylinder receiver compounds—Three-cylinder ordinary compounds—Triple compound, or triple expansive engines—Experimental demonstration of the saving in fuel to be effected by compounding—The distribution of the steam in the various types of compound engines—Actual indicator diagrams of the various types of compound engines—The manner of reducing the diagrams of compound engines—The mechanical advantages of compound as compared with simple expansive engines—Example of the curve of twisting moments on the crank of a triple compound engine—The relative sizes of the cylinders of compound engines—Means of equalising the power developed in the separate cylinders of compound engines—Table of cylinder ratios for various types of compound engines working with different pressures of steam.

IT was pointed out in the opening chapter that in the elementary form of steam engine the cylinder served the triple purpose of boiler, engine, and condenser. It was

also shown that the principal subsequent improvements consisted mainly in the separation of these functions, the generation and condensation of the steam being now invariably effected in separate vessels, while the cylinder is reserved for its true purpose—the conversion of the energy in the steam into mechanical power. In spite of these improvements, the cylinder, however, still acts in expansive engines, to a great extent both as a steam generator and condenser; and, moreover, it exercises these functions precisely at the wrong times, for, as will be shown presently, the metal sides and ends abstract heat from the entering steam, causing a partial condensation at the moment when it is most desirable to maintain the temperature; while, on the other hand, the steam thus condensed is partly re-evaporated during the whole period of expansion, when it can only give out a fraction of its original energy, and partly during the exhaust period, when it not only gives out no useful energy whatever, but also tends to impair the vacuum and to increase the back pressure. It is this peculiarity which accounts, in a great measure, for the wasteful performance of many engines which use steam expansively.

The foregoing statements have been confirmed by numerous experiments on steam engines, in which the feed water supplied to the boiler was carefully measured, and the steam used in the engines was calculated from indicator diagrams for the point before release, in the manner explained in Chapter VIII., p. 343. In engines which work expansively, the amount of steam used, calculated from the diagrams, is nearly always considerably less than that supplied by the boiler, even after every allowance is made for priming and for the condensation due to the mechanical equivalent of the work done. When no special precautions are taken the deficiency is sometimes enormous. This proves that besides the steam which issues at exhaust and which is accounted for by the indicator diagram, there is also present in the cylinder a quantity of water which must continue to evaporate during

the period of exhaust, and which may partly be carried over mechanically into the condenser, and which has no more useful effect than if it were passed straight from the boiler to the condenser.

This peculiarity of engines which use steam expansively is accounted for by the great difference which exists between the initial and final temperatures of the steam in the cylinder. Thus, if the entering steam had a pressure of 150 lbs. per square inch, and were expanded down to 20 lbs. absolute, the initial temperature would be 358°, and the final 228°, showing a difference of 130°; while, if the engine were condensing, the cylinder would be exposed, during the whole of the exhaust, to the temperature of the condenser, say 158°, showing a difference in this case of 200°. During the whole period of expansion and exhaust, the surface of the metal forming the sides and ends of the cylinder and piston is being cooled down by the combined effects of radiation to the moist steam—which readily absorbs radiant heat—and conduction to the condensed steam, which is re-evaporated from the metallic surfaces. The result is, that when the fresh steam re-enters the cylinder, it comes in contact with the relatively cool surfaces, and a large portion of it is condensed; the latent heat of the steam condensed raising the temperature of the sides up to that of the entering steam. In order to make up for the steam condensed, fresh steam rushes in from the boiler, and thus much more steam enters during the period of admission than is required to fill the cubic contents of this portion of the cylinder.

When the steam is cut off and commences to expand, its pressure and temperature fall, but, during the early period of expansion, its temperature is generally still higher than that of the sides of the cylinder, and more condensation takes place. At the same time, however, the reduction in pressure allows a portion of the water formed by condensation during the admission period to re-evaporate; for this water

has, when formed, the temperature of the entering steam, and therefore, when the pressure in the cylinder falls, it possesses too much heat to remain in the state of water, and part of it boils off, the evaporation being aided and increased by the supply of heat forthcoming from the hot sides and end of the admission portion. Thus, during the whole period of expansion, both condensation and re-evaporation go on. During the earlier period of the stroke the condensation predominates, but towards the middle and end of the stroke, as the temperature of the expanding steam approximates more closely to that of the sides of the cylinder, the condensation becomes less and less, and the re evaporation becomes more active, and consequently the latter predominates, while, during the exhaust, in consequence of the very low pressure, the circumstances are most favourable for re-evaporation.

It must be clearly understood that the action of the sides of the cylinder above described goes on, no matter how perfectly the outside of the cylinder may be protected by means of non-conducting coverings. The latter are only useful to prevent radiation of heat from the outer surface of the cylinder, and in no way prevent the condensation due to loss of temperature by the sides during expansion and exhaust, though, as will be subsequently shown, the failure to protect the outside of the cylinder may enormously increase the action of the sides.

The effects due to variation of temperature in the cylinder may perhaps best be understood by examining the case of a cylinder of given dimensions under circumstances which, though they never actually occur in practice, are nevertheless approximated to in very bad cases.

We will take as an example a cylinder the thickness of the metal of which is only $\frac{1}{100}$ of an inch, and which is supposed to be capable of instantly following the temperature of the steam. The cylinder is further to be lagged with a perfectly non-conducting material; the ends and

piston are supposed to be of the same thickness as the sides. The steam is cut off at one-tenth of the stroke. The following are the remaining data :—

Diameter of cylinder . .	= 2 feet
Length of stroke . . .	= 4 ,,
Length of admission portion .	= ·4 ,,
Area of ends	= 3·14 square feet
Area of piston . . .	= 3·14 ,,
Area of admission portion of sides	= 2·51 ,,
Initial pressure of steam . .	= 90 lbs. per sq. inch absolute
Initial temperature of steam .	= 320°
Exhaust temperature . .	= 157°·5

We will next suppose that the temperature of the cylinder when the steam enters is the same as that of the exhaust. The exposed surface is that of the cylinder end, the piston, and the admission portion of the sides, or, in all, 8·79 square feet, and weighing 3·5 lbs. If the metal is supposed to be iron, the specific heat will be ·11 nearly, and the number of thermal units required to heat up this weight of metal by $320° - 157°·5 = 162°·5$ is–

$$3·5 \times 162·5 \times ·11 = 62·56.$$

We must next find out what proportion of the heat contained in the entering steam goes towards raising the temperature of the sides, &c., of the cylinder. The volume of the admission portion of the cylinder is 1·256 cubic feet, and the weight of steam contained in this space is ·26 lb. Now the latent heat of 1 lb. of steam of 90 lbs. pressure is about 888 thermal units, and, therefore, $888 \times ·26 = 231$ thermal units is the latent heat of the steam which fills the admission space. Now, as we have seen, about 62·5 units must be applied to heating up the metal of the cylinder, that is to say 27 per cent., or over one-fourth of the steam condenses to water of the temperature of 320°, and a corresponding quantity of fresh steam from the boiler enters the cylinder in order to make good the deficiency.

Let us next consider the state of things when the steam

has expanded to double its bulk. It will then occupy a length of ·8 foot of the cylinder. Its final pressure will be about 45 lbs., its final temperature 275, and the mean temperature of the steam during expansion $= \frac{275+320}{2} = 297°{\cdot}5$ approximately, which is still about 138° above the temperature of the exhaust. Now, to heat up the 2·51 square feet area of side in which the expansion has taken place requires $2{\cdot}51 \times {\cdot}4 \times 138 \times {\cdot}11 = 15{\cdot}25$ thermal units, which quantity of heat must be supplied by the condensation of a portion of the expanding steam. But, in the meantime, the temperature imparted to the metal during the period of admission is 22°·5 above the mean temperature of the steam during the period of expansion which we have just been considering. The temperature of the 27 per cent. of ·26 lb. of condensed steam is also 22°·5 above the average, and is consequently too hot to remain as water at the reduced pressure, and consequently a portion of it re-evaporates. The supply of heat available for re-evaporation is $(3{\cdot}5 \times 22{\cdot}5 \times {\cdot}11) + ({\cdot}067 \times 22{\cdot}5 \times 1) = 10{\cdot}17$ thermal units; so that during the first stage of the expansion 15·25 units are abstracted from the steam, while only 10·17 units are given back to it, and consequently condensation largely preponderates over re-evaporation.

If the stroke were divided into ten equal portions, and the processes of condensation and evaporation followed in each, in the manner above indicated, it would be found that in the next stage the two processes would be nearly equal, and that throughout the remainder of the stroke the evaporation would preponderate.

If we suppose that the pressure of the steam before release is about 9 lbs. per square inch, its temperature will be about 188°, or about 30°·5 above the exhaust. The weight of the whole exposed surface of the cylinder and piston will be 12·5 lbs., and if this fall to the temperature of the condenser, $12{\cdot}5 \times 30{\cdot}5 \times {\cdot}11 = 42$ thermal units will be available

for re-evaporation during the exhaust period, without counting the surplus heat in whatever water may be present in the cylinder at the end of the stroke.

The above description of what goes on in a hypothetical cylinder differs very materially from the actual series of operations as they occur in steam engines. In the first place, the metal of which the cylinder is composed is greatly thicker than in the example given. When once the thick mass of metal has been heated up, it probably exercises a moderating influence on the maximum difference of temperature. Again, metal does not possess the power of *instantly* taking up the temperature of the steam with which it may be brought in contact. A certain interval of time is necessary, and hence the speed with which the series of operations is repeated, or, in other words, the velocity of the piston may materially affect both the initial condensation and the re-evaporation. Moreover, the dryness or wetness of the sides exercises an immense influence on the condensation. Suppose, for instance, that, in the above example, the admission space of the cylinder had been covered with a film of water of one-hundredth of an inch in thickness, of the same temperature as the condenser ; the number of heat units that would have been required to raise it to the temperature of the entering stream would have been 74·25, as against 62·56 required to heat up the metal sides to the same degree, and consequently the condensation in the latter case would have been more than double what it would be were the sides dry to start with.

In addition to the direct condensing effect which water in the cylinder exercises, it also acts as a most excellent medium for the interchange of heat between the steam and the sides. It has been already stated that perfectly dry steam is a very indifferent radiator and conductor of heat, while moist steam is comparatively good in both respects. Now the presence of water in the cylinder, by increasing the moisture of the steam, aids in the rapid transmission of heat

from the latter to the metal sides ; and, *vice versâ*, the power of the sides in transmitting heat to the exhaust steam is enormously increased by the presence of a film of water, which, as fast as it evaporates, takes up heat from the metal with great rapidity.

There are four principal causes for the existence of water in the cylinders, viz.—

Priming, or the carrying over of water in the fresh steam from the boiler.

Excess of initial condensation over re-evaporation, which always occurs when the expansion is carried beyond a certain point.

The disappearance of heat due to the work done by the steam.

Loss of heat by radiation from the external surface of the cylinder.

The latter cause of condensation always exists when the cylinder is uncovered, or but imperfectly protected. The great indirect harm caused by water in the cylinder accounts for the fact already alluded to, that want of proper lagging increases the loss of efficiency in expansive engines, the loss being enormously greater than the amount of heat which escapes by radiation. Of these causes, the first—priming—is probably not always as harmful as might be supposed. Very often the water comes over from the boiler in the condition of minutely fine spray, thoroughly mixed with and having the same temperature as the body of the steam. Under these circumstances priming is not so injurious as when comparatively large bodies of water come over at intervals, which accumulate in the cylinder, or form a film over the surfaces.

The second cause—excess of condensation over re-evaporation—is a most fruitful source of waste, and should be most carefully guarded against. It results in the continuous accumulation of water in the cylinder, and consequently causes an amount of waste which goes on increasing with each

stroke. It always exists when the expansion is carried beyond a certain limit, and this is one of the reasons why excessive expansion in a non-compound engine is unattended by any economy. The exact point to which expansion can be safely carried, so as to avoid the accumulation of water, depends largely upon the piston speed and the dimensions of the cylinder. It is also greatly affected by the supply of heat to, or its abstraction from, the steam during the progress of expansion. If the cylinder be kept warm artificially, the limit of economical expansion can be considerably extended; on the other hand, if radiation from the outer surfaces be permitted, the limit will be correspondingly reduced.

The third cause—disappearance of heat due to work done—is comparatively insignificant, and as the resulting condensation takes place throughout the mass of the steam, and does not form a film over the surfaces, the mischief due to this cause may be overlooked.

The fourth cause—loss of heat by external radiation—can always be effectually guarded against by properly covering the cylinders.

The initial condensation which takes place is, independently of all other considerations, dependent on the dimensions of the cylinder, the ratio of expansion, and the pressure of the steam. Let d be the diameter of the cylinder in feet; l the length of stroke; r the ratio of expansion; V the volume of one pound of steam at its initial pressure; and E the exposed surface of sides and ends of cylinder and of face of piston per pound weight of steam. Then we have—

Exposed surface of end and piston face $= \frac{\pi d^2}{2}$

„ admission portion of sides $= \frac{\pi d l}{r}$

Cubic contents of admission portion $= \frac{\pi d^2 l}{4r}$

Weight of steam in „ $= \frac{\pi d^2 l}{4rV}$

Therefore the exposed surface per pound of steam, or—

$$E = \frac{\pi d^2}{2} + \frac{\pi dl}{r} \div \frac{\pi d^2 l}{4rV}$$

$$= 2V\left(\frac{r}{l} + \frac{2}{d}\right).$$

That is to say, the exposed surface per pound weight of admitted steam, on which, other things being equal, the initial condensation depends, varies directly with V, the volume of one pound of the steam, and therefore varies inversely with the pressure. It also varies directly with r, the ratio of expansion, and inversely with l and d, the dimensions of the cylinder. Hence we see that initial condensation is diminished by increasing the pressure of the steam and the size of the cylinder, and by diminishing the expansion. One of the reasons why small steam engines are so uneconomical compared with large ones is the relatively large surface exposed by their cylinders per pound of steam admitted.

Experiments have been carried out upon steam engines in order to ascertain the loss due to condensation in cylinders, and to test the effects of the various methods of reducing the loss. Some of the results are given on page 453, and prove that in unfavourable cases, with unjacketed cylinders, the steam wasted from this cause may be very nearly equal to the steam accounted for by the indicator diagram.

Experiments have also been made by Mr. Emery, in America, to test the saving to be effected by using a non-conducting material for the cylinders in place of metal. Two cylinders were used of identical dimensions, one made of glass and the other of iron. They were worked under precisely similar conditions as to steam pressure, valves, ratio of expansion, &c. The steam used by each was condensed and carefully measured. The average of several

series of experiments proved that the iron cylinder consumed double as much steam on the average as the glass one.

Methods employed to diminish the Loss due to Liquefaction in the Cylinders and subsequent Re-evaporation.—The great majority of the improvements in the modern steam engine have been directed towards diminishing the losses which come under the above heading. The remedies all consist in methods, either for the addition of heat to the steam before it enters, or while it is in the cylinder; or else for reducing the range of temperature within the cylinder. These remedies are four in number, two belonging to each category. They may be, however, and often are, used in combination. They are as follows:—

1st, the use of superheated steam.

2nd, jacketing, or surrounding the cylinder with hot steam from the boiler.

3rd, cushioning, or compressing the exhaust steam in the cylinder before fresh steam is admitted.

4th, compounding, or allowing the steam to expand in two or more cylinders successively instead of in one, thus reducing the range of temperature in each.

Superheated Steam.—Steam is said to be superheated when heat is added to it after it reaches the ordinary or saturated condition. Superheated steam resembles a perfect gas in its behaviour. Its specific heat at constant volume is ·336 thermal unit. That is to say, the addition of about one-third of a thermal unit will raise the temperature of one pound of steam by one degree; so that a comparatively small addition of heat will raise the temperature of the steam very considerably. In practice steam is superheated by passing it, on its way from the boiler to the cylinder, through tubes or vessels surrounded by the hot gases as they pass from the furnace to the chimney.

If we revert to the example given on page 442, in which 62.56 units were abstracted from the entering steam, it will

be seen that we should need to add $62{\cdot}56 \div {\cdot}336 = 186°$ to the steam before entering the cylinder, in order that it might possess heat enough to bring up the temperature of the cylinder sides to that of steam of 90 lbs. pressure, and still remain in the condition of dry saturated steam. As, however, the natural temperature of steam of this pressure is 320°, the temperature of the superheated steam would have to be $320 + 186 = 506°$ in order effectually to prevent initial condensation. Such a high temperature as this, however, is quite unsuited for use in the steam engine, as it burns up and hardens all the lubricants, destroys the packing in the stuffing-boxes, and exercises a chemically corrosive effect on all brass and copper work with which it may come in contact. It is found in practice that whenever steam reaches the temperature of 400°, injury to the cylinder, &c., invariably follows. Hence we see that high-pressure steam cannot be sufficiently superheated to thoroughly cure initial condensation. The case is different, however, with steam of comparatively low pressure, such as 30 to 35 lbs. absolute per square inch. If steam of 30 lbs. pressure had been used in the example already given, the difference between the temperature of admission and exhaust would have been $259° - 157°{\cdot}5 = 101°{\cdot}5$. Consequently, only 39 thermal units would have been abstracted from the entering steam, the temperature of which would have needed the addition of $39 \div {\cdot}336 = 116°$, in order to provide for this loss. Thus the temperature of the superheated steam on entering the cylinder would be $259° + 116° = 375°$, which falls just within the safe limit. Of course the pressure as well as the temperature of steam is raised by superheating, and in the example, which has been given merely for the sake of illustration, the shape of the diagram and the power indicated up to the point of cut-off would be altered by the process.

It will be seen from the foregoing that superheating the boiler steam is impracticable for modern high-pressure

engines, but it is used with great advantage in the old type of low-pressure marine engine. When the boilers are given to priming, a moderate amount of heat may with advantage be added to the steam after it is formed, so as to insure its being thoroughly dry before it enters the cylinder. In this way much of the loss due to accumulation of water in the cylinders may be avoided.

Steam Jacketing.—One of the most common methods of checking the waste due to condensation is to supply heat to the steam while in the cylinder. This is done by surrounding the cylinder proper with a metallic casing, an interstice being left between the two, in which circulates steam of the boiler pressure. An example of a jacketed cylinder has been given on page 206, fig. 49. The effect of the jacket is to supply heat to the inner surface of the cylinder walls, to make up for that abstracted during expansion and exhaust. The result is that the difference between the temperatures of the sides is greatly reduced, and condensation is checked while re-evaporation is promoted, and a preponderance of condensation over re-evaporation is greatly hindered.

The heat supplied to the cylinder by the jacket causes a certain amount of the jacket steam to liquefy, so that the question may be asked, wherein lies the advantage of this system over that of allowing the entering steam to supply the required heat by its own liquefaction? It is occasionally urged that in the latter case a portion at least of the condensed steam does work by subsequent re-evaporation, whereas in the case of the jacket the condensed steam can do no work whatever. The answer to this question is, that the loss when the heat is supplied by the incoming steam is mostly due to the fact that the steam in the cylinder is rendered moist by the first condensation, and consequently becomes a good radiator and conductor of heat, and thus enormously increases the action of the sides. The mere alteration of the temperature of the sides would not cause much loss if only the steam could be kept dry even in a comparatively

cool cylinder, because in this case there would be little interchange of heat between the steam and sides. Consequently the benefit of a jacket lies in the fact that the condensation which does take place occurs for the most part outside the cylinder, and the inside steam is kept comparatively dry, and thus the interchange of temperature between the sides and the working steam is greatly checked, and the sides themselves are consequently prevented from rapidly taking up the temperature of the steam. Thus a jacket operates in two ways in keeping the temperature of the cylinder walls constant. First, by keeping the working steam comparatively dry, it reduces the power of the sides of receiving heat from and giving it out to the former, and thus deprives the sides of the power of taking up the extremes of temperature which would otherwise be possible ; and second, whatever differences of temperature would actually occur are further greatly reduced by the flow of heat from the jacket steam to the inner walls of the cylinder. It is only the heat supplied in the latter process which costs the jacket steam anything. The great gain due to the rendering the working steam non-conducting and non-radiating costs nothing whatever, seeing that it is an indirect effect of keeping the sides hot. Thus the steam jacket, though for half its time it is warming the exhaust, has proved in the majority of cases to be an undoubted source of economy. It would probably be much more perfect in its action were it not for the fact, that it is much more difficult to supply heat to the inner surfaces of the cylinder through the thick sides, than it is for the cool moist steam to abstract heat from these same surfaces with which it is in immediate contact. The sluggish action of the sides in conducting heat is further promoted by the difficulty of keeping up a really effective circulation of steam in the jacket.

Cases occasionally occur in which steam jackets have been proved to have had little or no effect upon fuel economy. It has been already mentioned that when expan-

sion is carried too far, condensation preponderates over re-evaporation. Now one of the effects of steam jacketing would undoubtedly be to increase the profitable range of expansion. If, however, the expansion were carried beyond this point, so that the condensation preponderated, water would of course accumulate in the cylinder, and the steam would become moist in spite of the action of the jacket, which latter would consequently be of little or no use.

Precautions to be observed in Jacketing Cylinders.—Many cases have occurred in practical experience in which jacketing has been carried out so as to be positively injurious instead of beneficial. One of the commonest mistakes was to allow the exhaust steam to expand into the jacket, so that the cylinder, instead of being surrounded by a medium hotter than its own mean temperature, was really immersed in steam to which it was in a condition to impart heat. Occasionally the fresh steam from the boiler is allowed to pass through the jacket before it enters the cylinder. This is a bad practice, although it ensures circulation in the jacket, because, as has been shown before, jacket steam to be effective must liquefy, and nothing can be worse than to cause the steam to become moist before it enters the cylinder. Very often it has been found that, in consequence of the absence of good automatic means for drawing away the water caused by condensation in the jackets, the latter have become filled with cold water, thereby aggravating all the evils which they were intended to mitigate. In every case jackets should have their own independent supply of boiler steam, and should be provided with trustworthy automatic means for carrying off the condensed steam.

Experiments on the effect of Steam Jackets.—The following table is extracted from the record of some experiments made by Mr. Charles Emery on the engines of the United States Coast Survey steamer 'Bache,' in the year 1874. The engines were compound, the high-pressure cylinder having a diameter of 15·98 inches, the low-pressure 25

	Simple Engine							Compound Engine						
	Without Jackets			With Jackets				Without Jackets			With Jackets			
Boiler pressure . .	81	79·6	78·1	80·8	81	79·5	30·9	82	80·3	80·3	81·4	80·3	80·2	80·1
Vacuum in condensers .	24	23·78	24·2	24·66	25·28	25·5	24	24	24·3	24·65	24·5	26·5	26·5	26·6
Revolutions per minute .	37·3	44·9	47	39·9	46·2	53·8	45·3	42·6	47·7	49·3	38·9	48·2	53·2	56·3
Indicated horse-power .	47·2	71·8	89·1	54·8	74·6	116	66	55·9	77	85	46·4	77·4	99·2	110·5
Ratio of expansion . .	11·8	7·6	5·3	12·6	8·57	5·1	2·18	9·14	6·65	5·63	16·8	9·2	6·97	5·7
Water used per I.H.P. per hour, including drainage from jackets and reservoir } lbs.	35	29·6	26·25	27·1	24	23·15	34	23·76	23·04	23·2	25·1	20·7	20·33	20·36
Ditto, as calculated from diagrams } lbs.	21	17·75	17·35	16·4	15·57	16·25	24	12·7	12·3	12·3	18·5	15·7	14·8	15·25

inches, and the stroke 24 inches. During part of the trials the large engine was used alone as a simple single cylinder engine with and without jacket. During the remainder of the trial the engines were worked compound, the large cylinder with and without jackets : the high-pressure cylinder had no jacket.

From an inspection of this table we see that under all circumstances, whether the engine was worked simple or compound, a considerable saving was effected by the use of the jacket, when corresponding rates of expansion are compared. For instance, when worked compound, about 3 lbs. of water were saved per horse-power per hour, being at the rate of about 13 per cent.; while in the case of the simple engine worked at the higher rates of expansion as much as 22·5 per cent. was saved.

The same experimenter carried out tests on other marine engines at about the same period, and the result, as given in the published statements, conclusively proved the efficacy of the jacket.

Cushioning, or compressing the Exhaust Steam in the Cylinder before the Readmission of Fresh Steam.—It has been already pointed out that in most engines the exhaust port is closed before the end of the stroke, and the steam then remaining in the cylinder is compressed at the expense of the energy in the steam on the other side of the piston. The advantages of compression in saving the steam required to fill clearance space and steam ports, and in improving the diagram of twisting moment on the crank, have been already made clear. There is, however, another advantage to be derived from compression, inasmuch as the steam while undergoing forcible diminution of volume rises in temperature, and partly restores the sides and end of the cylinder to their original condition. In condensing engines compression is rarely carried out sufficiently to be of any very great use ; but in high-pressure engines, with link expansion gear, the greater the ratio of expansion the greater must also of necessity be

the degree of compression (see fig. 109, and also diagrams, page 338), and it is probable that in these cases compression diminishes very greatly the tendency to initial condensation. Moreover, it is a remedy which, when link gearing is used, automatically proportions itself to the extent of the evil. For the tendency to condensation grows as the rate of expansion increases, and in the same ratio, as has just been pointed out, the compression increases also. This is probably one of the reasons which accounts for the economical performance of locomotive engines.

Compounding, or allowing the Steam to expand successively in two or more Cylinders.—The use of steam of very high pressures, worked at great rates of expansion, causes such severe variations in the temperature of the cylinder, especially in condensing engines, that the remedies above referred to, taken by themselves, fail to cure the evil of initial condensation. Hence it becomes necessary to divide the total expansion between two or more cylinders, so that each cylinder, in an ordinary two-cylindered engine, roughly speaking, ranges through about half the difference of temperature which it would were the expansion carried out completely in a single cylinder. The term compound is applied to all engines in which the steam is expanded partly in a small or high-pressure cylinder, and thence exhausted into a cylinder of larger diameter. In the most modern types of marine engines the pressure used is so great—reaching sometimes 160 lbs. per square inch above the atmosphere—that the range of temperature in each of the cylinders of an ordinary compound engine is still too great, and in such cases the steam is expanded successively in three, and even in four cylinders. Such engines are called, according to the number of the cylinders in which successive expansion takes place, triple or quadruple expansive engines.

There are many different arrangements of compound engines, depending upon the number of cranks used, and upon the angles which these form with one another.

Tandem Engines.—When only one crank is used the two cylinders must obviously be in the same straight line. This arrangement is illustrated in fig. 194, which shows the two cylinders connected to each other by a distance piece, or conical bracket. The large and small pistons are connected to the same piston rod, and the slide valves are in the same straight line, so that they can be both worked off the same eccentrics. It is usual to keep the two cylinders some little distance apart, in order that each cylinder cover may have its own stuffing-box, as otherwise direct leakage would in time take place between the cylinders. The pistons, of course, occupy simultaneously the same positions in their respective cylinders, and the steam exhausts directly from the one face of the high-pressure on to the opposite face of the low-pressure piston, there being no intermediate reservoir.

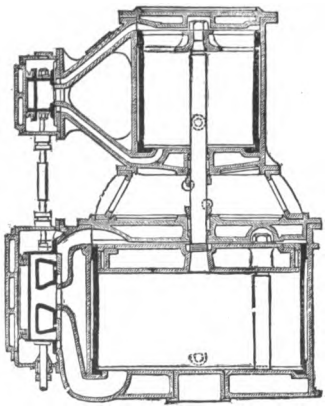

Fig. 194.

This type of compound engine has the merit of great simplicity. It provides perfectly for the expansion of the steam. It can be repeated as often as is wanted, so as to provide for cases where two or more cranks are required, and old-fashioned simple engines can be readily converted to compounds on this type. As usually made its chief disadvantage lies in the long steam passages in the high-pressure cylinder, caused by the necessity of bringing the valves of both cylinders into the same straight line. These long passages cause a considerable waste of steam.

Compound Engines with Cranks at Right Angles, and an Intermediate Receiver.—Very often it is found inconvenient

to allow the length for horizontal, or the height for vertical engines which is necessary when the tandem arrangement is adopted. Consequently, when the engine has two cranks at right angles to each other, as a rule a high and a low pressure cylinder are placed side by side, each of them driving its own crank. In such cases, however, the pistons cannot occupy simultaneously the same positions in their respective cylinders. For instance, when the high-pressure piston has completed its stroke, the steam, which has just done its work in the small cylinder, is ready to exhaust into the large cylinder. But the large cylinder is not at this moment ready to receive fresh steam, for its piston is only at or about half-stroke. Hence the steam from the high-pressure cylinder must in the first instance be discharged into an intermediate receiver, in which it is compressed by the advancing high-pressure piston, until the large piston has completed its stroke, when the low-pressure valve opens, and admits the steam from the receiver. Figs 195, 196 show the general arrangement of a compound receiver marine engine having the cranks at right angles. Fig 195 shows a longitudinal section through the cylinders, and fig. 196 is an end elevation, showing the arrangement of the engine-frames and of the condenser and air-pumps. The size of the receiver is a matter of some importance. It was formerly the practice to make it a large receptacle, in some cases surrounding the small cylinder. One of the consequences of having a large receiver is, that when communication is opened between it and the exhaust ports of the small cylinder there is a considerable drop in the pressure of the issuing steam. This drop of pressure takes place without the performance of any work, and there is a consequent loss of efficiency in the steam owing to its expansion not being continuous. This point will be referred to again when we come to deal with the indicator diagrams of compound engines. At the present time it is usual to limit the size of the receiver; in fact, it often consists merely of the

valve-box of the low-pressure cylinder, and the pipe connecting the latter with the high-pressure exhaust passages, together with that ever-changing portion of the small cylinder which is not cut off by the exhaust face of its valve.

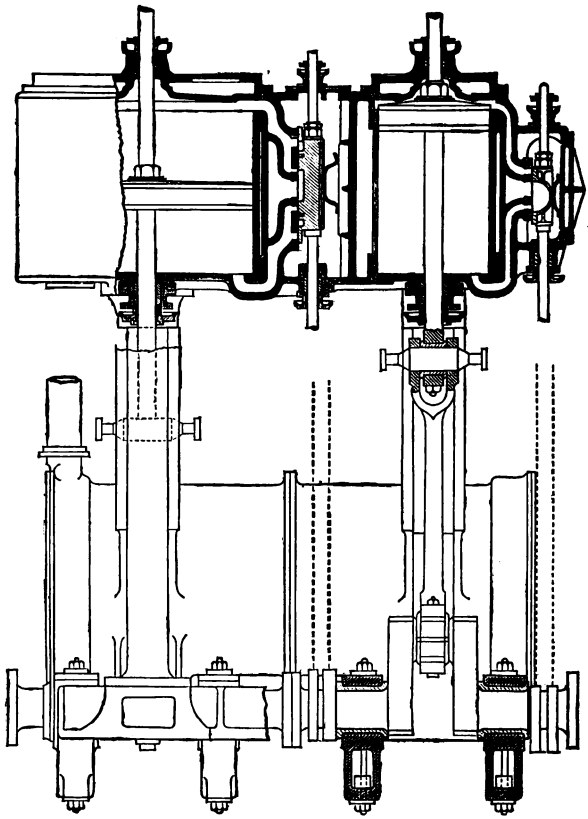

Fig. 195.

Whenever the power of the engines is so great that the low-pressure cylinder would become of very large diameter, it is usual to have two low-pressure cylinders, which draw

their steam from a common receiver. The cranks in such cases are usually at angles of 120° with one another, though sometimes, in order to secure a special distribution of the

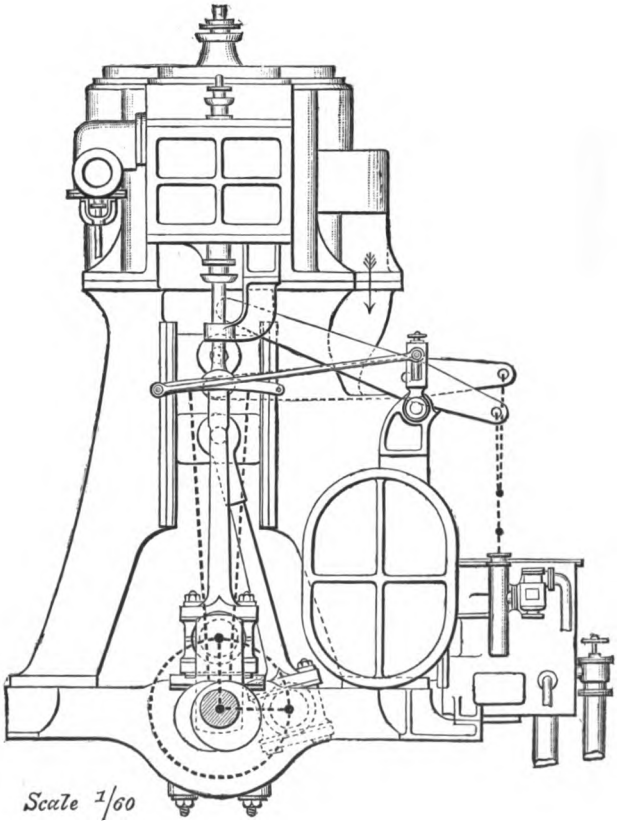

Fig. 196.

steam, or a more uniform curve of twisting moments on the shaft, the two low-pressure cranks are exactly opposite each other, while the high-pressure crank is at right angles to

them, and occasionally the high-pressure crank makes a right angle with one low-pressure crank, the remaining one being at 135° with each of the others. Figs. 197, 198 show the general arrangements of a three-cylinder compound engine where two low-pressure cylinders are used, the valves being

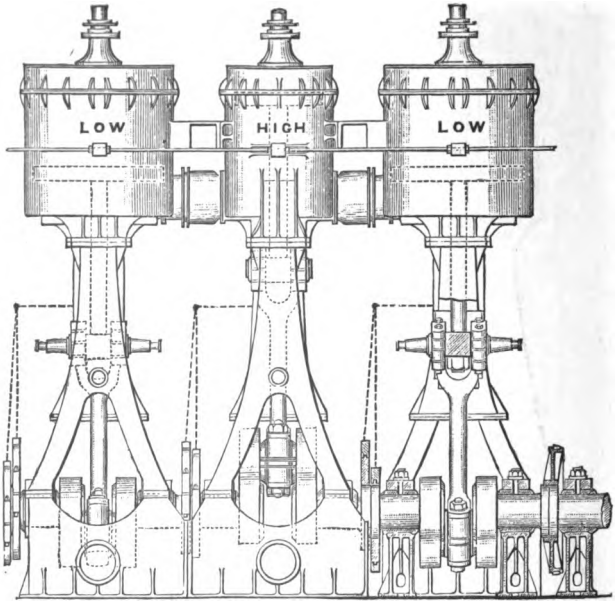

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Fig. 197.

at the sides instead of between the cylinders. In this example the small cylinder is placed between the other two, though occasionally the two low-pressure cylinders are together, and the small cylinder outside.

Fig. 197 is a sketch of the front elevation, and fig. 198 a section through one of the low-pressure cylinders with its

piston valve. These engines belong to the well-known Transatlantic mail steamer, the 'Arizona.'

Triple Expansive Engines.—The same arguments which justify expansion in two cylinders successively, when the steam pressure lies between 60 and 90 lbs. per square inch, render it advisable to expand in three cylinders successively

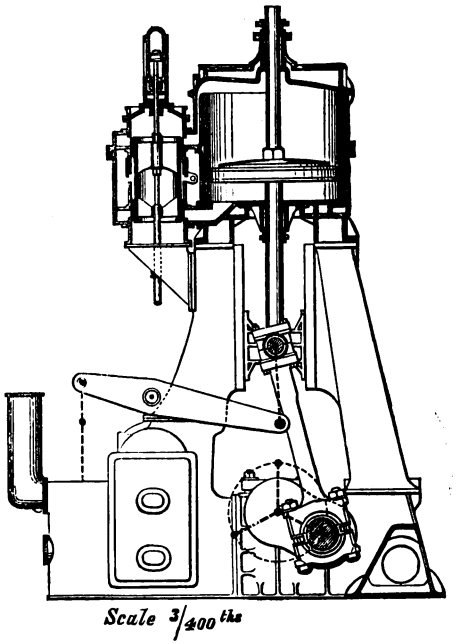

Fig. 198.

when still higher pressures are used. Such engines are called triple expansive, to distinguish them from three-cylinder ordinary compound engines. They were first introduced by Mr. A. C. Kirk.

The simplest arrangement of triple expansive engines is that illustrated in the frontispiece and figs. 199, 200, in which

the high, intermediate, and low pressure cylinders are side by side, each working a separate crank. Sometimes the high-

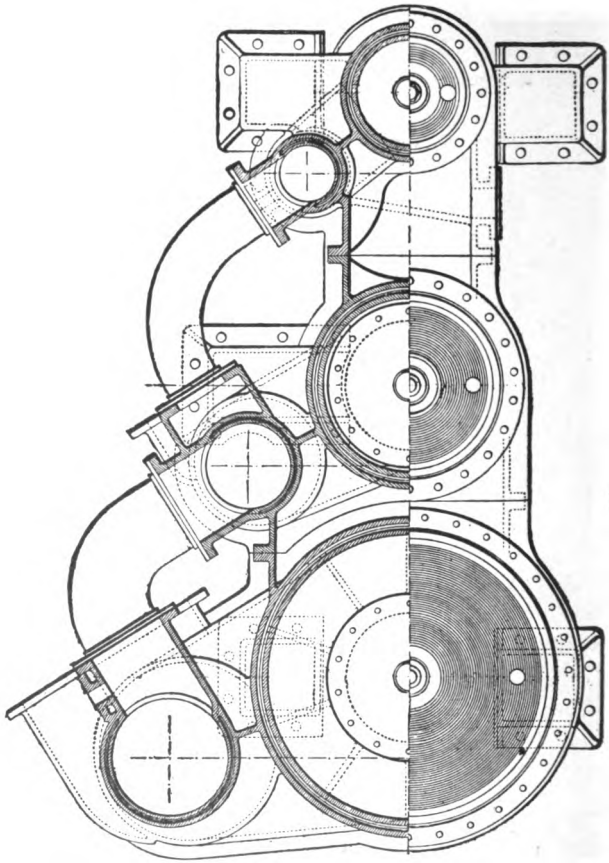

Fig. 199.

pressure is placed over the intermediate cylinder, tandem fashion, the two working on to one crank, while the low-pressure cylinder drives a separate crank. Occasionally, as

in the case of ordinary compound engines, the low-pressure cylinder is divided into two, in order to avoid excessive size.

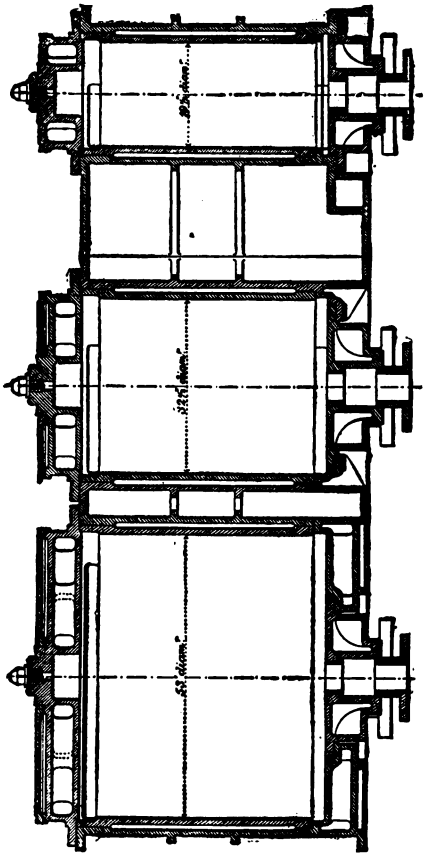

Fig. 200.

In this case the small cylinder is placed over one low-pressure cylinder, and the intermediate one over the other low-pressure, the engine resembling a double tandem with four

cylinders and two cranks. The arrangement of the cylinders is, in fact, a matter of convenience, and depends upon the height and length available in the engine-room, and on the approximation to uniformity which it is desired to effect in the curve of twisting moments. Fig. 200 is a vertical longitudinal section through the cylinders, and fig. 199 a plan and half horizontal section through the cylinders, showing the positions of the piston valves, and the arrangement of the steam pipes connecting the cylinders.

Saving of Fuel effected by Compounding.—The saving of fuel effected by the use of steam of the pressure of from 120 to 150 lbs. worked in triple expansion engines may be put down as about fifteen to twenty per cent. over the expenditure in ordinary compound engines using steam of from 70 to 90 lbs. pressure. One of the first marine triple expansion engines ever made developed 1800 horse-power on the trial trip of four hours, with an expenditure of 1·28 lb. of coal per hour. The ordinary consumption at sea may be put down as about 1·5 to 1·6 lb. of coal per I.H.P. per hour. The corresponding consumption of the older type of engine with the lower pressures given above is generally from 1·8 to 2·2 lbs.

The saving of fuel effected by compounding is well illustrated by the table on page 453. Comparing the simple and compound engines when both working with jackets, it will be noticed that, with a ratio of expansion of between 5 and 6, the simple engine consumed 23·15 lbs. of water and the compound 20·36 lbs. per I.H.P. per hour, showing a saving in favour of the compound of 12½ per cent. For ratios of expansion of between 7 and 9 the saving in favour of the compound was about 15 per cent., showing, as might be expected, that the greater the difference of temperature in the cylinder, the greater the benefit to be derived from compounding. The difference in the consumption of steam between the simple engine without, and the compound engine with jacket, is still more marked. For instance,

with a ratio of expansion of 7·6, the simple engine without jacket requires 9·3 lbs. of water per horse-power more than the compound with jacket, which shows that a saving of nearly 32 per cent. is in this particular instance due to the combination of the remedies of compounding and jacketing.

The distribution of the Steam in Compound Engines.—Before we examine the actual indicator diagrams of compound engines, or investigate their mechanical as distinguished from their thermal advantages, it is necessary to trace out the theoretical distribution of the steam in some of the types which occur in practice. In doing so, we will assume for the sake of simplicity that the steam is not released till the end of the stroke, that there is no compression of the exhaust steam, and that there is no resistance due to ports and passages.

Let v represent the volume of the small cylinder.

V	„	„	volume of the large cylinder.
R	„	„	ratio of the two cylinders.
V_R	„	„	volume of the receiver.
ρ	„	„	ratio of the volumes of the receiver and the small cylinder.
r	„	„	rate of expansion in the small cylinder.
r'	„	„	rate of expansion in the large cylinder.
E	„	„	total rate of expansion.
p	„	„	absolute initial pressure of the steam in the small cylinder.

Then we have the total rate of expansion $E = rR$.

The pressure of the steam at the end of the stroke in the large cylinder $= \frac{p}{E}$, and the volume which it occupies is V.

Hence, as the product of pressures and volumes is equal during hyperbolic expansion, we have

$$V \times \frac{p}{E} = \frac{v}{r} \times p \quad \therefore \quad \frac{V}{E} = \frac{v}{r}$$

Tandem Engines with a Receiver.—This is the first case which occurs in practice, because the passages between the face of the high-pressure valve and the back of the low-pressure valve, including the valve-box of the low-pressure cylinder, constitute in themselves a receiver the effect of which must not be neglected.

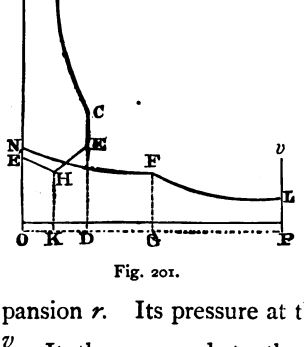

Fig. 201.

The steam is admitted into the small cylinder during the portion AB of the stroke, the length of the line AB being determined by the rate of expansion r. Its pressure at this point being p, and its volume $\frac{v}{r}$. It then expands to the end of the stroke C, when its volume is v and its pressure CD = $\frac{p}{r}$.

At this point communication is opened with the receiver, the volume of which is V_R, and which is filled from the last stroke with steam having the pressure p_R. Consequently we have now a total volume $v+V_R$ and two masses of steam, one having the pressure $\frac{p}{r}$ and the other the pressure p_R. These pressures at once equalise themselves and attain the value

$$\frac{\frac{p}{r}v + p_R V_R}{v + V_R}$$

Let this pressure be represented by DE. Hence we see that, unless at the moment of the opening of the exhaust the receiver is already filled with steam of the pressure in the small cylinder $\frac{p}{r}$, there will be a sudden drop in the diagram represented by CE. That is to say, expansion will take place

without any work being done, and the heat liberated in the process will be expended in superheating the steam in the receiver.

The pressure in the receiver is of course the initial pressure in the large cylinder. From the point O mark off ON′=DE. In drawing the low-pressure diagram it must be remembered that the volume of the large cylinder is R times that of the small cylinder; therefore on the line of volumes any given part of the stroke will be represented by a line R times as long as the same part of the stroke in the high-pressure diagram. Thus, for instance, the whole stroke is represented by OP = OD × R.

The steam now expands in the low-pressure cylinder and receiver together till the point of cut-off, which is $\frac{1}{r'}$ of the stroke. At this point the volume of the steam is made up of three parts—viz.

The portion of the large cylinder up to the cut-off point $= \frac{V}{r'}$.

The volume of the receiver $= V_R$.

The portion of the small cylinder which yet remains to be traversed $= v - \frac{v}{r'} = v\left(1 - \frac{1}{r'}\right)$.

The pressure is got, as usual, by remembering that the product of the pressure and volume at all points is constant. At the commencement of the stroke of the large cylinder this product was

$$\frac{\frac{pv}{r} + p_R V_R}{v + V_R} \times (v + V_R)$$

Therefore at the point of the stroke now under consideration, the pressure

$$FG = \frac{\frac{pv}{r} + p_R V_R}{\frac{V}{r'} + V_R + v\left(1 - \frac{1}{r'}\right)}$$

This is also the pressure of the point H in the diagram of the small cylinder, where

$$KD = \frac{OG}{R} = v - \frac{v}{r'} = v\left(1 - \frac{1}{r'}\right)$$

As soon as the steam is cut off in the low-pressure cylinder it expands to the end of the stroke, and as all the steam which is admitted to the high-pressure cylinder must eventually find its way to the low-pressure cylinder, its final pressure $PL = \frac{p}{E}$. At the same time the steam in the receiver is being compressed by the advancing piston of the small cylinder, and finally attains the pressure p_R which was assumed to exist in it just before the small cylinder exhausted into it.

The value of p_R can be easily expressed in terms of the known quantities. For, as we have seen, the volume of steam in the large cylinder when communication with the receiver was cut off was $\frac{V}{r'}$,

and its pressure

$$\frac{\frac{pv}{r} + p_R V_R}{\frac{V}{r'} + V_R + v\left(1 - \frac{1}{r'}\right)}$$

Now at the end of the stroke the volume is V and the pressure is $\frac{p}{E}$. Hence the products of these two quantities must be equal, or

$$\frac{\frac{pv}{r} + p_R V_R}{\frac{V}{r'} + V_R + v\left(1 - \frac{1}{r'}\right)} \times \frac{V}{r'} = V \times \frac{p}{E}$$

An expression from which p_R may be obtained.

The above equation may be simplified by substituting for

V_R	its equivalent	ρv
V	,,	Rv
E	,,	Rr

And then reducing we get

$$p_R = \frac{p}{E}\left(\frac{r'-1}{\rho} + r'\right)$$

An equation which gives the receiver pressure just before the small cylinder exhausts into it, in terms of the initial pressure, the total rate of expansion, the rate of expansion in the low-pressure cylinder, and the ratio which the volume of the receiver bears to that of the small cylinder.

In the diagram fig. 201 OE′ represents the pressure p_R, and the curve HE′ shows the line of compression as it affects the small piston.

In most cases the pressure in the receiver is less than the terminal pressure in the small cylinder, and consequently there is the sudden drop shown in the diagram by the line CE. It is obvious, however, from the above equation that the value of the receiver pressure can be varied by varying r', the rate of expansion in the low-pressure cylinder. If it be wished to adjust the cut-off in this cylinder, so that the receiver pressure may just equal the terminal pressure in the small cylinder, we can easily find the required rate of expansion by equating the value just found for the receiver pressure with the terminal pressure of the small cylinder—viz. $\frac{p}{r}$, and solving for r'.

Compound Engines with Receivers and Cranks at right angles.—In tracing the diagrams of the above type of engines we must remember that, neglecting the effect of the length of the connecting-rod, the large piston is always at mid-stroke when the small piston is at either end of its cylinder ; consequently the exhaust steam from the small cylinder enters the receiver when the large piston is at mid-stroke. Whether

it also simultaneously enters the large cylinder depends upon the point of cut-off in the latter; if this be before half-stroke, the steam cannot enter, but if after half-stroke, it must enter the large cylinder as well as the receiver.

In the high-pressure cylinder the steam enters during the portion AB of the stroke depending on the point of cut-off, and expands to C, the end of the stroke. At C the exhaust takes place, and the pressure falls to E. The next step depends on the rate of expansion in the large cylinder. If this be after half-stroke the steam will expand in receiver and large cylinder together till the cut-off takes place. Suppose this to happen when the small piston has travelled through DG of its return stroke, and the pressure has then fallen to F. After this point the small piston continuing to advance compresses the steam in the receiver till it reaches the point I corresponding to half-stroke, the pressure having then risen to H. Now when the small piston is at mid-stroke the large one is just about to commence its stroke, and the large cylinder is just commencing to take steam from the receiver. Hence the pressure in the latter will fall till the end of the stroke of the small piston, finally attaining the value OK.

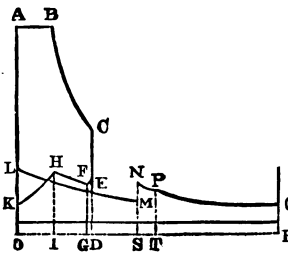

Fig. 202.

The diagram of the large cylinder must next be drawn, the line of volumes, as usual, corresponding with the ratio between the two cylinders. The initial pressure is the same as the receiver pressure when the small piston is at half-stroke, viz. HI. Set off, therefore, OL = HI to represent this pressure. From this point the steam expands in receiver and large cylinder till half-stroke, when the small cylinder exhausts into the receiver. At half-stroke, then, the pressure in the cylinder is the same as the pressure in the receiver

when the small piston has returned to the end of the stroke, viz. OK. Erect therefore SM = OK. At this point the pressure suddenly rises to N, so that SN = DE, and then the steam again expands in receiver and large cylinder till the point of cut-off T is reached, when the pressure TP is the same as FG. From this point onwards the steam expands in the large cylinder till the end of the stroke R, while the steam in the receiver is compressed as already shown by the line FH. At the end of the stroke the pressure in the large cylinder falls to that of the condenser.

To express these various pressures in terms of the initial pressure and the volumes of the two cylinders and the receivers, we proceed precisely as in the first example. Let k denote the portion DG of the stroke passed over by the small piston, when steam is cut off in the large cylinder; then $GO = (1-k)v$ is the fraction of the volume of the small cylinder which at that moment is open to the exhaust. The terminal pressures CD and QR are exactly the same as in the first example, viz. $\frac{p}{r}$ and $\frac{p}{E}$ respectively. Also the pressure in the large cylinder at the point of cut-off equals the terminal pressure multiplied by the rate of expansion; or $TP = \frac{pr'}{E}$, and this, as already shown, is the same as the pressure GF. At this point the volume occupied by the steam is made up of two parts, viz.

The contents of the reservoir $= V_R$
and the portion GO of the small cylinder $= (1-k)v$
and consequently the total volume $= V_R + (1-k)v$

As soon as steam is cut off in the low-pressure cylinder, it is compressed in the small cylinder and receiver along the line FH till the high-pressure piston is at half-stroke, when the volume occupied by the steam $= V_R + \frac{v}{2}$,

and the pressure

$$\text{IH} = p_R = \frac{pr'}{E} \times \frac{\{V_R + (1-k)v\}}{V_R + \frac{v}{2}} = \frac{pr'}{E} \times \frac{\rho + 1 - k}{\rho + \frac{1}{2}}$$

This is also the value of OL, the initial pressure in the large cylinder. From thenceforward the steam enters the large cylinder and expands in it and the receiver till half-stroke, when its volume $= V_R + \frac{V}{2}$.

To obtain its pressure SM at this point we have

$$\text{SM} \times \left(V_R + \frac{V}{2}\right) = \frac{pr'}{E} \times \frac{V_R + (1-k)v}{V_R + \frac{v}{2}} \times \left(V_R + \frac{v}{2}\right)$$

$$\therefore \text{SM} = \frac{pr'}{E} \times \frac{V_R + (1-k)v}{V_R + \frac{V}{2}} = \frac{pr}{E} \times \frac{\rho + 1 - k}{\rho + \frac{R}{2}}$$

It now only remains to find the pressure SN = DE, which is that of the receiver when the small cylinder exhausts into it. The resultant pressure and volume is derived from two components, one being that of the steam in the small cylinder, whose volume is v and pressure $\frac{p}{r}$, and the other that of the steam in the receiver and half the large cylinder, whose volume is $V_R + \frac{V}{2}$, and whose pressure SM is given above. Compounding these two we get

$$\frac{\frac{p}{r}v + \frac{pr'}{E} \times \frac{V_R + (1-k)v}{V_R + \frac{V}{2}} \times \left(V_R + \frac{V}{2}\right)}{v + V_R + \frac{V}{2}}$$

Substituting for V_R, V and r their values $v\rho$, vR, and RE, reducing, we get

$$\text{SN} = \text{DE} = \frac{p}{E} \frac{r'\{\rho + 1 - k\} + R}{1 + \rho + \frac{R}{2}}$$

Here, again, there is a considerable drop of pressure in the receiver, and an increase of pressure in the low-pressure diagram at half-stroke. In order that there should be no drop it would be necessary that the terminal pressure in the small cylinder should be equal to the receiver pressure. Then the resultant pressure would be the same as either of its components, and the line CD would equal DE. Equating the values previously given for these two pressures, and solving for R, we can if desired find out what must be the ratio between the two cylinders, with a given rate of expansion in the large cylinder, in order that the receiver pressure may be equal to the terminal pressure in the small cylinder.

When the cut-off in the low-pressure cylinder takes place *before* half-stroke the diagrams will differ somewhat from those explained in the preceding example.

At C, fig. 203, the high-pressure cylinder exhausts into the receiver, the pressure falling to DE. Now when the small piston is at the end of its stroke the large piston is at half-stroke, and therefore the steam is already cut off, and consequently the exhaust steam from the small cylinder only enters the receiver, and not the receiver plus half the large cylinder, as in the previous example. When the small piston makes the return stroke it compresses the exhaust steam before it and in the receiver till mid-stroke, the line EF being the curve of compression. When the small piston is at mid-stroke the large piston is at one end of its cylinder, and consequently draws steam from the receiver till the cut-off is effected and the steam expands in the receiver and the large cylinder along the curve FH. When the small piston occupies the position I the steam is cut off in the large cylinder, and the steam in the receiver is then

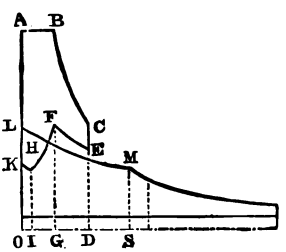

Fig. 203.

compressed by the small piston along the line HK till the end of its stroke.

The initial pressure OL in the large cylinder is of course equal to the receiver pressure GF. From the point L the steam expands in receiver and large cylinder together till the point of cut-off S is reached, and from this point expansion takes place in the large cylinder alone till the end of the stroke.

The values of the terminal pressures in the two cylinders are the same as in the previous example, viz. $\frac{p}{r}$ and $\frac{p}{E}$. At the point of cut-off in the large cylinder, the pressure SM equals the terminal pressure, multiplied by the rate of expansion in the large cylinder ; therefore

$$SM = \frac{p}{E} \times r' = HI$$

Immediately the cut-off is effected the volume of the receiver steam is made up of two parts, viz. the volume of the receiver V_R and the portion OI of the small cylinder which the piston has yet to travel at the moment of cut-off in the large cylinder. Now $OI = (1 - k)\, v$. Therefore the total volume occupied by the receiver steam is $V_R + (1 - k)\, v$. By the time the small piston reaches the end of its stroke this volume is reduced to V_R, and the pressure

$$OK = \frac{pr'}{E} \times \frac{V_R + (1 - k)\, v}{V_R}$$

The small cylinder next exhausts into the receiver, and a volume of steam having the above pressure and volume $= V_R$ becomes mixed with the contents of the small cylinder having a pressure $\frac{p}{r} = \frac{pV}{Ev}$ and volume $= v$.

The resulting pressure

$$DE = \frac{\frac{pr'}{E} \times \frac{V_R + (1 - k)\, v}{V_R} \times V_R + \frac{pV}{Ev} \times v}{V_R + v}$$

$$= \frac{\frac{p r'}{E}(V_R + (1-k)v) + \frac{pV}{E}}{V_R + v} = \frac{p}{E}\frac{r'(\rho + 1 - k) + R.}{\rho + 1}$$

This body of steam is compressed in the receiver by the advancing high-pressure piston till it reaches mid-stroke, when communication is opened with the large cylinder. Therefore, the volume is reduced at this point by half the contents of the small cylinder, and becomes $V_R + \frac{v}{2}$.

While its pressure

$$GF = \frac{p}{E}\frac{r'(V_R + (1-k)v) + V}{V_R + \frac{v}{2}} = \frac{p}{E}\frac{r'(\rho + 1 - k) + R}{\rho + \frac{1}{2}}.$$

This is also the initial pressure OL in the large cylinder.

In order to avoid drop in the receiver when the small cylinder exhausts into it, we should have, as before, to equate the values of DC and DE, and solve for R, which would give us the necessary ratio of the two cylinders for the given rate of expansion.

The value of k, the fraction of the stroke traversed by the piston of the small cylinder when steam is cut off in the large cylinder, is not the same in the two examples given. Let CE be the position of the crank of the large cylinder when steam is cut off in it, in the case of the first example, i.e., after half-stroke. Then the corresponding position of the high-pressure crank is found by drawing FC at right angles to EC; and H and G are the corresponding positions of the two pistons. Then $\frac{AG}{AB} = k$ is the fraction of stroke traversed by the small piston when steam is cut off in the large cylinder.

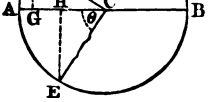

Fig. 204.

Calling the radius of the circle unity, we have—

$$AG = 1 - GC = 1 - \cos FCA = 1 - \sin\theta;$$

$$\text{and } \frac{AG}{AB} = k = \frac{1 - \sin\theta}{2}.$$

Also $\frac{AB}{BH} = r' =$ rate of expansion in large cylinder.

$$\therefore \frac{2}{1 + \cos\theta} = r'.$$

$$\therefore \cos\theta = \frac{2 - r'}{r'}.$$

Deducing the corresponding value of $\sin\theta$ in terms of r', we have—

$$k = \frac{r' - 2\sqrt{r' - 1}}{2r'}.$$

The case when steam is cut off in the low-pressure cylinder before half-stroke is represented by fig. 205, the same letters being used as in the previous example.

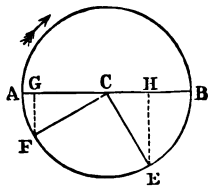

Fig. 205.

In this case it can easily be proved that

$$k = \frac{r' + 2\sqrt{r' - 1}}{2r'}.$$

Space will not permit of a full investigation being given for all the possible arrangements of cylinders, but the principles on which all such calculations proceed, having been fully illustrated in the three examples just given, the student will have no difficulty in applying them to the cases of ordinary compounds with two low-pressure cylinders and cranks at any given angles, or to the case of triple compound engines. In the case of ordinary compounds with two low-pressure cylinders and cranks set at angles of 120° with one another it is only necessary to bear in mind that three separate cases may occur. According as the small cylinder exhausts—

1. Into the receiver only; which it does when the cut-off in the large cylinder takes place before one-quarter stroke

2. Into the receiver and one of the large cylinders; which takes place when the cut-off in each low-pressure cylinder takes place between one-quarter and three-quarters stroke.

3. Into the receiver and both large cylinders; which takes place when the cut-off in the latter is after three-quarters stroke.

The latter case never occurs in practice, because the distribution of the steam would be very bad, as the high-pressure cylinder would discharge into one of the large cylinders when its piston was at one-quarter stroke, and into the other at three-quarters stroke, i.e. just before the cut-off took place. Hence, the amount of work done by each of the two large cylinders would be very unequal

Actual Indicator-diagrams of Compound Engines.—We give below a few specimens of indicator diagrams of various types of compound engines. The first is taken from a tandem, or direct expansion engine. The upper and larger diagram is from the small cylinder, while the lower one is from the low-pressure cylinder. The latter diagram, however, gives no idea to the eye of the relative work done by this cylinder, for it must be borne in mind that, though the pressures shown are low, the area of piston on which they act is, as a rule, from three to four times that of the high-pressure piston. In a subsequent example it will be shown how to combine the diagrams of the several cylinders of a compound engine, so that the work done by each may

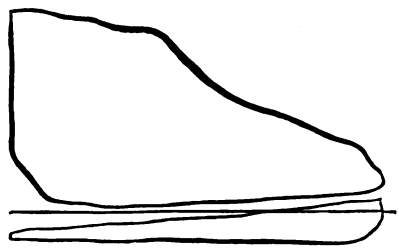

Fig. 206.

be directly compared, and the combined work compared with what would be done by the steam, supposing the whole expansion had taken place in the large cylinder only.

In the tandem type of engine the two cylinders are in direct communication during the whole stroke till compression begins in the small cylinder ; consequently the back pressure line of the top diagram is practically identical during the greater part of its length with the steam line of the lower pressure diagram. The gap between them represents loss of pressure due to the receiver formed by the pipes between the two cylinders and to the resistance of the passages.

Fig. 207 represents a pair of diagrams from a two-cylinder compound engine with receiver, the cylinders having diameters of 46 and 87 inches respectively, and the common stroke of 57 inches, and the ratio of the cylinders 3·6 to 1. The particulars as to the initial and mean pressures and the horse-power developed by each cylinder are given on the diagrams.

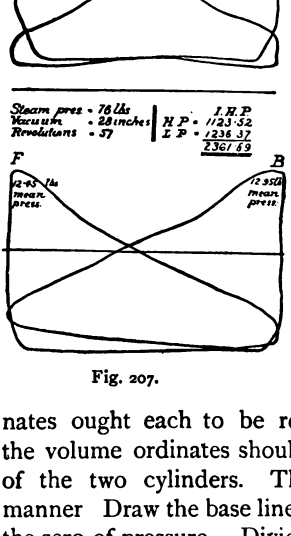

Fig. 207.

It will be noticed that the scales of pressure for each cylinder are quite different, that for the high-pressure being 60 lbs. to the inch, and for the low-pressure 16 lbs. to the inch. In order to make the diagrams comparable, the pressure ordinates ought each to be reduced to the same scale, and the volume ordinates should be in the ratio of the volumes of the two cylinders. This is effected in the following manner Draw the base line AB, fig. 208, corresponding with the zero of pressure. Divide each diagram, fig. 207, by any number of vertical ordinates. Divide the line AB at the

point C into two parts, so that AC : AB :: vol. of small cylinder to vol. of large cylinder :: 1 : 3·6. Divide AC and AB into the same number of parts as there are divisions in the original diagrams, and draw through these points a series of vertical ordinates. Then measuring from the base line AB mark off on these ordinates, to any convenient scale, the pressures as found from the corresponding ordinates on the original diagrams. And through the points thus found draw the new diagrams as shown on fig. 208.

In order to compare the work done by the steam in the compound engine with the work that would be done if the expansion took place all in the large cylinder, we must know first the rate of expansion in the compound engine. This in the present example is as nearly as possible ten. Hence, allowing an admission line DE of one-tenth the stroke, and drawing in the hyperbolic line of expansion EF, we obtain the approximate diagram for this ratio of expansion in a single cylinder, on the assumption that there is no loss from condensation. It will be seen that the two actual diagrams fit fairly well into the single approximate diagram after allowance has been made for the 'drop,' or fall of pressure at the end of the stroke of the high-pressure cylinder, and for the resistance due to passages between the cylinders. Instead of the hyperbola EF, the curve $pv^{1\cdot0646}=C$, which represents the adiabatic curve of expansion of dry saturated steam, is often made use of. Some engineers consider that $pv^{1\cdot2}=C$ represents more closely what takes place.

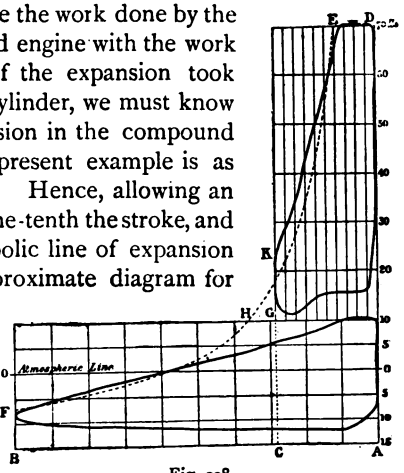

Fig. 208.

Fig. 209 is a set of indicator diagrams taken from a three-cylinder compound engine, having two low-pressure cylinders

and cranks set at equal angles. The particulars as to boiler and receiver pressure, vacuum, number of revolutions, horse-power, &c., are given on the diagrams. The engines from which these diagrams were taken belong to the Trans-atlantic steamer the 'Arizona' illustrated in figs. 197, 198.

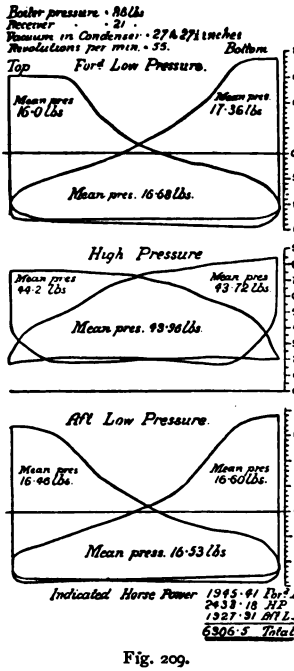

Fig. 209.

Fig. 210 gives the diagrams taken from a set of modern triple expansive marine engines, together with a combination diagram formed in the manner already explained. The dotted line shows the adiabatic expansion of steam with the same ratio of expansion in a single cylinder, and the gaps between the actual diagrams and the adiabatic line show the losses due to 'drop' in the two receivers, to the resistance of passages and to the condensation which takes place in the cylinders.

The mechanical advantages of compound engines.—In addition to diminishing the loss due to condensation in the cylinders, the compound engine possesses mechanical advantages over the older type of engine, in which the expansion takes place completely in a single cylinder. In the latter case there is a great difference between the initial and mean strains on the piston, whenever the rate of expansion is high; whereas in the compound engine the difference between the initial and mean strains in each cylinder is much reduced. Similarly the twisting moment

on the crank shaft is more nearly uniform in the case of the compound engine. Now the dimensions of the moving parts have to be designed to meet the maximum strains; hence

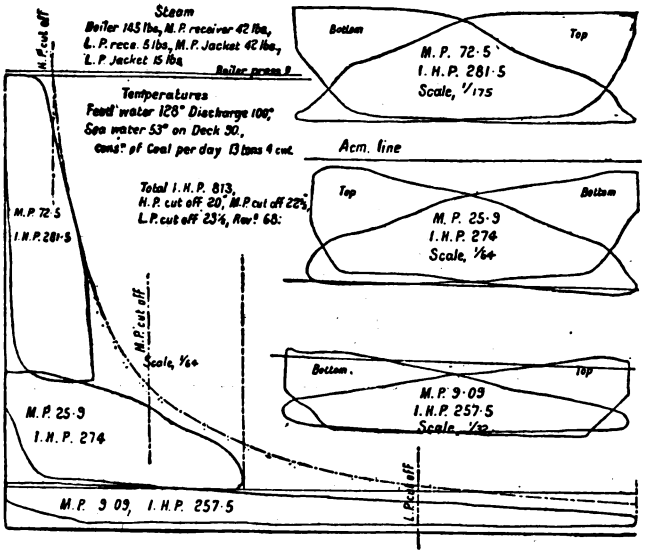

Fig. 210.

with compound engines a saving may be effected in the weight of these parts.

As examples we will compare, first, the case of a single cylinder expansive engine with a tandem compound; and, second, a two-cylinder expansive with a two-crank compound of the receiver type, neglecting clearance and compression and assuming hyperbolic expansion.

Let the initial pressure of the steam be 100 lbs. per square inch, absolute; the ratio of expansion 6, and the area of the piston in the ordinary expansive engine, A. Then, if both engines have the same stroke, the area of the low-pressure cylinder of the compound engine will also be

A, and if the ratio of the two cylinders be, say, 4, the area of the high-pressure cylinder will be $\frac{A}{4}$.

Taking first the simple expansive engine, the theoretical mean pressure for hyperbolic expansion is got by the formula.

$$\text{Mean pressure} = p\frac{1+\log_e E}{E} = 100\,\frac{1+1{\cdot}7918}{6} = 46{\cdot}5 \text{ lbs.}$$

If the back pressure be 4 lbs. absolute per square inch, then the mean effective pressure on piston $= (46{\cdot}5-4)$ A $= 42{\cdot}5$ A lbs., the initial effective pressure on piston $= (100-4)$ A $= 96$ A lbs.

Taking now the case of the compound tandem engine, we have, as in the first case,

The mean effective pressure $= 42{\cdot}5$ A lbs.

The initial pressure on the low-pressure piston equals the terminal pressure on the high, provided there is no loss.

Now the rate of expansion in the high-pressure cylinder $= \frac{6}{4} = 1{\cdot}5$.

Therefore the terminal pressure $= \frac{100}{1{\cdot}5} = 66{\cdot}6$ lbs.

Now the effective initial load on the small piston

$$= (100 - 66{\cdot}6)\frac{A}{4} = 8{\cdot}32 \text{ A.}$$

And the effective initial load on the large piston

$$= (66{\cdot}66 - 4) \text{ A} = 62{\cdot}66 \text{ A.}$$

Total initial load $= 71$ A, as against 96 A in the first case, the mean pressure being the same in each instance.

Take next the comparison between a two-cylinder simple expansive engine and a two-cylinder receiver compound, the area of the low-pressure cylinder of the latter being, as before, denoted by A, while the piston area of each cylinder of the simple engine is $\frac{A}{2}$. The other data are supposed to be unchanged. The mean pressure in the simple engine is the same as in the first example, viz. 46·5 lbs. per square inch.

The mean effective pressure on each piston

$$= (46{\cdot}5-4)\frac{A}{2} = 21{\cdot}25 \text{ A lbs.}$$

The initial effective pressure on each piston

$$= (100-4)\frac{A}{2} = 48 \text{ A lbs.}$$

Next take the case of the compound engine, and suppose the ratio of the cylinders to be 3 to 1. As the rate of expansion is 6, the cut-off in the small cylinder must be at half-stroke.

In an engine of this description the receiver pressure would probably be 28 lbs. Now to find out the point of cut-off in the low-pressure cylinder in order that this may be the receiver pressure at the commencement of the stroke of the large piston, we must remember that the total rate of expansion is 6, and therefore the final pressure in the large cylinder is $\frac{100}{6} = 16\frac{2}{3}$ lbs. Therefore the cut-off in this cylinder is $\frac{16\frac{2}{3}}{28} = {\cdot}59$ of the stroke. From these data we deduce the following particulars :—

Mean pressure in small cylinder

$$= 100\,\frac{1+\log_e 2}{2} = 84{\cdot}65 \text{ lbs. per square inch.}$$

Mean effective pressure in the small cylinder

$$= 84{\cdot}65-28 = 56{\cdot}65 \text{ lbs. per square inch.}$$

Mean effective load on small piston

$$= 56{\cdot}65 \times \frac{A}{3} = 18{\cdot}88 \text{ A lbs.}$$

Initial effective load on small piston

$$= (100-28)\frac{A}{3} = 24 \text{ A lbs.}$$

Mean pressure in large cylinder

$$= 28\,\frac{1+\log_e 1{\cdot}69}{1{\cdot}69} = 25{\cdot}2 \text{ lbs. per square inch.}$$

Mean effective pressure in large cylinder

$$= 25{\cdot}2-4 = 21{\cdot}2 \text{ lbs. per square inch.}$$

Mean effective load on large piston $= 21{\cdot}2 \times A$ lbs.
Initial effective load on large piston $= (28-4)\ A = 24$ lbs.

Hence we see that in the simple engine the initial load is to the mean as 2·16 to 1. Whereas in the small cylinder of the compound engine the ratio is 1·27 to 1, and in the large cylinder 1·13 to 1.

It is unnecessary to pursue this investigation further, for it is evident that in the cases of three-cylinder ordinary compounds, and triple expansive engines the ratios will be more in favour of the compound system than in the two examples given.

The foregoing calculations take no account of the strains caused by the inertia of the reciprocating parts, and they

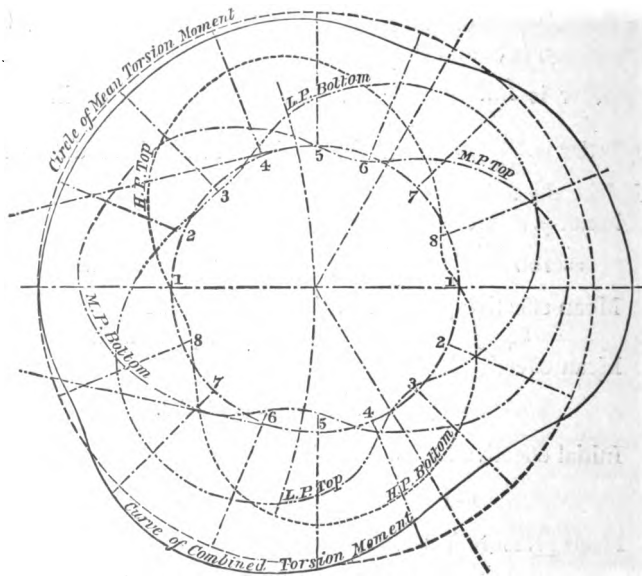

Fig 211.

therefore only apply to the strains when the engine is moving slowly, as, for instance, when stopping and starting. The

proper method of investigating the true pressures on the moving parts, when their inertia and the effects of compression are taken into account, is explained in Chapter V.

The curve of twisting moments on the crank axle is also more uniform in the case of the compound engines, and as the reciprocating parts are comparatively light, the influence exerted by their acceleration and retardation on the curve of moments is less than in the case of simple engines working at the same rate of expansion Fig. 211 is given as an example of the uniformity of twisting moment which may be obtained with a triple expansive engine having three cranks set at 120°, and fig. 212 is the same diagram drawn on a straight instead of a circular base. The indicator diagrams from which the curves were deduced are also given (see fig. 210).

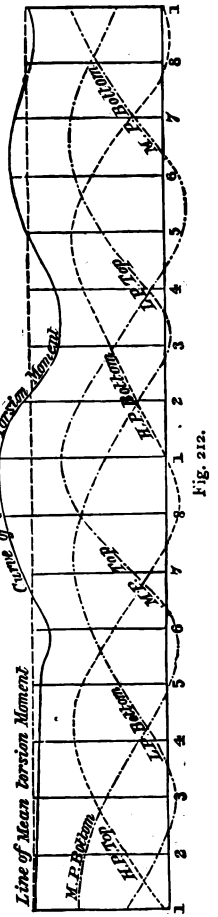

Fig. 212.

The relative sizes of the Cylinders in Compound Engines.—Within certain limits the engineer possesses considerable latitude in the choice of the ratio of cylinders in compound engines. The objects to be kept in view are—1, to divide the power as equally as possible between the cylinders—2, to avoid an excessive rate of expansion in either cylinder, which produces a high ratio of initial to mean pressure—and 3, to avoid excessive drop in the receiver.

It is always possible to equalise the power developed by the two cylinders, by altering the point of cut-off in the large cylinder, no

matter what the ratio may be between the two. The effect of increasing the rate of expansion in the large cylinder is to increase the receiver pressure and consequently the initial pressure, while at the same time it increases the back pressure in the small cylinder. Consequently the proportion of the power developed in the small cylinder is decreased.

As all marine engines, no matter what the initial pressure may be, expand down to about the same terminal pressure, the size of the large cylinder is fixed solely by the power to be developed, for it must be large enough to contain the whole volume of the steam at the terminal pressure. Hence the ratio of the two cylinders is determined by the size we choose to give the small cylinder. If we start with the assumption that the low-pressure cylinder is to exert half of the total power developed, then the mean pressure in it for a given power is always the same, and as the terminal pressure is also constant, the point of cut-off depends upon the pressure in the receiver.

Now the larger the high-pressure piston relatively, the smaller the mean pressure in it per square inch, for a given power. But as the initial pressure is fixed, we must have a more unfavourable ratio of initial to mean pressures with relatively large high-pressure cylinders than with small. Also the larger relatively the high-pressure piston, the smaller must be the terminal pressure in it, and consequently the smaller also the receiver pressure would be, unless it were kept up by an early cut-off in the large cylinder. Thus we see that when the high-pressure cylinder is relatively large, that is, when the ratio of the two cylinders is small, the initial pressure in each must be high, relatively to the mean pressures. While on the other hand the 'drop' in the receiver will be comparatively small.

As the boiler pressure increases, the rate of expansion should increase also; hence, either the cut-off must take place earlier in the small cylinder, or else the ratio of the two cylinders must be increased. If the proportions are

such that the cut-off in the small cylinder must take place before half-stroke, it will be necessary to provide this cylinder with a separate expansion valve.

The following[1] table gives the proportions of cylinders most generally in use so as to avoid the use of expansion valves, and to secure a favourable ratio of initial to mean pressures with a moderate 'drop' in the receiver.

Type of Engine		Boiler-pressure absolute								
		85	95	105	115	125	135	145	155	165
Tandem . .	Ratio of large to small cylinder	4 to 3·5	4	4·5	5	—	—	—	—	—
2 cylr. receiver .	Ditto	3	3·75	4	4·5	—	—	—	—	—
3 cylr. receiver .	Combined ratio of low-pressure to small cylinder		3·4	3·7	4	—	—	—	—	—
Triple expansive	Ratio of low-pressure to high-pressure cylinder	—	—	—	—	5	5·4	5·8	6·2	6·6

In the case of triple expansive engines the ratio of the large low-pressure to the intermediate cylinder should be half that of the low-pressure to the high.

[1] These figures are given on the authority of Mr. A. E. Seaton's *Manual of the Marine Engine.*

APPENDIX.

TABLE I.

THE PROPERTIES OF SATURATED STEAM.

Temperature, Fahrenheit. t.	Pressure in lbs. per square inch at sea level. p.	Heat required to raise 1 lb of Water from 32° to t° Foot lbs h.	Total heat of Evaporation. Foot lbs. H.	Volume of 1 lb. in cubic feet
100	·942			
101	·971			
102	1·001			
103	1·031			
104	1·062	55,612	859,793	312·8
105	1·094			
106	1·127			
107	1·160			
108	1·195			
109	1·230			
110	1·267			
111	1·304			
112	1·342			
113	1·381	62,560	861,908	244·
114	1·421			
115	1·462			
116	1·504			
117	1·547			
118	1·591			
119	1·637			
120	1·683			
121	1·731			
122	1·779	69,522	864,024	192·
123	1·829			
124	1·880			
125	1·932			
126	1·985			

Temperature, Fahrenheit. t.	Pressure in lbs. per square inch at sea level. p	Heat required to raise 1 lb. of Water from 32° to t°. Foot lbs. h.	Total heat of Evaporation. Foot lbs. H.	Volume of 1 lb. in cubic feet
127	2·047			
128	2·096			
129	2·154			
130	2·212			
131	2·273	76,484	866,139	152·4
132	2·334			
133	2·397			
134	2·461			
135	2·526			
136	2·594			
137	2·663			
138	2·733			
139	2·805			
140	2·878	83,459	868,254	122·
141	2·953			
142	3·030			
143	3·108			
144	3·188			
145	3·270			
146	3·354			
147	3·440			
148	3·527			
149	3·616	90,435	870,369	98·45
150	3·707			
151	3·800			
152	3·895			
153	3·992			
154	4·091			
155	4·192			
156	4·295			
157	4·401			
158	4·508	97,411	872,484	80·02
159	4·618			
160	4·730			
161	4·844			
162	4·961			
163	5·08			
164	5·20			
165	5·32			
166	5·45			
167	5·58	104,387	874,600	65·47
168	5·71			
169	5·85			

Temperature, Fahrenheit. t.	Pressure in lbs. per square inch at sea level. p.	Heat required to raise 1 lb. of Water from 32° to t°. Foot lbs. h.	Total heat of Evaporation. Foot lbs. H.	Volume of 1 lb. in cubic feet
170	5·98			
171	6·12			
172	6·26			
173	6·40			
174	6·55			
175	6·70			
176	6·85	111,363	876,715	53·92
177	7·01			
178	7·17			
179	7·34			
180	7·50			
181	7·67			
182	7·84			
183	8·01			
184	8·19			
185	8·37	118,353	878,830	44·70
186	8·56			
187	8·75			
188	8·94			
189	9·13			
190	9·33			
191	9·53			
192	9·74			
193	9·95			
194	10·16	125,357	880,945	37·26
195	10·38			
196	10·60			
197	10·82			
198	11·05			
199	11·29			
200	11·52			
201	11·76			
202	12·01			
203	12·26	132,360	883,060	31·26
204	12·51			
205	12·77			
206	13·03			
207	13·30			
208	13·57			
209	13·84			
210	14·12			
211	14·41			
212	14·70	139,363	885,175	26·36

Temperature, Fahrenheit. t.	Pressure in lbs. per square inch at sea level. p.	Heat required to raise 1 lb. of Water from 32° to t°. Foot lbs. h.	Total heat of Evaporation. Foot lbs. H.	Volume of 1 lb. in cubic feet
213	14·99			
214	15·29			
215	15·60			
216	15·91			
217	16·22			
218	16·54			
219	16·87			
220	17·20			
221	17·53	146,380	887,290	22·34
222	17·87			
223	18·22			
224	18·57			
225	18·93			
226	19·29			
227	19·66			
228	20·03			
229	20·41			
230	20·80	153,412	889,405	19·03
231	21·19			
232	21·59			
233	21·99			
234	22·40			
235	22·82			
236	23·25			
237	23·67			
238	24·11			
239	24·55	160,429	891,520	16·28
240	25·00			
241	25·46			
242	25·92			
243	26·39			
244	26·87			
245	27·35			
246	27·84			
247	28·34			
248	28·85	167,460	893,635	14·00
249	29·36			
250	29·88			
251	30·41			
252	30·94			
253	31·48			
254	32·03			
255	32·59			

Temperature, Fahrenheit. t.	Pressure in lbs. per square inch at sea level. p.	Heat required to raise 1 lb. of Water from 32° to t°. Foot lbs. h.	Total heat of Evaporation. Foot lbs. H.	Volume of 1 lb. in cubic feet
256	33·15			
257	33·73	174,505	895,751	12·09
258	34·31			
259	34·90			
260	35·50			
261	36·11			
262	36·72			
263	37·35			
264	37·98			
265	38·62			
266	39·27	181,564	897,866	10·48
267	39·93			
268	40·60			
269	41·27			
270	41·96			
271	42·65			
272	43·35			
273	44·07			
274	44·79			
275	45·53	188,637	899,981	9·124
276	46·27			
277	47·02			
278	47·78			
279	48·55			
280	49·33			
281	50·13			
282	50·93			
283	51·74			
284	52·56	195,711	902,096	7·973
285	53·39			
286	54·24			
287	55·09			
288	55·96			
289	56·83			
290	57·72			
291	58·62			
292	59·53			
293	60·45	202,798	904,211	6·992
294	61·38			
295	62·33			
296	63·29			
297	64·25			
298	65·23			

Temperature, Fahrenheit. t.	Pressure in lbs. per square inch at sea level. p.	Heat required to raise 1 lb. of Water from 32° to t°. Foot lbs. h.	Total heat of Evaporation. Foot lbs. H.	Volume of 1 lb. in cubic feet
299	66·22			
300	67·22			
301	68·24			
302	69·27	209,885	906,327	6·153
303	70·31			
304	71·36			
305	72·42			
306	73·50			
307	74·59			
308	75·69			
309	76·81			
310	77·94			
311	79·08	216,986	908,442	5·433
312	80·23			
313	81·40			
314	82·59			
315	83·78			
316	84·99			
317	86·21			
318	87·45			
319	88·70			
320	89·97	224,087	910,557	4·816
321	91·25			
322	92·54			
323	93·85			
324	95·17			
325	96·51			
326	97·86			
327	99·23			
328	100·62			
329	102·02	231,216	912,672	4·280
330	103·43			
331	104·86			
332	106·31			
333	107·77			
334	109·25			
335	110·74			
336	112·24			
337	113·76			
338	115·30	238,358	914,787	3·814
339	116·86			
340	118·43			
341	120·02			

Temperature, Fahrenheit. t.	Pressure in lbs. per square inch at sea level. p.	Heat required to raise 1 lb. of Water from 32° to t°. Foot lbs. h.	Total heat of Evaporation. Foot lbs. H.	Volume of 1 lb. in cubic feet
342	121·63			
343	123·26			
344	124·89			
345	126·55			
346	128·23			
347	129·93	245,501	916,902	3·410
348	131·64			
349	133·37			
350	135·11			
351	136·87			
352	138·65			
353	140·45			
354	142·27			
355	144·10			
356	145·95	252,658	919,017	3·057
357	147·82			
358	149·72			
359	151·63			
360	153·56			
361	155·51			
362	157·48			
363	159·46			
364	161·47			
365	163·49	259,829	921,132	2·748
366	165·53			
367	167·60			
368	169·69			
369	171·79			
370	173·92			
371	176·07			
372	178·23			
373	180·42			
374	182·63	267,013	923,247	2·476
375	184·86			
376	187·11			
377	189·38			
378	191·67			
379	193·98			
380	196·32			
381	198·68			
382	201·06			
383	203·46	274,198	925,362	2·236
384	205·88			

Temperature, Fahrenheit. t.	Pressure in lbs. per square inch at sea level. p.	Heat required to raise 1 lb. of Water from 32° to t°. Foot lbs. h.	Total heat of Evaporation. Foot lbs. H.	Volume of 1 lb. in cubic feet
385	208·33			
386	210·79			
387	213·28			
388	215·79			
389	218·32			
390	220·88			
391	223·46			
392	226·07	281,394	927,478	2·025
393	228·70			
394	231·35			
395	234·02			
396	236·72			
397	239·44			
398	242·19			
399	244·96			
400	247·75			
401	250·57	288,634	929,593	1·838

In order to reduce the figures contained in columns 2 and 3 of the above table to thermal units, it is only necessary to divide by 772, the number of foot pounds corresponding to one thermal unit.

To obtain the latent heat for any temperature we have only to subtract the figures in column 3 from the corresponding figures in column 4.

To find the pressure for any temperature intermediate to those given in the table, as for example 310°·25.

Find by the table the difference between the pressures for 310° and 311°. This is 1·14. Multiplying 1·14 by ·25, and adding the result to the pressure corresponding to 310°, we get 77·94 + ·285 = 78·225 lbs. per square inch as the pressure corresponding to 310°·25.

To find the temperature corresponding to 100 lbs. per square inch from the tables we find—

Pressure corresponding to 328° = 100·62 lbs. per square inch.
" " 327° = 99·23
Difference = 1·39

Now, if for 1·39 lbs. difference of pressure the difference of temperature is 1°, what will be the difference of temperature for ·62 lb. difference of pressure?

$$1{\cdot}39 : 1 : {\cdot}62 : x.$$

$$\therefore x = \frac{{\cdot}62}{1{\cdot}39} = {\cdot}446.$$

Then the temperature corresponding to 100 lbs. pressure is $328° - {\cdot}446° = 327°{\cdot}554$.

Similarly to find the value of h. or H. for any temperature intermediate between those for which the values are given in the table. For example, find the total heat of evaporation of steam having the temperature 332°.

Total heat of evaporation corresponding to	338° =	914,787
,, ,, ,,	329° =	912,672
Diff. for	9° =	2,115
∴ Diff. for	1° =	235

Now 332° is 3 degrees above 329°. Therefore the total heat of evaporation for $332° = 912{,}672 + 3 \times 235 = 913{,}377$ foot lbs.

In a similar manner can be solved such a problem as the following. How much heat is required in order to raise the temperature of feed-water from that of the hot well 122° to that of the boiler 365°. The values of h. in this instance are taken from column 3.

This method of interpolation is only permissible because the values of h. and H. vary so slowly with the temperature. The method would not, for instance, be applicable to column 5.

TABLE II.

HYPERBOLIC LOGARITHMS.

The hyperbolic logarithm of a number is found by multiplying the common logarithm of the number by 2·30258.

Example:—The common logarithm of 7 is 0·8450980, which multiplied by 2·30258505 gives 1·9459100, the hyperbolic logarithm.

No.	Logarithm	No.	Logarithm	No.	Logarithm	No.	Logarithm
1·01	·0099503	1·26	·2311116	1·51	·4121095	1·76	·5653138
1·02	·0198026	1·27	·2390169	1·52	·4187103	1·77	·5709795
1·03	.0295588	1·28	·2468601	1·53	·4252676	1·78	.5766133
1·04	·0392207	1·29	·2546422	1·54	·4317825	1·79	·5822156
1·05	·0487902	1·30	·2623643	1·55	·4382549	1·80	·5877866
1·06	·0582690	1·31	·2700271	1·56	·4446858	1·81	·5933268
1·07	·0676586	1·32	·2776317	1·57	·4510756	1·82	·5988365
1·08	·0769610	1·33	·2851788	1·58	·4574249	1·83	·6043159
1·09	·0861777	1·34	·2926696	1·59	·4637339	1·84	·6097655
1·10	·0953102	1·35	·3001046	1·60	·4700036	1·85	·6151856
1·11	·1043600	1·36	·3074847	1·61	·4762341	1·86	·6205764
1·12	·1133286	1·37	·3148108	1·62	·4824261	1·87	·6259384
1·13	·1222175	1·38	·3220835	1·63	·4885801	1·88	·6312717
1·14	·1310284	1·39	·3293037	1·64	·4946961	1·89	·6365768
1·15	·1397618	1·40	·3364722	1·65	·5007752	1·90	·6418538
1·16	·1484200	1·41	·3435897	1·66	·5068176	1·91	·6471033
1·17	·1570038	1·42	·3506568	1·67	·5128237	1·92	·6523251
1·18	·1655144	1·43	·3576744	1·68	·5187938	1·93	·6575200
1·19	·1739534	1·44	·3646431	1·69	·5247285	1·94	·6626879
1·20	·1823215	1·45	·3715635	1·70	·5306282	1·95	·6678294
1·21	·1906204	1·46	·3784365	1·71	·5364933	1·96	·6729445
1·22	·1988508	1·47	·3852623	1·72	·5423242	1·97	·6780335
1·23	·2070141	1·48	·3920420	1·73	·5481214	1·98	·6830968
1·24	·2151113	1·49	·3987762	1·74	·5538850	1·99	·6881346
1·25	·2231435	1·50	·4054652	1·75	·5596157	2·00	·6931472

HYPERBOLIC LOGARITHMS—*continued.*

No.	Logarithm	No.	Logarithm	No.	Logarithm	No.	Logarithm
2·01	·6981347	2·36	·8586616	2·71	·9969486	3·06	1·1184148
2·02	·7030974	2·37	·8628899	2·72	1·0006318	3·07	1·1216775
2·03	·7080357	2·38	·8671004	2·73	1·0043015	3·08	1·1249295
2·04	·7129497	2·39	·8712933	2·74	1·0079579	3·09	1·1281710
2·05	·7178399	2·40	·8754686	2·75	1·0116009	3·10	1·1314021
2·06	·7227059	2·41	·8796266	2·76	1·0152306	3·11	1·1346227
2·07	·7275485	2·42	·8837675	2·77	1·0188473	3·12	1·1378330
2·08	·7323678	2·43	·8878912	2·78	1·0224509	3·13	1·1410330
2·09	·7371640	2·44	·8919980	2·79	1·0260415	3·14	1·1442227
2·10	·7419373	2·45	·8960881	2·80	1·0296193	3·15	1·1474024
2·11	·7466880	2·46	·9001613	2·81	1·0331844	3·16	1·1505720
2·12	·7514160	2·47	·9042181	2·82	1·0367368	3·17	1·1537315
2·13	·7561219	2·48	·9082585	2·83	1·0402766	3·18	1·1568811
2·14	·7608058	2·49	·9122826	2·84	1·0438040	3·19	1·1600209
2·15	·7654679	2·50	·9162907	2·85	1·0473189	3·20	1·1631508
2·16	·7701082	2·51	·9202827	2·86	1·0508215	3·21	1·1662708
2·17	·7747271	2·52	·9242589	2·87	1·0543120	3·22	1·1693813
2·18	·7793248	2·53	·9282193	2·88	1·0577902	3·23	1·1724821
2·19	·7839015	2·54	·9321640	2·89	1·0612564	3·24	1·1755733
2·20	·7884573	2·55	·9360934	2·90	1·0647107	3·25	1·1786549
2·21	·7929925	2·56	·9400072	2·91	1·0681531	3·26	1·1817271
2·22	·7975071	2·57	·9439058	2·92	1·0715836	3·27	1·1847899
2·23	·8020015	2·58	·9477893	2·93	1·0750024	3·28	1·1878434
2·24	·8064758	2·59	·9516578	2·94	1·0784095	3·29	1·1908875
2·25	·8109303	2·60	·9555113	2·95	1·0818051	3·30	1·1939224
2·26	·8153647	2·61	·9593502	2·96	1·0851892	3·31	1·1969481
2·27	·8197798	2·62	·9631743	2·97	1·0885619	3·32	1·1999647
2·28	·8241754	2·63	·9669838	2·98	1·0919233	3·33	1·2029722
2·29	·8285518	2·64	·9707789	2·99	1·0952733	3·34	1·2059707
2·30	·8329090	2·65	·9745596	3·00	1·0986124	3·35	1·2089603
2·31	·8372467	2·66	·9783260	3·01	1·1019400	3·36	1·2119409
2·32	·8415671	2·67	·9820784	3·02	1·1052568	3·37	1·2149127
2·33	·8458682	2·68	·9858167	3·03	1·1085626	3·38	1·2178757
2·34	·8501509	2·69	·9895411	3·04	1·1118575	3·39	1·2208299
2·35	·8544154	2·70	·9932518	3·05	1·1151415	3·40	1·2237754

HYPERBOLIC LOGARITHMS—*continued.*

No	Logarithm	No.	Logarithm	No.	Logarithm	No	Logarithm
3·41	1·2267122	3·76	1·3244189	4·11	1·4134230	4·46	1·4951487
3·42	1·2296405	3·77	1·3270749	4·12	1·4158531	4·47	1·4973883
3·43	1·2325605	3·78	1·3297240	4·13	1·4182774	4·48	1·4996230
3·44	1·2354714	3·79	1·3323660	4·14	1·4206957	4·49	1·5018527
3·45	1·2383742	3·80	1·3350010	4·15	1·4231083	4·50	1·5040773
3·46	1·2412685	3·81	1·3376291	4·16	1·4255150	4·51	1·5062971
3·47	1·2441545	3·82	1·3402504	4·17	1·4279161	4·52	1·5085119
3·48	1·2470322	3·83	1·3428648	4·18	1·4303112	4·53	1·5107219
3·49	1·2499017	3·84	1·3454723	4·19	1·4327007	4·54	1·5129269
3·50	1·2527629	3·85	1·3480731	4·20	1·4350845	4·55	1·5151272
3·51	1·2556160	3·86	1·3506671	4·21	1·4374626	4·56	1·5173226
3·52	1·2584609	3·87	1·3532544	4·22	1·4398351	4·57	1·5195132
3·53	1·2612978	3·88	1·3558351	4·23	1·4422020	4·58	1·5216990
3·54	1·2641266	3·89	1·3584091	4·24	1·4445632	4·59	1·5238800
3·55	1·2669475	3·90	1·3609765	4·25	1·4469189	4·60	1·5260563
3·56	1·2697605	3·91	1·3635373	4·26	1·4492691	4·61	1·5282278
3·57	1·2725655	3·92	1·3660916	4·27	1·4516138	4·62	1·5303947
3·58	1·2753627	3·93	1·3686395	4·28	1·4539530	4·63	1·5325568
3·59	1·2781521	3·94	1·3711807	4·29	1·4562867	4·64	1·5347143
3·60	1·2809338	3·95	1·3737156	4·30	1·4586149	4·65	1·5368672
3·61	1·2837077	3·96	1·3762440	4·31	1·4609379	4·66	1·5390154
3·62	1·2864740	3·97	1·3787661	4·32	1·4632553	4·67	1·5411590
3·63	1·2892326	3·98	1·3812818	4·33	1·4655675	4·68	1·5432981
3·64	1·2919836	3·99	1·3837912	4·34	1·4678743	4·69	1·5454325
3·65	1·2947271	4·00	1·3862943	4·35	1·4701758	4·70	1·5475625
3·66	1·2974631	4·01	1·3887912	4·36	1·4724720	4·71	1·5496879
3·67	1·3001916	4·02	1·3912818	4·37	1·4747630	4·72	1·5518087
3·68	1·3029127	4·03	1·3937763	4·38	1·4770487	4·73	1·5539252
3·69	1·3056264	4·04	1·3962446	4·39	1·4793292	4·74	1·5560371
3·70	1·3083328	4·05	1·3987168	4·40	1·4816045	4·75	1·5581446
3·71	1·3110318	4·06	1·4011829	4·41	1·4838746	4·76	1·5602476
3·72	1·3137236	4·07	1·4036429	4·42	1·4861396	4·77	1·5623462
3·73	1·3164082	4·08	1·4060969	4·43	1·4883995	4·78	1·5644405
3·74	1·3190856	4·09	1·4085449	4·44	1·4906543	4·79	1·5665304
3·75	1·3217559	4·10	1·4109869	4·45	1·4929040	4·80	1·5686159

HYPERBOLIC LOGARITHMS—*continuea.*

No.	Logarithm	No.	Logarithm	No.	Logarithm	No.	Logarithm
4·81	1·5706971	5·16	1·6409365	5·51	1·7065646	5·86	1·7681496
4·82	1·5727739	5·17	1·6428726	5·52	1·7083778	5·87	1·7698546
4·83	1·5748464	5·18	1·6448050	5·53	1·7101878	5·88	1·7715567
4·84	1·5769147	5·19	1·6467336	5·54	1·7119944	5·89	1·7732559
4·85	1·5789787	5·20	1·6486586	5·55	1·7137979	5·90	1·7749523
4·86	1·5810384	5·21	1·6505798	5·56	1·7155981	5·91	1·7768458
4·87	1·5830939	5·22	1·6524974	5·57	1·7173950	5·92	1·7783364
4·88	1·5851452	5·23	1·6544112	5·58	1·7191887	5·93	1·7800242
4·89	1·5871923	5·24	1·6563214	5·59	1·7209792	5·94	1·7817091
4·90	1·5892352	5·25	1·6582280	5·60	1·7227655	5·95	1·7833912
4·91	1·5912739	5·26	1·6601310	5·61	1·7245507	5·96	1·7850704
4·92	1·5933085	5·27	1·6620303	5·62	1·7263316	5·97	1·7867469
4·93	1·5953389	5·28	1·6639260	5·63	1·7281094	5·98	1·7884205
4·94	1·5973653	5·29	1·6658182	5·64	1·7298840	5·99	1·7900914
4·95	1·5993875	5·30	1·6677068	5·65	1·7316555	6·00	1·7917595
4·96	1·6014057	5·31	1·6695918	5·66	1·7334238	6·01	1·7934247
4·97	1·6034198	5·32	1·6714733	5·67	1·7351891	6·02	1·7950872
4·98	1·6054298	5·33	1·6733512	5·68	1·7369512	6·03	1·7967470
4·99	1·6074358	5·34	1·6752256	5·69	1·7387102	6·04	1·7984040
5·00	1·6094379	5·35	1·6770965	5·70	1·7404661	6·05	1·8000582
5·01	1·6114359	5·36	1·6789639	5·71	1·7422189	6·06	1·8017098
5·02	1·6134300	5·37	1·6808278	5·72	1·7439687	6·07	1·8033586
5·03	1·6154200	5·38	1·6826882	5·73	1·7457155	6·08	1·8050047
5·04	1·6174060	5·39	1·6845453	5·74	1·7474591	6·09	1·8066481
5·05	1·6193882	5·40	1·6863989	5·75	1·7491998	6·10	1·8082887
5·06	1·6213664	5·41	1·6882491	5·76	1·7509374	6·11	1·8099267
5·07	1·6233408	5·42	1·6900958	5·77	1·7526720	6·12	1·8115621
5·08	1·6253112	5·43	1·6919391	5·78	1·7544036	6·13	1·8131947
5·09	1·6272778	5·44	1·6937790	5·79	1·7561323	6·14	1·8148247
5·10	1·6292405	5·45	1·6956155	5·80	1·7578579	6·15	1·8164520
5·11	1·6311994	5·46	1·6974487	5·81	1·7595805	6·16	1·8180767
5·12	1·6331544	5·47	1·6992786	5·82	1·7613002	6·17	1·8196988
5·13	1·6351057	5·48	1·7011051	5·83	1·7630170	6·18	1·8213182
5·14	1·6370530	5·49	1·7029282	5·84	1·7647308	6·19	1·8229351
5·15	1·6389967	5·50	1·7047481	5·85	1·7664416	6·20	1·8245493

HYPERBOLIC LOGARITHMS—*continued.*

No.	Logarithm	No.	Logarithm	No.	Logarithm	No.	Logarithm
6·21	1·8261608	6·56	1·8809906	6·91	1·9329696	7·26	1·9823798
6·22	1·8277699	6·57	1·8825138	6·92	1·9344157	7·27	1·9837562
6·23	1·8293763	6·58	1·8840347	6·93	1·9358598	7·28	1·9851308
6·24	1·8309801	6·59	1·8855533	6·94	1·9373017	7·29	1·9865035
6·25	1·8325814	6·60	1·8870697	6·95	1·9387416	7·30	1·9878743
6·26	1·8341801	6·61	1·8885837	6·96	1·9401794	7·31	1·9892432
6·27	1·8357763	6·62	1·8900954	6·97	1·9416152	7·32	1·9906103
6·28	1·8373699	6·63	1·8916048	6·98	1·9430489	7·33	1·9919754
6·29	1·8389610	6·64	1·8931119	6·99	1·9444805	7·34	1·9933387
6·30	1·8405496	6·65	1·8946168	7·00	1·9459100	7·35	1·9947002
6·31	1·8421356	6·66	1·8961194	7·01	1·9473376	7·36	1·9960599
6·32	1·8437191	6·67	1·8976198	7·02	1·9487632	7·37	1·9974177
6·33	1·8453002	6·68	1·8991179	7·03	1·9501866	7·38	1·9987736
6·34	1·8468787	6·69	1·9006138	7·04	1·9516080	7·39	2·0001278
6·35	1·8484547	6·70	1·9021075	7·05	1·9530275	7·40	2·0014800
6·36	1·8500283	6·71	1·9035989	7·06	1·9544449	7·41	2·0028305
6·37	1·8515994	6·72	1·9050881	7·07	1·9558604	7·42	2·0041790
6·38	1·8531680	6·73	1·9065751	7·08	1·9572739	7·43	2·0055258
6·39	1·8547342	6·74	1·9080600	7·09	1·9586853	7·44	2·0068708
6·40	1·8562979	6·75	1·9095425	7·10	1·9600947	7·45	2·0082140
6·41	1·8578592	6·76	1·9110228	7·11	1·9615022	7·46	2·0095553
6·42	1·8594181	6·77	1·9125011	7·12	1·9629077	7·47	2·0108949
6·43	1·8609745	6·78	1·9139771	7·13	1·9643112	7·48	2·0122327
6·44	1·8625285	6·79	1·9154509	7·14	1·9657127	7·49	2·0135687
6·45	1·8640801	6·80	1·9169226	7·15	1·9671123	7·50	2·0149030
6·46	1·8656293	6·81	1·9183921	7·16	1·9685099	7·51	2·0162354
6·47	1·8671761	6·82	1·9198594	7·17	1·9699056	7·52	2·0175661
6·48	1·8687205	6·83	1·9213247	7·18	1·9712993	7·53	2·0188950
6·49	1·8702625	6·84	1·9227877	7·19	1·9726911	7·54	2·0202221
6·50	1·8718021	6·85	1·9242486	7·20	1·9740810	7·55	2·0215475
6·51	1·8733394	6·86	1·9257074	7·21	1·9754689	7·56	2·0228711
6·52	1·8748743	6·87	1·9271641	7·22	1·9768549	7·57	2·0241929
6·53	1·8764069	6·88	1·9286186	7·23	1·9782390	7·58	2·0255131
6·54	1·8779371	6·89	1·9300710	7·24	1·9796212	7·59	2·0268315
6·55	1·8794650	6·90	1·9315214	7·25	1·9810014	7·60	2·0281482

HYPERBOLIC LOGARITHMS—*continued.*

No.	Logarithm	No.	Logarithm	No.	Logarithm	No.	Logarithm
7·61	2·0294631	7·96	2·0744290	8·31	2·1174596	8·66	2·1587147
7·62	2·0307763	7·97	2·0756845	8·32	2·1186622	8·67	2·1598687
7·63	2·0320878	7·98	2·0769384	8·33	2·1198634	8·68	2·1610215
7·64	2·0333976	7·99	2·0781907	8·34	2·1210632	8·69	2·1621729
7·65	2·0347056	8·00	2·0794414	8·35	2·1222615	8·70	2·1633230
7·66	2·0360119	8·01	2·0806907	8·36	2·1234584	8·71	2·1644718
7·67	2·0373166	8·02	2·0819384	8·37	2·1246539	8·72	2·1656192
7·68	2·0386195	8·03	2·0831845	8·38	2·1258479	8·73	2·1667653
7·69	2·0399207	8·04	2·0844290	8·39	2·1270405	8·74	2·1679101
7·70	2·0412203	8·05	2·0856720	8·40	2·1282317	8·75	2·1690536
7·71	2·0425181	8·06	2·0869135	8·41	2·1294214	8·76	2·1701959
7·72	2·0438143	8·07	2·0881534	8·42	2·1306098	8·77	2·1713367
7·73	2·0451088	8·08	2·0893918	8·43	2·1317967	8·78	2·1724763
7·74	2·0464016	8·09	2·0906287	8·44	2·1329822	8·79	2·1736146
7·75	2·0476928	8·10	2·0918640	8·45	2·1341664	8·80	2·1747517
7·76	2·0489823	8·11	2·0930984	8·46	2·1353491	8·81	2·1758874
7·77	2·0502701	8·12	2·0943306	8·47	2·1365304	8·82	2·1770218
7·78	2·0515563	8·13	2·0955613	8·48	2·1377104	8·83	2·1781550
7·79	2·0528408	8·14	2·0967905	8·49	2·1388889	8·84	2·1792868
7·80	2·0541237	8·15	2·0980182	8·50	2·1400661	8·85	2·1804174
7·81	2·0554049	8·16	2·0992444	8·51	2·1412410	8·86	2·1815467
7·82	2·0566845	8·17	2·1004691	8·52	2·1424163	8·87	2·1826747
7·83	2·0579624	8·18	2·1016923	8·53	2·1435893	8·88	2·1838015
7·84	2·0592388	8·19	2·1029140	8·54	2·1447609	8·89	2·1849270
7·85	2·0605135	8·20	2·1041341	8·55	2·1459312	8·90	2·1860512
7·86	2·0617866	8·21	2·1053529	8·56	2·1471001	8·91	2·1871742
7·87	2·0630580	8·22	2·1065702	8·57	2·1482676	8·92	2·1882959
7·88	2·0643278	8·23	2·1077861	8·58	2·1494339	8·93	2·1894163
7·89	2·0655961	8·24	2·1089998	8·59	2·1505987	8·94	2·1905355
7·90	2·0668627	8·25	2·1102128	8·60	2·1517622	8·95	2·1916535
7·91	2·0681277	8·26	2·1114243	8·61	2·1529243	8·96	2·1927702
7·92	2·0693911	8·27	2·1126343	8·62	2·1540851	8·97	2·1938856
7·93	2·0706530	8·28	2·1138428	8·63	2·1552445	8·98	2·1949998
7·94	2·0719132	8·29	2·1150499	8·64	2·1564026	8·99	2·1961128
7·95	2·0731719	8·30	2·1162555	8·65	2·1575593	9·00	2·1972245

EXAMPLES.

1. Give reasons for the supposition that heat is not a substance.

2. Define the meaning of the terms 'work' and 'energy.'

3. Give an account of Davy's reasons for believing that heat is a form of energy.

4. Define a horse-power. What is the distinction between 33,000 foot-pounds and one horse-power?

5. A coal-mine 250 feet deep must, in order to keep the workings dry, have 108,000 gallons of water pumped out of it every hour. What horse-power must the engines exert merely to raise this water without taking any account of the friction of the machinery, &c.

6. State the distinction between potential' and 'kinetic' energy, and give an example of each.

7. Describe any experiment with which you are acquainted which proves that work may be done by the expenditure of heat.

8. State what is meant by 'temperature.' Describe how temperature is commonly measured. How many scales of temperature are there in common use in Europe? A thermometer registers 364° on the Fahrenheit scale: what would be the corresponding numbers on the Centigrade and Réaumur scales?

9. What is meant by the term 'specific heat'?

10. Describe an experiment which proves that the same quantity of heat imparted to equal weights of different substances affects the temperatures of the substances unequally.

11. A pound of cast-iron (specific heat = ·130) is made red-hot and plunged into two gallons of water of the temperature 60°. In quenching the iron the temperature of the water rises 9°: what was the original temperature of the hot iron?

12. What is meant by the mechanical equivalent of heat? What is the equivalent in foot-pounds of the British thermal unit? Work at the rate of a horse-power is expended for an hour in creating friction, the heat generated by which is all communicated to 10 cubic feet of water

contained in a non-conducting tank. The original temperature of the water was 60°: what will be its temperature at the end of the hour?

13. State Boyle's law connecting the pressure and volume of gas. Show how the law may be represented graphically. Prove that the curve which represents the varying pressures of a portion of gas when the volume is changed and the temperature is kept constant is a rectangular hyperbola.

14. A cylinder containing air is fitted with a gas-tight piston by means of which the contained air is compressed to one-fourth of its original volume. Will the final pressure be four times the original pressure immediately after the compression takes place? Give your reasons for your conclusion.

15. State what is meant by an isothermal line of a gas.

16. What is the general effect of raising the temperature of a portion of gas, first, when the volume is kept unchanged, and second, when the pressure is kept constant? State Charles' law, and give the formula which expresses it. A cylinder of 18 inches diameter and of indefinite length contains a cubic foot of air enclosed by a gas-tight piston. The cylinder is plunged into water which is kept at the temperature of 175°: to what height above the bottom of the cylinder will the piston be moved after the inclosed air has attained the temperature of the water?

17. State the distinction between Charles' and Dalton's laws.

18. Describe the air-thermometer, and state what is meant by the term 'absolute temperature.' Show how to deduce the number of degrees which the absolute zero is below the zero of the Fahrenheit scale on the assumption that Charles' law is true.

19. Show how to deduce from Boyle's and Charles' laws a formula connecting the volume, the pressure, and the absolute temperature of a portion of gas.

A pound of air of the temperature 32° and atmospheric pressure is heated up to 100°: what is the product of its pressure and volume at the latter temperature?

20. Establish the ratio between the specific heat of air heated, first, at constant volume, and, second, at constant pressure.

21. What is meant by the term 'latent' as distinguished from 'sensible' heat? When water is turned into steam, state the various ways in which heat is expended.

22. Is Boyle's law applicable to the case of expanding steam? State your reasons for your answer. Make a sketch of the isothermal line of steam of, say, 212°, and explain what the different portions of the line represent.

23. A portion of gas is enclosed in a cylinder under pressure, and is expanded so as to do work. How can you secure that the curve of expansion shall be a rectangular hyperbola?

24. Describe what is meant by a 'cycle of operations.'

25. Give a numerical expression for the quantity of heat required to raise the temperature of air from temperature t_1 to t_2: first, the volume of the air being kept constant; and, second, the pressure being kept constant.

26. A portion of gas is expanded isothermally from volume v_1 to v_2, the initial and final pressures being p_1 and p_2, and the temperature t. State how much heat is expended in doing external work; how much in doing internal work; and how much heat must be supplied to the gas in order that the condition of isothermal expansion may be fulfilled.

27. A portion of gas is expanded from volume v^1 to v^2, the equation of the curve of expansion being $p\,v^n = \text{constant}$. Deduce an expression for the total quantity of heat expended during the operation.

28. When gas expands adiabatically, prove that the equation of the curve of expansion is $p\,v^\gamma = \text{constant}$.

29. What will be the final temperature τ_2 of gas expanded adiabatically, the original temperature being τ_1 and the ratio of expansion r? What will be the total loss of heat by the gas during the expansion?

30. As a numerical application of the above, find the final temperature of air expanded adiabatically to double its volume, the initial temperature being 539° Fahrenheit. Find also the final temperature when the ratios of expansion are 3, 4, and 5. (A table of logarithms will be required for the solution.) The student should notice the great fall in temperature of gas expanding adiabatically as compared with steam, and draw his own conclusions as to the suitability of air as a medium for the working of heat-engines.

31. A cubic foot of air of the pressure 100 pounds per square inch and temperature 539° Fahrenheit is expanded adiabatically till its volume is doubled: construct a table showing the pressures and corresponding temperatures for the volumes 1·1, 1·2, 1·3, 1·4, &c. up to 2.

32. What are the essential conditions of working to realise a theoretically perfect heat-engine? Prove that if the essential conditions are realised the efficiency of the engine is represented by the fraction $\frac{\tau_1 - \tau_2}{\tau_1}$ where τ_1 and τ_2 are the absolute temperatures of the sources of heat and cold respectively.

33. A pound of coal will generate during combustion 14,000 units of heat. Supposing that a theoretically perfect heat-engine consumed one pound of coal per minute and that its limits of working temperature

were $\tau_1 = 2{,}440°$ and $\tau_2 = 60°$, what would be the horse-power developed by the engine?

34. What is the meaning of the terms 'specific volume' and 'relative volume' of steam? How may the relative volume be calculated when the specific volume is given? State Zeuner's law connecting the pressure and volume of dry steam.

35. State an approximate formula for the total heat of steam formed from water of the temperature t, the temperature of the steam when formed being T. Of the above a certain quantity is expended in doing external work: give Zeuner's approximate formula for the heat which thus disappears.

36. If p be the pressure of steam, and p_b be the back pressure in pounds per square inch; also, if H be the total heat of formation of a pound of steam, and v its specific volume: deduce expressions for the heat expended, the external work done, and the heat rejected per cubic foot of the contents of the cylinder in a non-expansive steam-engine.

37. A condensing non-expansive engine uses steam of the pressure of 60 pounds absolute; the back pressure is 2 pounds per square inch. How many pounds of water must be evaporated for this engine per effective indicated horse-power per hour?

N.B. The specific volume of steam of 60 pounds pressure is 7·037 cubic feet. *Ans.* 33·7.

38. Supposing the feed-water in the above engine is taken from the condenser and has the temperature of 100°, what is the ratio of heat expended to work done?

N.B. The total heat of formation of steam of 60 pounds pressure is 1171·3 thermal units.

39. In an engine using steam expansively the pressure during admission is P_1 pounds per square inch, the volume when steam is cut off is V_1, the rate of expansion is r, and the back pressure, which is supposed to be uniform, is P_b. Find an expression for the effective work done, and also for the mean pressure, on the assumption that the expansion takes place in accordance with Boyle's law, and that there is no clearance.

40. In an engine using steam expansively the pressure during admission is $P_1 = 95$ pounds per square inch absolute. The back pressure $P_b = 3$ pounds per square inch. The volume of one pound of steam of pressure p_1 is v_1 cubic feet. The rate of expansion $r = 2$.

Find out the effective work per pound of steam in thermal units; and the total weight of steam supplied to the cylinder per effective indicated horse-power per hour.

The expansion is supposed to be hyperbolic, and clearance neglected and the steam dry at the end of the stroke.

Solution.—Let p be the final and p_m the mean pressure in pounds per square inch. Also, let v_2 = the specific volume of steam of pressure p_2. Then the total work done

$$= 144\, p_2 v_2 (1 + \log_e r)$$

The effective work done equals the foregoing minus the work done in overcoming the back pressure

$$= 144\, p_2 v_2 (1 + \log_e r) - 144\, p_b v_2 = 144\, p_2 v_2 \left(1 + \log_e r - \frac{p_b}{p_2}\right)$$

Now, multiplying v_2, i.e. the specific volume of dry steam of 42·5 pounds per square inch by $144\, p_2$, and reducing to thermal units, we obtain the number 77·7. Hence the effective work done

$$= 77{\cdot}7 \left(1{\cdot}696 - \frac{3}{42{\cdot}5}\right) = 126{\cdot}3 \text{ thermal units per pound of steam.}$$

Now one horse-power per hour $= \frac{33000 \times 60}{772} = 2565$ thermal units per hour

$$\therefore \frac{256}{126{\cdot}3} = 20{\cdot}3 \text{ pounds}$$

equals the weight of steam which must be supplied to the cylinder.

41. Find out what quantity of heat must be added to the steam during expansion in the above example, in order that the condition may be realised that it should be dry, and saturated at the end of the stroke.

Solution.—The symbol P is used throughout to denote the pressure per square foot corresponding with the pressure p per square inch. If steam of pressure p_2 be dry at the end of the stroke, it must have had imparted to it not only the heat of formation H_2 of dry steam of this pressure, but also the equivalent of heat corresponding to the work done over and above whatever work done is included in H_2. Now the work done included in H_2 equals the pressure 144 P_2 multiplied by the corresponding volume v_2 (see page 99). But the total work done equals the mean pressure P_m multiplied by the final volume v_2. Therefore the difference, or $P_m v_2 - P_2 v_2 = (P_m - P_2) v_2$, must be added to H_2. Hence the total heat in the steam at the end of the expansion, provided it be then dry and saturated,

$$= H_2 + (P_m - P_2) v_2.$$

Now the steam when admitted into the cylinder was at the pressure P_1, and its total heat of formation is H_1.

$$\therefore H_2 + (P_m - P_2) v_2 - H_1$$

must be added to the steam during expansion.

Substituting for P_m its value, the above expression becomes

$$P_2 v_2 \log_e r + H_2 - H_1$$

The value of $H_2 - H_1$ can be obtained from Table I.; or, if no table be at hand, its approximate value is ·305 thermal units for every degree of difference of temperature between steam of the pressures p_1 and p_2.

Applying these results to the case in hand, we have

$$P_2 v_2 \log_e r + H_2 - H_1 = 77{\cdot}7 \times {\cdot}696 - {\cdot}305(324^\circ - 271^\circ)$$
$$= 37{\cdot}9 \text{ thermal units per pound of steam used.}$$

But, as we proved in the previous example, 20·3 pounds of steam are used per effective horse-power per hour;

$\therefore$ $20{\cdot}3 \times 37{\cdot}9 = 769{\cdot}37$ thermal units per horse-power per hour must be supplied to the cylinder from a steam-jacket in order to keep the steam dry at the end of the stroke. This would be supplied by the liquefaction in the jacket of about ·7 pound of steam per hour. Hence the theoretical consumption of water in this engine working under the above conditions would be 21 pounds per hour, as against 33·7 in the case of Example 37.

42. What would be the theoretical quantity of water required in the above example if the steam-engine were a perfect engine working between the limits of temperature corresponding to the initial and the back pressures of the steam?

43. State in what respects the action of a steam-engine differs from that of a perfect heat-engine.

44. State Navier's formula for the expansion of steam, giving the numerical constants.

45. Investigate an expression for the work done during expansion, using Navier's formula (instead of, as heretofore, assuming hyperbolic expansion), and taking account of the effect of clearance.

46. State De Pambour's theory of the steam-engine, and reduce it to mathematical form.

47. Analyse the nature of the resistances to the motion of stationary and locomotive engines.

(For numerical examples of the application of De Pambour's theory see pages 136 and 139).

48. Steam of 75 lbs. pressure above the atmosphere is used in a cylinder. It expands down to 4 lbs. absolute at the point of release. What is the ratio of expansion, supposing the clearance to occupy a space equivalent to 5 per cent. of the stroke, and the release to take place at a point 7 per cent. of the stroke, before the end?

49. The stroke of a piston is 3 feet 6 inches, the ratio of expansion is 3·5: at what pressure must the steam be admitted in order that at the release, which is supposed to take place at the end of the stroke, the steam may have expanded down to 5 lbs. absolute, the clearance being equal to 6 per cent. of the stroke?

50. A crane is employed to lift a maximum weight of one ton. The chain is wound round a barrel 2 feet in diameter, to which is made fast a spur wheel 4 feet in diameter, driven by a 9 inch pinion. The pinion is keyed to the crank axle of a two-cylindered engine, the diameter of each cylinder being 6 inches, and the stroke 1 foot. What mean pressure of steam will be required in order to lift the weight, without taking any account of the other resistances to the motion of the piston?

51. A locomotive weighing 32 tons is drawing a train of 150 tons up an incline of 1 in 180, at a speed of fifteen miles an hour. The diameter of each cylinder is 18 inches, the stroke 24 inches, and the diameter of the driving-wheel 6 feet. What is the mean pressure of the steam required in order to overcome the resistance of the engine and train, without taking account of the other resistances to the motion of the piston?

52. How is mass measured, and what units of mass are adopted in practice?

For examples on the application of the laws of motion see pages 148 to 153.

For examples on fly-wheels see page 155.

53. Calling the weight of a body w, v the velocity with which it moves in a circle of radius r, prove that the centrifugal force $F = \frac{w v^2}{g r}$. Also, deduce an expression for the centrifugal force when you are given the number of revolutions per minute (N), instead of the circular velocity.

54. Explain what is meant by the twisting moment on a crank shaft, and show how the variation in the twisting moments during a revolution may be represented graphically by a curve on a straight base.

55. Show by the principle of work that there is no loss of power in converting rectilinear into circular motion by means of a crank and connecting-rod.

The mean pressure in the cylinder is P lbs. per square inch: what is the mean tangential pressure on the pin of a crank of radius $= r$?

56. Supposing the motion of the crank-pin in its circle to be practically uniform, what influence has the fact that the connecting-rod is

finite in length, on the velocity of the piston in each of the four quarters of a revolution?

57. Show how to obtain the twisting moment on the crank graphically when the pressure on the piston is known, and the ratio of length of connecting-rod to length of crank is given, for any position of the crank-arm.

58. Explain the general effect of the inertia of the reciprocating parts in modifying the twisting moments on the crank. Explain the nature of a graphic diagram for exhibiting the pressures absorbed and restored by the reciprocating parts at different parts of the stroke, stating how you would calculate the initial and final pressures, taking no account of the length of the connecting-rod. Also explain how this graphic diagram would be altered, first, in the case of vertical engines by the effect of gravity on the reciprocating masses; and second, in the case of horizontal engines, when the ratio of the length of the connecting-rod to that of the crank is given.

59. Explain in detail the various steps to be taken in order to construct an exact curve of twisting moments, when you are provided with a pair of indicator diagrams, and the necessary data concerning weights and dimensions.

60. Explain the methods by which in practice uniformity of twisting moment is approximated to.

61. What are the essential data necessary in order to determine the weight of the fly wheel of an engine?

62. Explain the objects of cushioning the exhaust steam.

63. What area of passage would you give to a steam port for a cylinder of 24 inches diameter, 36 inches stroke, the engine making 45 revolutions per minute?

64. What are the advantages of making the working barrel of a cylinder of a separate detachable piece, called a liner?

65. Under what circumstances would you fit a steam-jacket to a cylinder, and what precautions would you adopt in designing the jacket?

66. Make a sketch of the general arrangements of the cylinder of a locomotive engine, showing a section through the cylinder and valve box.

67. Give a description, with sketch, of the piston-packings of a marine engine.

68. A locomotive piston of 18 inches diameter is provided with three half-inch packing-rings, so adjusted as to exert a pressure on the piston sides of 3 lbs. per square inch. The stroke is 24 inches, and the diameter of the driving-wheels 6 ft. 6 in. What horse-power is exerted in overcoming the friction of the pistons when the engine is

running at the speed of 48 miles an hour, the co-efficient of friction between the rings and sides of the cylinder being taken as ·085?

69. Prove that so long as an engine runs in one direction pressure is only exerted upon one of the slide bars.

70. In designing motion blocks and slide bars, what should be the maximum pressure per square inch of bar surface allowed for?

71. Given the steam pressure, the diameter of cylinder, and the ratio of length of crank to connecting-rod, what is the maximum pressure on the slide bar?

72. What are the principal disadvantages of making the connecting-rod short relatively to the crank arm?

73. The diameter of a cylinder is 30 inches, the steam pressure 40 lbs. per square inch. What is the maximum strain in the connecting-rod when the latter is 4 times the length of the crank?

74. Make sketches of the big ends of connecting-rods (1) when the brass steps are held in place by a strap, and (2) when the end is solid. What effect on the length of the rod is produced driving in the cotter in each of these cases?

75. You are required to drive a slide valve having a travel of four inches from a main shaft of the diameter of seven inches: make a sketch of the method you would adopt, giving dimensions.

76. Investigate the moment of resistance of a hollow shaft, the exterior diameter of which is R and the interior r, and prove that with a given weight of metal you can turn out a stronger shaft by making it hollow rather than solid.

77. The indicated horse-power of an engine is 500, the number of revolutions 50 per minute. What should be the diameter of the crank shaft, the metal being steel having a shearing strength of 80,000 lbs. per square inch, and the factor of safety being 6?

78. Explain the action of Watt's governor and prove that the speed of revolution of the engine is inversely proportional to the square root of the height of the cone of revolution. What is the object of crossing the arms of a governor?

79. Explain the nature of the objections to extreme sensitiveness in a governor.

80. Make a sketch showing how the rate of expansion of an engine may be controlled by the governor.

81. A fly-wheel has a mean radius of 10 feet, and weighs 10 tons, the whole of which is supposed to be concentrated at the mean radius. The section of the rim is 160 square inches. What is the maximum safe speed the wheel can be run at, on the assumption that the tensile

strength of cast-iron is 15,000 lbs. per square inch, and that the factor of safety is 5? Also at what speed would the wheel burst asunder?

82. Sketch a D slide valve, and explain the action of outside and inside lap on the valve.

83. What is the meaning of the term 'lead'? Given a slide valve with a travel of $2\frac{1}{2}$ inches, outside lap of $\frac{1}{2}$ inch and lead of $\frac{1}{8}$ inch, what will be the throw and the angle of advance of the eccentric?

84. Make a sketch of an arrangement for reversing an engine fitted with a slide valve.

85. Make a sketch of Meyer's valve gear, and state under what circumstances it is desirable to fit a separate expansion valve to an ordinary slide valve. State clearly all the functions of the main and the expansion valves.

86. What are the disadvantages of slide valves? and make a sketch showing how these disadvantages are obviated in the Corliss engine. What is the object of the separate exhaust valves in the latter engine?

87. Make a sketch of the piston valve of a marine engine, and state what its advantages are.

88. Explain how the slide valves of marine engines are usually relieved of a portion of the pressure on their backs.

89. The back of the slide valve of a locomotive exposes 180 square inches of area. The pressure in the valve box is 140 pounds per square inch, which is partly balanced by the pressures acting on the under surfaces of the valve so that the average net force pressing the valve down is 130 pounds. The coefficient of friction has been experimentally proved to be ·22. The travel of the valve is 4 inches. All the other data of the engine are the same as in Example 68. What is the horse-power absorbed when the engine is running at 48 miles an hour in overcoming the friction of the valves?

90. Make a sketch of Joy's valve gear, and explain some of the advantages which it possesses over the ordinary eccentric gear.

91. Explain the principle of Zeuner's valve diagrams, and show, choosing any dimensions of valves &c. you like, how the diagram may be made to indicate the positions of the piston at which the steam admission, the cut-off, the release, the compression, take place.

92. Prove that with an eccentric valve motion the point at which compression commences must vary with the rate of expansion.

93. The travel of a slide valve is 8 inches, the outside lap 2 inches, the inside lap $\frac{1}{4}$ inch, the angle of advance 40°. Construct a Zeuner's diagram showing the positions of the crank when the admission takes place, the steam is cut off and released, the exhaust closed, and also the amount of the lead.

94. The travel of a slide valve is 4 inches, the angle of advance is 40°, the ratio of expansion is 1·25, the steam is released when the piston has still 3 per cent of the stroke to travel. Find the outside and inside lap, the lead, and the position of the crank when the steam is admitted and the exhaust closed. The ratio of length of connecting-rod to crank is to be neglected.

95. In an engine with a 3-feet stroke the length of the connecting-rod is 6 feet, the steam is cut off when the crank is at angles of 60° from the line of dead centres: what is the ratio of expansion in the forward and in the back stroke?

Numerous other problems in simple valve setting and designing will be found on pages 281 to 290.

96. In a Stephenson's link motion with open arms, you are required to fix the positions of the notches in the reversing lever quadrant by a geometrical construction, so that steam may be cut off when the crank makes angles of 45°, 60°, 90°, 120° and 135°, with the dead centres, choosing any dimensions you think proper for the various parts of the valves and gear.

97. What is the general effect on the lead and the point of compression of increasing the rate of expansion in Stephenson's link motion, (1) when the arms are open, (2) when the arms are crossed? Illustrate your answer by means of Zeuner's diagrams, the angles of advance of the virtual eccentrics corresponding with the various rates of expansion being found by geometrical method. Choose any convenient dimensions for the gear.

98. Show that when Zeuner's diagram is applied to the elucidation of Meyer's valve gear, a resultant circle can always be found the chords of which represent the distances apart of the centres of the two valves.

99. Is Meyer's valve gear suitable for use with engines which have to be reversed frequently? Illustrate your reply by means of a Zeuner's diagram, and state the best position for the eccentric of the expansion valve so as to secure the most uniform steam distribution for running in both directions.

100. Describe Richards' indicator with the help of illustrative sketches.

101. What points connected with the working of steam engines are revealed by indicator diagrams?

102. Being given the diagram of an expansive engine, state how you would estimate the mean pressure. What data in addition to the diagram would you require before you could calculate the power exerted by the engine?

103. Make a sketch of the theoretical diagram of a condensing engine, and show what modifications in the outline of the latter are to be expected in practice.

104. What effect has clearance on the shape of the diagram?

105. What are the leading characteristics of the diagrams of locomotive engines working at high rates of expansion?

106. Do the pressures recorded by indicator diagrams give the actual forces urging the piston? Give your reasons for your reply.

107. Why is it that when every care is taken in the valve-setting, the diagrams from the two ends of a cylinder often differ considerably in area and shape? Under what circumstances may this peculiarity be turned to account?

108. Explain exactly how you would draw the combined diagram of the two cylinders of a compound engine when you are provided with a diagram from each cylinder.

109. What are the principal causes which affect the back-pressure line of the diagram?

110. What is the meaning of the terms 'gross' and 'net' indicated power?

111. Are high rates of expansion economical in non-condensing engines? Give your reasons for your reply.

112. State how to ascertain if the valves and piston of an engine are steam-tight.

113. Explain how to measure the expenditure of steam accounted for by the diagram.

114. How many units of heat are obtained by the combustion of one pound of carbon with sufficient air to form, (1) carbonic oxide, (2) carbonic anhydride?

115. What is the minimum weight of air necessary to effect the complete combustion of one pound of carbon, and what should be the temperature of the products of combustion?

116. When a chemical combination of carbon and hydrogen is burnt in oxygen how would you estimate the heat of combustion?

117. What are the principal constituents of fuel?

118. A firegrate is 4 feet 6 inches long, and 3 feet wide; twenty pounds of coal are burnt on it per square foot of area per hour, with a supply of 24 lbs. of air per lb. of fuel. What is the temperature, and what the volume in an hour of the products of combustion, (1) as formed in the furnace; and (2) in the chimney, supposing the latter to be maintained at the temperature most suitable for draught-creation?

119. State the principal causes of the waste of fuel in boilers.

120. Describe a modern Lancashire boiler, and give illustrative sketches.

121. What are the principal ends gained by the use of Galloway tubes?

122. What precautions would you observe in placing the gusset stays in the flat ends of Lancashire boilers?

123. What are the principal peculiarities of locomotive boilers?

124. The firegrate area of a locomotive is 20·5 square feet. It is intended to burn on the average 50 lbs. of fuel per square foot of grate-surface per hour. How much heating-surface would you provide?

125. A modern marine high-pressure boiler has to supply steam to an engine indicating 560 horse-power, and which consumes 18 pounds of water per I.H.P. per hour. How much grate-area would you think it necessary to provide, and how much heating-surface, ordinary draught being used?

126. Give sketches showing the general arrangements and the approximate dimensions of the boiler which you would provide for the above purpose, the pressure being 90 lbs. absolute.

127. If the shell-plates were made of steel, what thickness would you employ, having reference to the Board of Trade rules?

128. What area of opening of safety-valves would you allow?

129. Why are the ends of tubes furthest from the furnace or combustion chamber of comparatively little use in absorbing heat when a boiler is new? and why are they likely to be more useful after the boiler has been worked for a time?

130. An engine is required to give out very varying quantities of power during the course of every hour. Would you provide for it a boiler of comparatively large or of comparatively small cubic contents? State your reasons.

131. Investigate the strength of a hollow cylinder with flat ends pressed from within, and prove that the strain in the plane of the axis is double that in a plane at right angles to it. Does the shape of the ends affect the strain transmitted by them to the boiler body?

132. How are internal furnaces and flues constructed so as to allow for expansion and contraction, and to provide against collapse?

133. On what does the strength of a hollow cylinder to resist collapse principally depend? Why are hollow cylinders pressed from without more liable to destruction than the same cylinders pressed with equal force from within?

134. State what you consider to be the principal advantages of Fox's corrugated flues.

135. Explain the action of, and illustrate by sketches the Bourdon pressure gauge.

136. Describe any of the structural arrangements with which you are acquainted for attaching the ends to the body of a boiler, and for strengthening them; also for attaching the furnace tube to the ends. State fully what precautions must be adopted in strengthening the flat ends.

137. A cylindrical land boiler is 7 feet 6 inches in diameter, and has to sustain a pressure of steam of 90 lbs. by the gauge. What thickness of mild steel shell-plates would you adopt for the shell?

138. The effect of punching rivet-holes is to compress a thin layer of the metal all round the hole, and to greatly increase its tensile strength, and, at the same time, to diminish its stretching power. When the holes are drilled the metal remains in its normal condition. What conclusions would you draw from these facts as to the relative strength of punched and riveted joints, and state your reasons?

139. Make sketches, including sections, of single and double riveted lap-joints.

140. State the methods in which a single riveted lap-joint may give way when subjected to tensile strain.

Ans. 1. The plate may tear asunder where its area is reduced, between the rivet-holes. 2. The rivets may shear asunder. 3. The metal between the rivet-holes and the edge of the plate may be crushed. 4. The plate may break across in front of the rivet-holes and at right angles to the edge. (N.B. The two latter causes of fracture may be provided against by giving the plates a sufficient depth of lap. As a rule, the portion of the plates which overlap should be not less than three times the diameter of the rivet-hole.)

141. Show how to proportion a single-riveted lap-joint so that the resistance of the plates to tearing may just equal the resistance of the rivets to shearing.

Ans. Let d = diameter of rivet in inches, t = thickness of plates, $\frac{\pi d^2}{4}$ = area of a rivet, p = pitch of rivets, i.e. distance apart from centre to centre, S = shearing strength of rivets per square inch, T = tensile strength of plates per square inch. Then area of section of plate between any two holes, multiplied by tensile strength of plate, must equal area of one rivet multiplied by the shearing strength of the material.

$$\therefore (p-d)t\mathrm{T} = \frac{\pi d^2}{4}\mathrm{S}.$$

As a general rule, the tensile and shearing strengths are equal for iron plates and iron rivets.

$$\therefore (p-d)t=\frac{\pi d^2}{4} \qquad \therefore p=\frac{\pi d^2}{4t}+d=\cdot 785\frac{d^2}{t}+d.$$

Which formula gives the pitch in terms of the thickness of plates and the diameter of the rivets.

142. In a single-riveted joint the plates are $\frac{3}{4}$ inch thick, the rivets are $1\frac{1}{8}$ inch diameter. What should be the pitch on the suppositions (1) that the shearing and tensile strengths are equal, and (2) that the safe tensile strength is 25 per cent. greater than the safe shearing strength?

143. What must be the diameter of the rivet in a single-riveted lap-joint so that the resistance of the rivet to crushing and shearing may be equal?

Ans. The resistance of the rivet to crushing equals its diameter multiplied by the thickness of the plate, multiplied by the resistance of the metal to crushing per square inch. The resistance of iron rivets in iron plates to crushing is double the resistance to shearing. The resistance of the rivet to shearing equals its area multiplied by the shearing strength of the metal per square inch.

$$\therefore td\,2\mathrm{S}=\frac{\pi d^2}{4}\mathrm{S} \qquad \therefore d=\frac{8t}{\pi}=2\cdot 55\,t.$$

The diameter thus obtained is, however, far larger than is admissible in practice.

N.B. A practical rule for the diameter of the rivet in terms of the thickness of the plate is

$$d=1\cdot 2\sqrt{t}.$$

If the diameters of rivets progress by 16ths of an inch, then for single-riveted joints we may take the number of 16ths next above the diameter, as given by the formula, and for double-riveted joints the number of 16ths next below.

144. In a double-riveted lap-joint, find the pitch of the rivets so that the shearing and tensile strength of the joints may be equal, for iron plates and rivets.

Ans. Referring to *Ex.* 141, when the joint is double-riveted, we have the area of two rivets to shear instead of one.

$$\therefore (p-d)\,t=\frac{\pi d^2}{2} \qquad \therefore p=1\cdot 57\frac{d^2}{t}+d.$$

N.B. By making use of this equation $d=1\cdot 2\sqrt{t}$, and substituting this value of t in the above equation, we can obtain an expression for the pitch in terms of the thickness of the plate alone. When the pitch

and diameter of the rivets are known the strength of the plate between the rivet-holes can be ascertained, and the strength of the joint compared with that of the whole plate can be calculated.

145. Make sketches, including sections of single-and double-riveted butt-joints, in both single and double shear.

146. Is there any difference between the strength of a single-riveted lap-joint and the corresponding butt-joint in single shear?

147. Show how to proportion a double-riveted butt-joint in double shear, so that the tensile strength of the plate between the rivet-holes may be equal to the shearing strength of the rivets (iron plates and rivets).

Ans. In this case (referring to *Ex.* 1) we have four rivet areas to shear instead of one.

$$\therefore (p-d)\,t = \pi d^2 \text{ and } p = \frac{\pi d^2}{t} + d.$$

N.B. In practice it is not found safe to calculate on the area of four rivets, and by the Board of Trade rules only $3\frac{1}{2}$ are allowed. Hence, taking this into consideration, and making use of the formula $d = 1{\cdot}2\sqrt{t}$, we get

$$p = 3{\cdot}95 + d.$$

For plates of less than $\frac{3}{4}$ inch thickness, the results got by the above formula are greater than those adopted in practice, because the diameter of the rivets is generally less than what would be indicated by theory.

148. Enumerate the fittings required for a Lancashire boiler.

149. Describe the principle on which the injector works, and make a sectional sketch of an injector, showing how the steam and water supply can be adjusted.

What are the objections to creating a draught in the ordinary way by means of a funnel or chimney?

150. Make a sketch illustrating the ordinary weighted lever safety-valve. The diameter of opening of a safety-valve is 4 inches; the distance from the fulcrum to the centre of the valve is 5 inches; the lever is 21 inches long, weighs $3\frac{1}{2}$ lbs., and its centre of gravity is 8 inches from the fulcrum; the valve weighs 5 lbs. What weight must be hung one inch from the end of the lever so that steam may blow off at 50 lbs. absolute per square inch?

151. Explain the reasons which led to the abandonment of jet condensers for marine engines, and show how surface condensation has rendered possible the use of high-pressure steam in marine boilers.

152. Describe fully, and illustrate with sketches, a modern surface condenser for a marine engine, together with its air and circulating pumps.

153. A marine engine indicating 4,000 H.P. uses 17 pounds of steam per I.H.P. per hour, and expands down to 5 pounds absolute: what surface would you provide in the condenser; how much cooling water of initial temperature of 60° would be required per hour; and what should be the capacity of the air-pumps (single-acting), the engine running at 56 revolutions per minute?

154. Why must the capacity of the air-pump of a surface condenser be so largely in excess of the volume of the water into which the steam condenses per stroke?

155. How is the condition of the vacuum in a condenser recorded? and what is the relation between the vacuum as recorded and the absolute pressure in pounds per square inch?

156. What precautions must be adopted in making the tubes of surface condensers, so as to avoid the possibility of carrying over corrosive metallic salts into the boiler?

157. Make a sketch of any of the packings for condenser tubes in common use.

158. Describe a jet condenser for a high-speed stationary engine, and state how the use of a separate air-pump can be avoided.

159. An engine indicating 60 H.P. uses 20 pounds of steam per I.H.P. per hour, and expands down to 6 pounds absolute: how much injection water will be required per hour, the original temperature of which is 60°, the temperature of the hot well being 106°?

160. Make a sketch, and give a description, of an ejector condenser for a two-cylinder engine.

161. The metal of a cylinder is capable of rapidly receiving and transmitting heat: explain why this property is the cause of serious loss of efficiency in the expansive steam engine. How does it come to pass that the use of the steam-jacket greatly diminishes this loss of efficiency in spite of the fact that, to keep the cylinder always hot, steam is constantly being condensed in the jacket, the greater part of the heat thus liberated being uselessly transmitted to the exhaust.

162. What is the object of superheating steam? Is it possible to obtain any practical good effect in modern high-pressure engines by superheating the steam? Give your reasons for your reply.

163. Examine the effect of the dimensions of the cylinder, the initial pressure of the steam, and the rate of expansion on the initial condensation which takes place in the cylinder.

164. What are the four principal causes for the presence of water in the cylinders of steam engines?

165. What is the object of compounding or expanding the steam successively in two or more cylinders instead of in one? Why is it

that in modern marine engineering triple and quadruple expansive engines are now so largely used?

166. Illustrate by outline diagrams the principal methods of arranging the cylinders in ordinary and triple expansive engines.

167. A simple and a compound engine work at the same rate of expansion, and develope the same power. What is the size of the low-pressure cylinder of the compound compared with the cylinder of the simple engine? Give your reasons for your reply.

168. State the mechanical advantages of compound over simple expansive engines, and investigate the ratio of maximum to mean pressures, in (1) a pair of simple expansive engines, and (2) a compound engine of the same power, and working with the same initial pressure and the same ratio of expansion, on the supposition that the steam expands hyperbolically and that the effects of early release, compression, and clearance are neglected. The initial pressure is 115 lbs. absolute. The ratio of expansion = 12, the area of each of the high-pressure cylinders = A. The ratio of area of low to area of high-pressure cylinder in the compound engine = 4·5. The received pressure 24 lbs. per square inch.

169. Explain how in practice the powers developed in the two cylinders of a compound engine may be made approximately equal.

170. Make a sketch diagram illustrating the distribution of the steam in both cylinders of an ordinary two-cylinder receiver compound engine, choosing any symbols you like to represent the governing data. The steam in the large cylinder is supposed to be cut off before half stroke, and the expansion to take place hyperbolically. The effects of clearance, early release, and compression are to be neglected.

171. A triple expansive engine works at a consumption of 1·3 lb. of coal per I.H.P. per hour. The boilers evaporate 8 pounds of water per hour per pound of coal from the temperature of the feed 105°, and at the temperature of the initial pressure of the steam, viz. 165 lbs. above the atmosphere. (Corresponding temperature 366°.) What is the consumption of steam per horse-power per hour? What would it be if the engines were theoretically perfect and working between the above limits of temperature? What is the efficiency of the engine compared to a perfect engine? What is its absolute efficiency? What is the efficiency of the boiler compared to that of a perfect boiler which cools the products of combustion down to the temperature of the feed, the heat of combustion of one pound of coal being put down as 14,000 thermal units? Finally, what proportion of the total heat-supply is wasted by the boiler and what by the engine?

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